RESEARCH PROBLEM

1. A. A. Mullin: *Properties of mutants.*

Let $(A, *)$ denote a nonempty set A together with a closed binary composition law "$*$" defined on A. By a *mutant* of $(A, *)$ is meant a subset M of A that satisfies the condition that $M * M \subseteq \overline{M}$, where $M * M = \{a * b : a \in M \text{ and } b \in M\}$ and $\overline{M}$ is the set of all of the elements of A not in M. If all of the elements of A are idempotent with respect to "$*$" let the empty set be the only mutant of $(A, *)$.

(i) What are necessary and sufficient conditions to be imposed upon the structure of an algebraic system in order that all of its maximal mutants shall have the same cardinality? It is known that all of the maximal mutants of any infinite cyclic group have the same cardinality. The author has counterexamples to the general case, when no structure is imposed upon the algebraic system. (ii) Is there a "significant" algebraic system in which all of the maximal mutants are not of the same cardinality? It is known that the only nonmutant coset of a group is the subgroup itself. (iii) What kind of groups permit their mutant cosets to be maximal? Clearly the additive group of integers permits this condition. (Received August 29, 1960.)