ence or authority. The reviewer knows of no published proof of this result,—and believes that in such circumstances, even in an elementary text, appropriate authority (e.g. "correspondence") or qualification should be indicated. Indeed the reviewer would wish for more complete and specific references in general in a work where so much is asserted without proof.

Chapter IV. The discussion of the second incompleteness theorem is erroneous: two statements of the forms "'(for all x) [x not proof of f]' is provable" and "(for all x) ['x not proof of f' is provable]" are confused. The latter, asserted to be unprovable in Z, can be proved rather simply in Z by considering the alternative cases that Z be consistent or inconsistent. In introducing the proof of Tarski et al., "validity" is used, without definition, in their sense (=provability) rather than in any sense previously explained by the author.

Chapter V. The syntactical role of abstraction is not clear. Contextual definition should be earlier and more complete. Definition and use of "$y(x)$" on p. 95 is inconsistent; the definition is appropriate to a function but not to a general relation.

A final comment. The reviewer would have preferred that indication be given of logic-algebra and general recursive function theory as two of the most active research areas in foundations today, though the author might reasonably maintain that this is beyond his announced historical aims.

Among the less trivial typographical errors:

p. 45, "Sx" for "x" in last two occurrences in l. 16;
p. 67, l. 10b, "sentence calculus" for "predicate calculus";
p. 69, ll. 5–6, "$P=Q$" for "$G=0$" and "$F=G$" for "G";
p. 95, l. 13, "→" for "&".

HARTLEY ROGERS, JR.

Geometric algebra. By E. Artin. New York, Interscience Publishers, Inc., 1957. 10+214 pp. $6.00.

When Hilbert's *Grundlagen der Geometrie* and other texts on the foundations of geometry appeared around the turn of the century, the approach was almost purely geometric. It is typical of the development of mathematics in the intervening years that, in this latest book on geometry, the approach is almost entirely algebraic.

In the preface the author states that his aim is to offer a text (based on lecture notes of a course he has given at New York University) which would be of a geometric nature yet distinct from a course in linear algebra, topology, differential geometry, or algebraic geometry. This aim then accounts for the several different topics discussed in the book. For the student whose knowledge of modern algebra is meagre, the first chapter offers a compilation of algebraic theorems (and their proofs) which he is most likely to need. In particular, a thorough discussion is given of the "pairing" of a left vector space W and a right vector space V over a (not necessarily commutative) field k which is given by a product (that is, bilinear form) $AB \in k$ for all $A \in W$ and $B \in V$.

Chapter II is devoted to affine and projective geometry. Here the approach is the following: given a plane geometry whose objects are points and lines, and where certain axioms of a geometric nature are assumed true, is it then possible to find a field k such that the points of the given geometry can be described by coordinates from k and the lines by linear equations? The author chooses four axioms and then sets about constructing the field. In so doing he has a chance to study the group of dilatations and its invariant subgroup, the group of translations. Then it is shown that the set k of all trace preserving homomorphisms is a field. (Hilbert's theorem that the field k is commutative if and only if Pappus' theorem holds is later proved.) Coordinates are introduced by use of a fixed point and two translations with different traces. This particular axiomatic study of affine geometry was previously given in the author's paper, *Coordinates in affine geometry*, Notre Dame, 1940.

Next the converse problem is attacked, in that a field k is given and an affine geometry is constructed. The fourth axiom, in the presence of the other three, is equivalent to Desargues' theorem in the plane. Thus the geometry which has been considered is Desarguian geometry. Actually, non-Desarguian planes are not discussed in the book but references to literature on the subject are given.

The fundamental theorem on ordered geometries is proved: an ordering of a plane geometry induces canonically a weak ordering of the field k, and conversely. Archimedean ordering of a Desarguian plane is discussed briefly and, in particular, it is noted that in an archimedean plane the theorem of Pappus holds.

The chapter concludes with a discussion of projective spaces; in particular, the fundamental theorem of projective geometry is proved. Axioms for projective geometry in a plane are given and some mention is made of axioms needed if the dimension of the space is greater than 2.

Symplectic and orthogonal geometry are discussed in Chapter III. Here one is dealing with a vector space V of finite dimension over a commutative field k. A product $XY \in k$ is said to define a metric structure on V, and gives a pairing of V and V into k. Assume that $AB=0$ (orthogonality) implies $BA=0$. It is shown that two cases arise: (1) symplectic geometry, where it is postulated that $X^2=0$ for all $X \in V$, which is, of course, equivalent to the study of skew symmetric bilinear forms; (2) orthogonal geometry, where $XY=YX$ for all X, $Y \in V$ which, if the characteristic of k is different from 2, is equivalent to the study of quadratic forms. A section is devoted to features common to symplectic and orthogonal geometry; notably, Witt's theorem is proved. Then each geometry is studied separately. Geometries over finite fields and orthogonal geometry over an ordered field are also discussed.

Chapter IV, devoted to the general linear group, can be read as a unit separate from the rest of the book if the student's knowledge of algebra is sufficient. The author first gives Dieudonné's extension of the theory of determinants to noncommutative fields. He then uses this idea in studying the structure of the general linear group and its subgroup, the unimodular group. Special attention is paid to the case where k is finite.

In Chapter V the structure of the symplectic and orthogonal groups is studied. The discussion of the symplectic group is straightforward. However, since the structure of the orthogonal group is a problem whose solution is only partly known, the presentation here leads to a study of some special topics: the orthogonal group of euclidean space (except for dimension 4, which is treated separately later in the chapter), elliptic spaces, the Clifford algebra, and the spinorial norm. With the mention of many unsolved problems, many conjectures, and some known counter examples, this last chapter should prove very stimulating to the student.

The text is very well printed and the exposition is clear. The author makes every effort to encourage the reader by pointing out to him the easier parts of the book and by suggesting spots he may skip on a first reading. The text contains a number of exercises.

The beginning graduate student, or very advanced undergraduate, will find this book an admirable introduction to material which is treated from a more advanced point of view and more extensively in Baer's *Linear Algebra and Projective Geometry* and Dieudonné's *La géométrie des groupes classiques*. Mathematicians will find on many pages ample evidence of the author's ability to penetrate a subject and to present material in a particularly elegant manner.

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