The theory of matrices. By F. R. Gantmacher. Trans. from the Russian by K. A. Hirsch, vols. I and II. New York, Chelsea, 1959. 10+374 pp.; 9+276 pp. $6.00 each.

Applications of the theory of matrices. By F. R. Gantmacher. Trans. from the Russian by J. L. Brenner, with the assistance of D. W. Bushaw and S. Evanusa. New York, Interscience, 1959. 9+317 pp. $9.00.

The original edition of Gantmacher's *Teoriya Matrits* consists of a single volume with fifteen chapters. The Chelsea volumes are translations of Chapters I through X and Chapters XI through XV respectively. The Interscience volume is a translation of Chapters XI through XV.

In the first ten chapters the reader is introduced to the classical matrix theory in the classical way. The following brief summary may give an indication of the author's point of view and the scope of the treatment: The book begins with a definition of matrices and the usual manipulations associated with them; the Gauss elimination scheme is studied in great detail. Next, vector spaces and linear transformations are introduced and the connection with matrices is given. This is followed by a thorough treatment of the characteristic and minimal polynomial and functions of matrices. Two independent treatments of similarity theory are given—one based on matrices with polynomial entries and the other (as Gantmacher puts it, after Krull) based on the decomposition of a vector space into cyclic subspaces relative to a linear transformation. Now one is ready to study matrix equations and among those treated are $AX-XB=C$, $X^m=A$, and $e^X=A$. The last two chapters are devoted to that part of matrix theory that has grown out of the study of quadratic and hermitian forms. Unitary and euclidean spaces are defined and hermitian, unitary, symmetric, skew-symmetric, orthogonal, and normal linear transformations are studied; the canonical forms for the corresponding matrices are obtained and some of the standard factorization theorems appear. The reduction of quadratic forms to sums of squares is given detailed treatment; and the extremal properties of the eigenvalues of a regular pencil of forms are studied; Chapter X closes with a section on Hankel forms.

There are no exercises for the reader although there are many worked-out numerical examples—particularly in the treatment of similarity theory. There are several applications to differential equations. No mention is made of dual spaces or tensor products—a knowledge of elementary determinant theory is a prerequisite.

The author has given a masterful presentation of this particular approach to matrix theory—there is a tremendous amount of information in these ten chapters. Two rather surprising omissions are Schur's theorem stating that each complex matrix can be brought to triangular form by a unitary similarity and the fact that each positive definite matrix is uniquely the product of a triangular matrix with positive diagonal entries and its transpose.

Each of the last five chapters is a gem of exposition devoted to a special topic. (These chapters are numbered I through V in the Interscience book.) Chapter XI deals with the orthogonal similarity theory of symmetric, skew-symmetric and orthogonal matrices with complex entries. Chapter XII presents Kronecker's solution of the problem of determining a criterion for the simultaneous equivalence of pairs of matrices. The chapter closes with an application to linear systems of differential equations. Chapter XIII is concerned with matrices with non-negative entries. Wielandt's proof of the Perron-Frobenius theorem is given and applications are then made to the eigenvalues and eigenvectors of stochastic matrices and oscillatory matrices (a totally non-negative matrix is *oscillatory* if some power of it is totally positive). In Chapter XIV we find the applications of matrix theory to linear systems of differential equations with variable coefficients. These are built around Volterra's Produkt-integral. Chapter XV contains, along with related topics, a beautiful treatment of the problem of determining the number of zeros of a polynomial in the right halfplane. Two proofs of the Routh-Hurwitz theorem are given and in addition one finds the criteria of Liénard-Chipart which greatly reduce the number of determinantal inequalities of the Routh-Hurwitz criterion.

Both translations are clear—there is little difference on the common chapters. In the Chelsea volumes there are new versions of a few paragraphs supplied by Professor Gantmacher. Thus what seems to be a very old error is choked off in the Chelsea version. (The statements concerning hermitian forms in Theorems 6 and 7 of Chapter II in the Interscience volume should be deleted.) The second Chelsea volume makes free use of the results in the first volume while the Interscience volume has five appendices added by the translator and these, together with some footnotes, make the book reasonably self-contained for a reader with some experience in elementary matrix theory.

The translators and publishers deserve a great vote of thanks for making this work available in English, but this duplication seems a terrible waste and one would hope that such a thing will not happen again.

JACK MCLAUGHLIN