$$p(u - v) \leqq \delta^p + \gamma[\eta(C)] + \int_{c(x)}^{C} \eta(s)ds.$$

The proof follows by setting $\mu'(s) = \eta(s)$, $y = m - \mu[c(x)]$, where m is a constant so chosen that the function $p(u-v) - y$ does not assume a positive maximum on ∂B.

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COMPLETE LOCALLY AFFINE SPACES AND ALGEBRAIC HULLS OF MATRIX GROUPS

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Let M be a complete Riemann manifold with curvature and torsion zero. If $\pi_1(M)$ denotes the fundamental group of M, then Bieberbach [3; 4] proved that $\pi_1(M)$ contains an abelian normal subgroup of finite index. Moreover, if M is compact then M is covered by a torus.

In recent years the study of general affine connections has led to the study of the following problem: How can one classify the manifolds which possess a complete affine connection with curvature and torsian zero? Such manifolds will be called complete locally affine spaces.

It was Zassenhaus [6] who first gave a general setting to the Bieberbach theorem. He showed a special case of the following theorem:

THEOREM 1. *Let G be a connected Lie group with its radical R simply connected, $\rho: G \rightarrow G/R$ the projection, and L a closed subgroup of G. If the identity component L_0 of L is solvable, then the identity component of the closure of $\pi_1(L)$ is solvable.*

This theorem in this generality is due to H. C. Wang [5] and his

[1] With partial support from the N. S. F.

proof is a modification of a proof given for this theorem by the author [1] when L was required to be discrete.

Now Bieberbach's theorem follows trivially from Theorem 1. However, Theorem 1 is still not strong enough to treat the general case of complete locally affine manifolds. In order to treat this general problem, we will need the following theorem.

THEOREM 2. *Let M be a complete locally affine manifold with fundamental group $\pi_1(M)$. Assume further that the holonomy group is discrete and isomorphic to $\pi_1(M)$. Then $\pi_1(M)$ is abelian.*

Using the above two theorems one can show the following:

THEOREM 3. *Let M be a complete locally affine manifold with fundamental group $\pi_1(M)$. Then $\pi_1(M)$ contains a normal solvable subgroup of finite index. Further if $\pi_1(M)$ is compact, M is finitely covered by a compact solvmanifold.*

COROLLARY. *Every complete locally affine manifold (compact or not) has Euler characteristic zero.*

ADDED IN PROOF

THEOREM 4. *Let M_i, $i=1, 2$, be a compact complete locally affine manifold with fundamental group $\pi_1(M_i)$. Assume G_i is the algebraic hull of $\pi_1(M_i)$ in the group of affine transformations. Then if σ is an isomorphism of $\pi_1(M_1)$ onto $\pi_1(M_2)$ then σ can be extended uniquely to an isomorphism of G_1 onto G_2.*

COROLLARY. *If M_1 and M_2 are compact complete locally affine manifolds with isomorphic fundamental groups, then M_1 is homeomorphic to M_2.*

The material announced in this paper is obviously much stronger than the results already announced in [2]. A detailed account of these results will be published elsewhere.

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