the usual Oxford Press product (although, for example, Schur's theorem cannot be found with its help).

This reviewer believes several objections to the omission of topics stem from more than personal preference. For instance, nowhere is a group mentioned: holonomy and motions are equally absent; symmetric spaces are left dangling with a tensor definition. Despite the jacket blurb, it is hard to see how this book would be useful to physics or engineering students. But advanced undergraduates and beyond should find it an excellent introduction. Indeed, it is the only book in English of its kind.

LEON W. GREEN

Aufbau der Geometrie aus dem Spiegelungsbegriff. By Friedrich Bachmann. Die Grundlehren der mathematischen Wissenschaften, vol. 96, Berlin-Göttingen-Heidelberg, Springer, 1959. 15+311 pp. DM 49.80.

This remarkable book is essentially an elaboration of an idea of G. Thomsen (*The treatment of elementary geometry by a group-calculus*, Math. Gaz. vol. 17 (1933) p. 232). The idea is to let the symbol for a point A or for a line a be used also for the corresponding involutory isometry, namely, A is the reflection in the point (or rotation through π about the point), and a is the reflection in the line. If A and B are two points, AB is the translation along their join through twice the distance between them. If a and b are two intersecting lines, ab is the rotation about their point of intersection through twice the angle between them. The point A and line a are incident if the corresponding transformations are commutative. The same property applied to two lines makes them perpendicular. In this case, if the lines are a and b, $ab(=ba)$ is their point of intersection! If $abc=cba$, the three lines a, b, c belong to a pencil, and then the line $d=abc$ is another member of the same pencil, namely the line for which the transformation ab (which is a rotation if a and b intersect) is equal to dc.

A good instance of the power of this method is the following generalization of the notion of isogonal conjugacy with respect to a triangle (p. 16): "If $ba'c=a''$, $cb'a=b''$, $ac'b=c''$, while $a'b'c'$ is involutory, then also $a''b''c''$ is involutory. *Proof.* $a''b''c''=ba'c\cdot cb'a\cdot ac'b=(a'b'c')^b$."

One soon begins to realize that such a geometry is not necessarily Euclidean. It is more like the "absolute" geometry of Bolyai, in which a pencil of lines having a common perpendicular is not necessarily the same as a pencil of parallels. In fact, the geometry may be regarded as a special kind of abstract group whose generators, called "points" and "lines," are involutory, with the distinction that, although the product of two lines may be a point, the product of two points is never a line. Even this restriction is later waived so as to cover the case of a generalized elliptic geometry which admits an absolute polarity.

Two proofs are given for the "Höhensatz:" The three altitudes of a triangle belong to a pencil. Still more remarkably, the author gives an "absolute" proof of the concurrence of medians (p. 74). This is followed by a concise and yet very readable account of the general projective plane, with Pappus's Theorem taken as an axiom. Naturally there is no space for all the proofs; in particular, the author refers to Hessenberg and F. Schur for the deduction of Desargues's Theorem and the Fundamental Theorem, respectively. He defines a collineation to be a transformation that preserves collinearity, and a projective collineation to be one which transforms every range or pencil projectively. He proves that such an effect on a single range (or pencil) suffices to make the collineation projective. It follows, in particular, that every perspective collineation is projective. When the center and axis are incident, he calls the perspective collineation a Translation, mentioning in a footnote "O. Veblen und J. W. Young sagen *Elation*." Perhaps he would have consented to adopt the word Elation had he noticed that it was used in German (by Lie in 1893) long before it was used in English!

Following von Staudt, the author defines a polarity to be a correlation of period two, and distinguishes an elliptic polarity, which has no self-conjugate points, from a hyperbolic polarity which has at least one.

After this excursion into projective geometry he returns to the absolute plane, which he proceeds to embed in the projective plane by regarding pencils of lines as ideal points. In order to remain in two dimensions, he makes use of Hjelmslev's *half-rotation* (Halbdrehung), which we must take care not to confuse with the familiar "half-turn" (the author's "reflection in a point"). Any rotation S (transforming each point P into P^S) determines a half-rotation which transforms P into the midpoint between P and P^S. Though not an isometry, this is a collineation. Any given ideal line can be transformed, by a suitable half-rotation, into an ordinary line. This makes it possible to verify, in this extended plane, all the projective axioms.

The author considers geometries over various fields. For instance, he gives (p. 123) a numerical proof that it is impossible to give an elliptic metric to a finite projective plane.

In the hyperbolic plane, two lines a and b are said to be parallel if they have neither a common point nor a common perpendicular. The author develops this "negative" definition into a very remarkable criterion. From an arbitrary point on b, draw g perpendicular to a, and e perpendicular to g. From an arbitrary point on e, draw perpendiculars to a, b, and let h join their feet. Then a and b are parallel if and only if e and h are perpendicular. Following Hilbert, he calls a pencil of parallels an *end*. He uses the above criterion to prove that any two ends determine a line. As an instance of the application of projective geometry to hyperbolic geometry, he points out that Seydewitz's theorem provides an immediate proof for the following property of a trebly-asymptotic triangle: the perpendiculars from any point on one side to the other two sides are perpendicular to each other.

In the elliptic plane, the effect of the absolute polarity is to make the reflection in a line a equivalent to the reflection in its pole A. Every isometry is expressible as the product of two such reflections. Isometries are represented as points in elliptic space, in a manner resembling §§7.3–7.5 of the reviewer's *Non-euclidean geometry* (3rd ed., University of Toronto Press, 1957). The author considers, in this group-space, a plane hexagon $P_1Q_2P_3Q_1P_2Q_3$ whose vertices lie alternately on two lines p and q. He draws lines through P_1, P_2, P_3 left-parallel to q, and lines through Q_1, Q_2, Q_3 right-parallel to p, so as to form Dandelin's *Hexagramme mystique* (cf. H. F. Baker, *Principles of geometry*, vol. 3, Cambridge University Press, 1923, p. 44) consisting of six generators of a Clifford surface. This enables him to prove Pappus's theorem in the plane pq. The deduction is illustrated by a particularly fine perspective view on page 254.

The above remarks may serve to suggest something of the flavor of this unusual book, which is well written, well printed, well indexed, and "chock full" of unfamiliar results. All geometers and most algebraists will be glad to keep it on an accessible shelf.

H. S. M. COXETER

Statistical independence in probability, analysis and number theory. By Mark Kac. Carus Mathematical Monographs, no. 12. New York, Wiley, 1959. 14+93 pp. $3.00.

This delightful little book is an example of informal pedagogy at its best. It entertains, it stimulates interest, it educates (somewhat haphazardly) and it challenges current dogma, all in such deceptively simple style that one feels as if he is reading a popularization from Scientific American. In fact it is popularization, on a very high