Analytic function theory, vol. 1. By Einar Hille, Boston, Ginn and Co., 1959. 11+308 pp. $6.50.

This is the first volume of a two volume work on analytic functions. It is an excellent book having several features which should make it attractive to the teacher and the student.

The first is the appearance of historical footnotes containing information about important theorems and their discoverers. One learns for example that the revival of Italian mathematics is conventionally dated from the journey which Enrico Betti, Francesco Brioschi and Felice Casorati made to France and Germany in 1858. Another feature is the skillful threading of ideas through the book. One first encounters an idea in an exercise, later it appears in an illustration, then it receives specific notice in a remark or discussion and finally is formalized in the statement of a definition or theorem.

This type of development is well illustrated by the treatment of inverse functions which originates in Chapter 3 and culminates in the last section of the book with a derivation of the properties of the k roots near a zero or order k. The end of volume 1 finds many of these ideas still developing, for example that of analytic continuation. Perhaps some will not reach maturity. Singular integrals may fall in this category. In any event the student will not fail to find many directions in which to continue his studies, aided by references at the end of each chapter. A contrary objective is fulfilled by the exercises at the end of each section which are designed primarily to illustrate the material in the section.

While the author emphasizes the historical origins of the subject the reader is also introduced to more recent points of view through the description of the algebraic structure of the various function classes as they arise. Another conspicuous merit is the elegance of detail and construction. One may point to the admissible functions introduced in the discussion of double series or the parallel curves used in the discussion of analytic continuation across a smooth arc.

As stated in the foreword the core of this volume consists of Chapters 4, 5, 7, 8, and 9. The remaining material in four chapters and three appendices is either introductory or supplementary. Chapters 1, 2 and 3 introduce the real and complex numbers, the topology of the complex plane and Möbius transformations. The section on Möbius transformations comprises only twelve pages yet contains discussions of the important subgroups: rigid rotations of the sphere, transformations leaving the unit circle invariant and transformations leaving two points fixed. Parabolic, elliptic, hyperbolic and loxodromic transformations are described. A proof of the Jordan theorem for polygons is contained in an appendix. Several threads of ideas originate in these chapters; for example the stereographic projection is shown to be isogonal and a pure magnification. This decomposition of the properties of conformality is carried through the discussions of differentiability in Chapter 4 with the presentation of some of the results from the theory of monogeneity.

Chapter 4 is concerned with the basic properties of holomorphic functions. A difficulty with the derivation of the chain rule for a composite function could be avoided by making use of the increment formula for a differentiable function. The increment formula for Stolz-Fréchet differentiable functions as used in the last theorem of the chapter could be used with advantage in an earlier differentiability argument. The chapter contains the Goursat proof of the inverse function theorem by the method of successive approximations, a discussion of conformal mappings with a brief mention of area, and definitions of various function spaces.

After three sections on infinite series, the elementary properties of power series are derived in Chapter 5; an example (Fejér) of a power series which converges uniformly but not absolutely on the circle of convergence is included. Direct analytic continuation is explained, singularity of a power series is defined and an example (Lerch) of a noncontinuable series is given. The elementary functions are studied in Chapter 6. The Riemann surfaces of the logarithm and inverse trigonometric functions are described with some care.

Chapter 7 commences the systematic study of complex integration although frequent use has already been made of an integral defined on a line segment. An appendix is devoted to the development of the Riemann-Stieltjes integral of complex valued functions. The complex integral on a rectifiable curve is then defined as the appropriate Stieltjes integral. This separation of the integration theory and the geometric interpretation is a desirable arrangement. Cauchy's theorem is proved for a simply connected domain with a rectifiable boundary and extended to doubly connected domains; the result for a domain of connectivity n is assumed. The integral formulas are derived and applied to the reflection principle, maximum principle and uniform convergence. Cauchy's principal value and integrals of Cauchy type are introduced.

Chapter 8 assembles Taylor's series, the Laurent expansion, the Mittag-Leffler principal part expansion of a meromorphic function and the Weierstrass factorization theorem. The maximum principal appears again in a third form and a long section is devoted to the Gamma function. Application of Taylor's series is made to obtain a result on the uniqueness of analytic continuation but a pending question on the existence of a singular point of a series on the circle of convergence remains unanswered. The answer as well as a formal definition of continuation will evidently appear in the second volume. This delay seems to be the only disadvantage of an otherwise effective instructional arrangement.

Chapter 9 contains the residue theorem together with applications to the principal of the argument, summation theorems, inverse function theorems and a proof of a case of the general implicit function theorem.

The errors in this book are very rare. The format and binding are attractive. The author has composed a lucid account of the basis for analytic function theory. Analysts will look forward to the appearance of volume two.

GUY JOHNSON

Topological analysis. By Gordon Thomas Whyburn. Princeton, Princeton University Press, 1958. 12+119 pp. $4.00.

The purpose of this book is to apply certain topological methods to the study of the theory of functions of a complex variable and to the generalization of some of these results to mappings that are light and open. The book is well written and the author is to be commended for an excellent job in the blending of topology and classical analysis. The first three chapters consist of topics selected from the author's *Analytic topology* (Amer. Math. Soc. Colloquium Publications, vol. 28, New York, 1942). However some changes in methods have been introduced. For example, by first showing that the continuous monotone image of the unit interval is a simple arc the author uses this to show that every locally connected continuum is arcwise connected and then uses this result to prove the characterization of a simple arc. The fourth chapter is concerned with standard definitions and elementary topics of complex variables. The fifth chapter is concerned with defining and developing properties of the topological index. This is the main topological tool used in the sequel. In the sixth and seventh chapters the topological index is used to develop results summed up in the following statement. If the complex valued function $f(z)$ is nonconstant and differentiable everywhere in a region R of the complex plane then f is light, f is strongly open, f has scattered inverse points, f is locally topological at a point z_0 in R if and only if $f'(z_0)\neq 0$ and f is locally equivalent to a power mapping. These methods have not as yet shown that $f'(z)$ is continuous or open but do show that $f'(z)$ has closed and scattered point inverses. In