

by the present reviewer (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 798–799). Volumes 2 and 3 of the original edition appeared in 1951 and 1954, respectively, and were duly reviewed. Now we have the first volume of the revised edition. The biggest change is that all the exercises have been omitted. The author says that a separate publication of exercises and their solutions is intended.

The general arrangement of the book has not been changed very much, but there has been extensive rewriting in places all through this first volume, with greater clarity and simplicity as the aim.

We mention some of the more significant changes.

In the discussion of the real numbers, Dedekind's axiom has been replaced by the following "Trennungssaxiom." Let A and B be two classes of real numbers such that $a \leq b$ if $a \in A$ and $b \in B$. Then there exists at least one number s such that $a \leq s \leq b$ if $a \in A$ and $b \in B$. As a criticism, I note that it should be specified that A and B are nonempty classes.

All of the diagrams in the new edition have been redrawn. The level of excellence in the diagrams is higher and more uniform than before.

Some material on infinite series and on the study of curves has been moved into volume 1 from volume 2.

The idea of an operator is introduced (a transformation of one function into another). The notion of a distributive operator is also discussed; the integral with variable upper limit is cited as an example.

Other concepts which are new in this edition are: (1) the concept of a majorant in infinite series and (2) the Lipschitz condition.

There is more about inequalities: Specifically, Jensen's inequality, and the inequalities of Hölder and Minkowski are included.

Purists of modernism in mathematics may note that the function concept is not stated in terms of a set of ordered pairs. Nor does the author write f instead of $f(x)$ for a function.

The book is a fine example of exposition. It has the stamp of the author's personality and distinction in its style and in the historical footnotes.

ANGUS E. TAYLOR

Mathematical aspects of subsonic and transonic gas dynamics. By Lipman Bers. Surveys in applied mathematics, no. 3, John Wiley and Sons, Inc., 1958. \$7.75.

The theory of compressible fluids leads to problems which evoke great interest among mathematicians, especially among those investigating various chapters in the theory of linear and nonlinear partial differential equations. During the last twenty years a great amount of material accumulated and there was a definite need for a survey of

all the work that has been done along these lines. The book by Bers represents a comprehensive survey of this activity. The author was able to present this large and involved material in a satisfactory manner.

The book belongs to a series of surveys in topics which lie on the border of pure mathematics and applications in physics. In the preface, Dr. Weyl, Director of the mathematical branch of the Office of Naval Research, discusses the aims and stresses the importance of this series.

In the introduction the author indicates that he limits his representation to the case of two-dimensional steady potential flows. Reference is made to a remark by von Kármán that the mathematician's help in solving aerodynamical problems is of little value: "I have had the experience that the mathematician may exactly prove existence and the uniqueness of solutions in cases in which the answer is evident to the physicist or engineer for physical reasons. On the other hand, if there is really serious doubt about the answer, the mathematician is of little help."¹ In answer, the author describes the task of a mathematician and shows that cooperation between aerodynamicists and mathematicians is necessary and valuable.

The reviewer wants to remark that in the old controversy, physicist versus mathematician, the use of mathematical methods in aerodynamics has been stressed by various aerodynamicists, e.g., R. von Mises, Sauer and others. In addition, it seems that one of the difficulties encountered in using mathematical methods for solving problems in physics, namely, the great amount of numerical computation needed for the evaluation of formulae, can be overcome in the future by the use of modern computing machines.

Chapter I deals with the differential equations of a potential gas flow. The equations and boundary conditions are derived and various methods for the determination of flow patterns are described. The hodograph method, approximate equations and simplified equations are derived. Various procedures for generating particular solutions are presented.

Chapter II deals with the mathematical background of the subsonic flow theory. The author discusses here elliptic equations arising in the theory and indicates applications of the theory of quasi-conformal mappings, of pseudo-analytic functions, of fixed point theorems and variational methods.

Chapter III discusses the behavior of a flow at infinity and meth-

¹ T. von Kármán, *Solved and unsolved problems of high speed aerodynamics*, Polytechnic Institute of Brooklyn (1955) pp. 11-40.

ods for the determination of flows around profiles, flows in channels and flows with a free boundary.

Chapter IV deals with the mathematical background of transonic gas dynamics. Partial differential equations of mixed type are derived and uniqueness and existence theorems are discussed.

In Chapter V some problems in transonic flows are described. In a short appendix numerical methods are considered. Finally, an extensive bibliography of about 400 papers is included.

The proofs are omitted; however, the main ideas on how to obtain the results are in most cases indicated. Further, the author formulates various new problems which arise in the theory.

In the opinion of the reviewer, the book is a valuable survey for engineers and mathematicians.

STEFAN BERGMAN

An introduction to the theory of numbers. By Ivan Niven and Herbert S. Zuckerman. New York, John Wiley and Sons, Inc., 1960. 8+250 pp. \$6.25.

This textbook provides the senior or first year graduate student with a lucid and inviting introduction to number theory. In the first eight chapters a variety of fundamental topics are systematically expounded; the remaining three chapters contain more specialized material. The chapter headings are: 1. Divisibility; 2. Congruences; 3. Quadratic reciprocity; 4. Some functions of number theory; 5. Some Diophantine equations; 6. Farey fractions; 7. Simple continued fractions; 8. Elementary remarks on the distribution of primes; 9. Algebraic numbers; 10. The partition function; 11. Density of sequences of integers. There are many praiseworthy features in the book. The style is pleasant and perspicuous; the motivation for ideas and methods is presented with didactic skill; definitions are exact; proofs are accurately stated.

The authors have taken great pains to answer a question that frequently perplexes the beginning student. "How does one solve a problem in the theory of numbers?" To this end they have furnished extensive lists of problems. (Several of these are of recent American Mathematical Monthly vintage.) Each set is pedagogically ordered, the transition from simple numerical exercise to difficult theoretical problem being swift but not violent. The authors adhere to the doctrine of separation of text and exercises. Nowhere does the proof of a theorem depend upon the results of a problem.

The approach to number theory in this book is analytical rather than historical. Only too infrequently (in the reviewer's opinion) do