Abelian groups. By L. Fuchs. Budapest, Publishing house of the Hungarian Academy of Sciences, 1958. 367 pp.

This encyclopedic work on the theory of abelian groups will certainly further stimulate the interest in this subject which has been growing steadily since the publication of Part II of Kuroš' *Group theory* (1953) and Kaplansky's *Infinite abelian groups* (1954). This is the first book which includes, along with the fundamental structure theory which appears in the earlier mentioned works, the large body of material contributed in the last several years by the Hungarian school of group theorists. The material in Kuroš is entirely covered. The generalizations to modules over a principal ideal domain and a complete discrete valuation ring, as well as the applications to topological groups, which are features of Kaplansky's monograph, are not included except in the exercises. The author believes that a module-theoretic treatment would imply either almost trivial generalizations and unnecessary complications in the discussions, or deeper extensions which are of a ring-theoretic rather than a group-theoretic nature. The author has been eminently successful in giving a complete, detailed, and easily understandable account of the present status of the theory of abelian groups with special emphasis on results concerning structure problems.

Chapter I includes a summary of the basic concepts of group theory which appears in any textbook on modern algebra, and the notation and terminology is established. The types of groups which play a central role in the structure theory are described in detail. The rank $r(G)$ of a group G is defined as the cardinal number of a maximal independent set (in G) containing only elements of infinite and prime power order. This definition includes those given by other authors for torsion free groups and p-groups, and the proof of the invariance of $r(G)$ clarifies the relations between these definitions. It should be noted that Prüfer's definition of rank which has been followed by many authors is equivalent to what is here defined as the reduced rank.

Chapter II is devoted to direct sums of cyclic groups and includes the Fundamental Theorem on finitely generated groups, Kulikov's criterion for a p-group to be a direct sum of cyclic groups, and some recent results of Kertész, Szele, and the author giving criteria for the existence of a basis.

Chapter III discusses the properties and structure theory of divisible groups. An alternate proof of the direct summand property of divisible groups is obtained, using Gacsályi's results on the solution of linear equations over divisible groups.

Chapter IV is an exposition of direct summands and pure subgroups. Besides the well-known material, this chapter features the results of Łoś on groups G which are direct summands of every group H which contains G as a pure subgroup, and a discussion of certain generalizations of pure subgroups.

Chapter V gives a detailed account of the theory and applications of the concept of a basic subgroup of a p-group, one of the important contributions of Kulikov. For p-groups which are not a direct sum of cyclic groups, a basic subgroup of a p-group G can be used to define a set of generators for G which can be used as a substitute for a basis in the investigation of the properties of p-groups of arbitrary cardinality. In Chapter VI, this investigation is carried out, after a complete account is given of the Prüfer-Ulm-Zippin theory for countable p-groups.

Chapter VII is devoted to torsion free groups. The characterization due to Baer, of torsion free groups of rank one in terms of types of elements is given. Following Kuroš, a system of invariants for torsion free groups of countable rank consisting of classes of row-finite matrices with p-adic elements is developed. The discussion of indecomposable groups includes the construction of an indecomposable group of rank $2^{2^{\aleph_0}}$ based on a recent result of de Groot. A large part of Baer's work on completely decomposable and separable groups is given.

Conditions for the splitting of a mixed group into a direct sum of a torsion and torsion free group and a classification of mixed groups in terms of infinite matrices comprise the principal material of Chapter VIII.

Chapters IX, X, and XI deal with homomorphism groups, endomorphism rings, automorphism groups, group extensions, and tensor products.

Chapter XII, on the additive group of rings, is the first systematic account of this interesting theory. There are two principal problems: (1) given a group G, determine all rings R with additive group isomorphic to G; and (2) given a ring property, find all groups G such that there exists a ring with the given property which has additive group isomorphic to G. All of the results related to these problems, many of them due to the author, which had appeared at the time of publication of this book are included. A short account of the multiplicative groups of fields is given in Chapter XIII.

Chapter XIV is concerned with the lattice of subgroups of an abelian group, and it is mainly an exposition of Baer's fundamental work on this subject.

The concluding Chapters XV and XVI contain results on the decomposition of a group into a direct sum of subsets, groups with generating systems with the property that each subsystem with the same cardinality is also a generating system, universal homomorphic images, and universal subgroups.

Among many notable features of this book which should be mentioned are the excellent bibliography, the exercises with a wide range of difficulty which cover virtually every topic presented, and the statement of eighty-six unsolved problems which already have led to new contributions to the theory of abelian groups. The book is printed in the same large clear type and format as the Hungarian mathematical journals and is remarkably free of misprints.

This book is an important addition to mathematical literature and is highly recommended to anyone whose interests touch the theory of abelian groups.

R. A. BEAUMONT

Special functions. By Earl D. Rainville. New York, Macmillan, 1960. 12+365 pp. \$11.75.

This work of 365 pages and 21 chapters introduces the reader to a large number of "special" functions and their properties, and with this purpose (the author informs us) the material of the book has been the basis of a course given by him since 1946. Most of these functions are classical: the Gamma function, Hypergeometric function, Bessel functions, Elliptic functions (including the Jacobi elliptics), and the important orthogonal polynomials.

With regard to these long-known and deeply studied functions one merit of the book lies in bringing under one cover, at less than encyclopedic length, this large variety of important tools of classical analysis. There also are results on generating functions that are perhaps not well known, so that one comes upon relatively new material; and in addition the book discusses, in some detail, Generalized hypergeometric functions, with special reference to polynomials defined in terms of these. Much of this information is not easily accessible elsewhere, and represents a valuable part, and in a certain sense the most interesting part, of the volume.

Chapters 1 and 3 are short introductions to infinite products and asymptotic series; and Chapter 2 discusses the Gamma and Beta functions. Chapters 4 and 5 take up the hypergeometric function and its generalizations. Here one finds the author's own results on contiguous function relations. Chapter 6 treats Bessel functions briefly (there are so many available sources of information for these); and briefly again, 7 touches on the confluent hypergeometric function.

With Chapter 8, on Generating Functions, we enter the region of the author's special interest. If $\{f_n(x)\}$ is an infinite sequence of func-