RESEARCH PROBLEMS

39. Joseph Hammer: *Matrix Theory.*

Given a set of n^2 numbers (not necessarily all different), find a method to decide whether it is possible to arrange them in an $n \times n$ matrix so that the value of its determinant will be a preassigned value.

40. Richard Bellman: *Algebra.*

A great deal has been done on the composition of algebraic forms; (see C. C. MacDuffee, *On the composition of algebraic forms of higher degree*, Bull. Amer. Math. Soc. **51** (1945), 198–211).

What is the situation for matrices whose elements are algebraic forms? If $Q(x) = (q_{ij}(x))$, $i, j = 1, 2, \cdots, R$, $x = (x_1, x_2, \cdots, x_N)$, when does a relation of the form $Q(x)Q(y) = Q(z)$ hold, where, as usual, $z_i = \sum_{j,k} a_{ijk} x_j y_k$ and the a_{ijk} are independent of x, y, z?

When can a given quadratic form $q(x) = (x, Ax)$ be imbedded in a composition matrix so that $q(x) = q_{11}(x)$?

For $R = 2$, we have relations of the form

$$q_{11}(x)q_{11}(y) + q_{12}(x)q_{21}(y) = q_{11}(z),$$

which means $q_{11}(x)q_{11}(y) = q_{11}(z)$ for all x such that $q_{12}(x) = 0$ or for all y such that $q_{21}(y) = 0$, a *relative composition*. When do relative compositions hold?

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