BOOK REVIEWS

Nonlinear problems in random theory. By Norbert Wiener. The Technology Press of Massachusetts Institute of Technology, 1958. 9+131 pp. $4.50.

Professor Wiener gave a seminar of fifteen lectures to a group of faculty and graduate students in Electrical Engineering at the Massachusetts Institute of Technology in 1958. This little book consists of that set of lectures, according to the author's preface, set down almost verbatim. The central idea of these lectures is the systematic use of polynomial functionals of Brownian motion in the solution of nonlinear problems from electrical engineering, physics, and elsewhere. An unusual variety of applications is considered, including frequency modulation, brain waves, coupled oscillators, nonlinear electrical networks, coding and decoding (in the information-theoretic sense), quantum theory, and statistical mechanics. The style of the book is highly informal. Informality of itself is perhaps refreshing, but here the reviewer feels that too often the informality has degenerated to sloppiness. One of several noticeable examples is a phrase appearing on page 45, " $\cdots$ since $F(T^t\alpha)\overline{F(\alpha)}$ is obviously a function which will be L^2, being the product of two functions in L^2." A more careful mathematical proofreading would have been desirable.

The first four lectures, comprising about the first third of the book, are used to develop heuristically the necessary theory for the applications to follow. A brief, easily readable, intuitive account of Wiener measure and the Wiener integral, patterned somewhat after a section of the author's Acta Mathematica paper of 1930, is given in the first lecture. Following this, polynomial functionals of Brownian motion are introduced (as e.g. $\int \cdots \int K(\tau_1, \cdots, \tau_n)dx(\tau_1, \alpha) \cdots dx(\tau_n, \alpha)$ where $dx(t, \alpha)$ denotes integration with respect to τ, and α is the stochastic variable). The calculus for finding the averages of such polynomials is demonstrated, and a reduction to a canonical form is developed, whereby to each symmetric kernel $K(\tau_1, \cdots, \tau_n)$ in L_2 is assigned an nth degree nonhomogeneous polynomial functional orthogonal to all polynomial functionals of lower degree. This reduction permits expansion of general nonlinear functionals in L_2 (Wiener measure) in orthogonal series of polynomial functionals (closely related to expansions obtained by Cameron and Martin in 1947), and is the essential tool used throughout the book.

We shall mention here just a couple of the applications. In the lectures on frequency modulation and brain waves, there is a discussion of the functionals

$$\exp\left[i \int \phi(t+\tau)dx(\tau, \alpha)\right]$$

and

$$\exp\left[i\,\epsilon \int\int K(t+\tau_1, t+\tau_2)dx(\tau_1, \alpha)dx(\tau_2, \alpha)\right].$$

These are expanded in orthogonal series and their covariances and spectra determined (it should be noted that some of these results can be found easily by standard methods). In two other lectures the author applies his method to the analysis and synthesis of a four-terminal electrical network. An empirical analysis is sketched which involves the ingenious device of generating Laguerre polynomials electrically with lattice networks, to use as kernels for the polynomial functionals.

This book provides some rather novel methods of attack on a class of difficult problems. In the reviewer's opinion, it should prove to be stimulating and useful reading for a considerable group of applied mathematicians, engineers, and physicists.

WILLIAM L. ROOT

Information theory and statistics. By Solomon Kullback, New York, John Wiley and Sons, Inc., 1959. 18+395 pp. $12.50.

Kullback and Leibler elaborated a definition of the information contained in an experiment to distinguish between two hypothetical distributions on a sample space. The original definition was given by Wiener in his *Cybernetics* and is formally a generalization of the one by Shannon, which is motivated by communication theory considerations and is justified by the nontrivial coding theorem for channels. The book under review does not attempt to generalize Shannon's theory but is concerned with the properties of the generalized information measure as a statistic in testing of hypotheses.

SEYMOUR SHERMAN

Probability and related topics in physical sciences. By Mark Kac, with special lectures by G. E. Uhlenbeck, A. R. Hibbs and Balth. van der Pol. Lectures in Applied Mathematics, vol. 1. New York, Interscience, 1959. 13+266 pp. $5.60.

What is the probability that a mathematics book (chosen at random?) will be informative, clearly written, and delightful to read? Although Professor Kac does not consider this question in his recent book, he clearly demonstrates that the set of such books is nonempty.