

projection from one plane to another [Yaglom 2, p. 17]. I did not check too many among these references, but I noticed a few others of this kind: p. 78. If $OP > k/2$ (O the center of the inversion, k the radius of the invariant circle) the inverse of P can easily be constructed by the use of compasses only, without a ruler [Forder 1, p. 222]. The proof is given. It is not mentioned that the theorem is true without the restriction on OP . p. 92: A sphere with center N and radius NS inverts the plane σ (tangent in S) into a sphere σ' on NS as diameter [Johnson 1, p. 108].

H. FREUDENTHAL

Neuere Methoden und Ergebnisse der Ergodentheorie. By Konrad Jacobs. *Ergebnisse der Mathematik und ihrer Grenzgebiete, neue Folge, Heft 29.* Springer Verlag, Berlin, 1960. 6+214 pp. DM 49.80.

This is a concise and elegant introduction to some of the new methods and results of ergodic theory. Its table of contents (translated and annotated) runs as follows.

Introduction. (Motivation, basic definitions, and a bird's eye view of the entire subject; written for the non-expert.) 1, *Functional-analytic ergodic theory.* (Mean ergodic theorem, first for unitary operators on Hilbert space, ultimately for semigroups on Banach spaces; emphasis on almost periodicity; norm convergence for martingales.) 2, *Markov processes.* (The work of Doebelin; heavy use of such modern methods as the Riesz convexity theorem and the Krein-Milman theorem.) 3, *The individual ergodic theorem.* (Birkhoff's theorem, the Dunford-Schwartz generalization; the Hurewicz theorem; the almost everywhere martingale theorem.) 4, *Global properties of flows.* (Recurrence, ergodicity and mixing; decomposition into ergodic parts; flows under a function; the problem of invariant measure, Ornstein's solution.) 5, *Topological flows.* (Considerations involving both measure and topology; typically, the work of Krylov and Bogoliubov.) 6, *Topological investigations in the space of measure-preserving transformations.* (The work of Halmos and Rohlin.) 7, *Non-stationary problems.* (Random ergodic theorem, non-stationary Markov processes.) 8, *Functional-analytic methods.* (Zorn's lemma, topological and metric spaces, topological vector spaces and Banach spaces, semigroups, Banach lattices, Hilbert spaces.) 9, *Measure and integral. Special vector spaces.* (Fields of sets, measures, measurable transformations, integration, L^p spaces, convergence theorems, conditional expectations, product spaces. Chapters 8 and 9 are called an appendix; their purpose is to fill gaps in the reader's prerequisites.) *Bibliography.*

(A scholarly compilation of eighteen pages.) *Index*. (Detailed and usable.)

A few words are in order on what the book does *not* contain. It does not contain "topological dynamics," the theory of geodesic flows, and most of the connections of the subject with spectral theory. These omissions are deliberate, presumably in order to keep the volume finite. The book also does not contain the work of Chacon and Ornstein (on the Dunford-Schwartz generalization of the ergodic theorem) and the work of De Leeuw and Glicksberg (on almost periodicity). The author mentions these omissions sadly; they are caused by a mixture of considerations involving space and timing. Finally, the book does not contain (not even by bibliographic mention) the recent work of Kolmogorov, Rohlin, Sinai, and others on the concept of entropy. This omission is most regrettable.

The style of the main body of the book is condensed, but clear and readable. The organization is excellent. The work as a whole is a must for every serious student of ergodic theory.

P. R. HALMOS

Topologische Räume. By H.-J. Kowalsky. Birkhauser Verlag (Mathematische Reihe Band 26), Basel und Stuttgart, 1961. 271 pp. DM 35.

The book treats many topics briefly: the basic ideas and results of general topology, and varied further developments. Within its limitations it is a remarkably well-organized and stimulating introduction to the field. The viewpoint is generally analytic (no homotopy, practically no dimension theory). Partial order is very heavily stressed; this affects definitions, choice of topics, and the order of introduction of the basic ideas.

Chapter I consists of three sections: naive set theory, lattices, filters. Emphasis is laid on the lattice of all filters in a set (how distributive is it? how many atoms are there in it?). Chapter II defines a topology by filters of neighborhoods. The usual equivalent definitions are established, but separation axioms, metric topologies, and order topologies are formulated filterwise. The main result of the chapter is the complete normality of linearly ordered spaces.

In Chapter III we find compactness, called "vollkompaktheit"; however, the partial compactnesses (except countable compactness), disappear after six pages. Paracompactness is rather fully treated (including the standard equivalences). Next there are short sections on connectedness and local connectedness, to be continued in Chapter V.