RESEARCH PROBLEMS

5. V. L. Klee, Jr.: *Convex sets.*

In Euclidean n-space E, let C be convex, B the boundary of C, S the unit sphere $\{x: \|x\| = 1\}$, and D the set of all points of S which represent directions of line segments in B. (Specifically, $d \in D$ provided $\|d\| = 1$ and the segment $[-d, d]$ is parallel to some segment contained in B.) Then D is an F_σ set in S. Must D be of the first category in S? Must the $(n-1)$-measure of D be zero? (The answer is affirmative for $n = 2$, for then D is countable.) The same questions may be asked for more general classes of surfaces B. (Received January 18, 1957.)

6. V. L. Klee, Jr.: *Convex sets.*

In Euclidean n-space E, let K be a family of sets which includes all open convex sets, includes the union of each increasing sequence of its members, and includes the intersection of each decreasing sequence of its members. Must K include all convex Borel sets in E? (For $n = 2$, the answer is affirmative. See Duke Math. J. vol. 20 (1953) pp. 105–112.) (Received January 18, 1957.)

7. J. R. Isbell: *Commuting mappings of trees.*

Let T be a tree, i.e. a compact locally connected space in which every two points are joined by a unique arc. It can be seen that every commutative group Γ of homeomorphisms of T has a common fixed point. [By Zorn's lemma, it suffices to show that there is a proper subcontinuum which is mapped onto itself by every element of Γ. Observe that for f, g, in Γ, g maps the set S of fixed points of f into itself. So does g^{-1}; hence $g(S) = S$. Similarly g leaves invariant the least subcontinuum containing S. But this is all of T only if f leaves every end point of T fixed.] Is this true for commutative semigroups of continuous mappings? It is not known even for a semigroup generated by two mappings on an arc. (Received June 7, 1957.)