no groups of any sort are introduced. It thus opposes the widely held opinion that algebra can clarify and simplify topology. However that may be, modern topology does involve a lot of algebra; and if indeed there is anything tricky about the application of algebra to topology, then the sooner its utility is gently illustrated to the student the better. There is, of course, considerable topological activity concerned with phenomena in lower-dimensional Euclidean spaces in which it appears possible to do quite well without some of the tools being sharpened in algebraic topology; and for such research this book will prepare the student quite well.

If the needs of the future algebraic topologist were thus as a matter of pedagogic (not sectarian) policy left to one side, the needs of the future topologic algebraist were overlooked altogether. Uniform structures are presented only with a countability restriction; function spaces are treated almost not at all. Infinite topological products are presented merely as an application of the axiom of choice, and Tychonoff's theorem appears as a variant of this axiom. (The treatment of the axiom of choice is marred by a preoccupation with what seems to be an unnecessary axiom of finite choice.) Metrizability of product spaces is not discussed (and therefore the Hilbert cube has to be carefully circumvented in the earlier section of metrization). One would have expected the theorem on the connectedness of product spaces. These defects can easily be repaired by a competent instructor, and are thus minor. We invite anyone preparing a course in topology to consider this work on the merit of the substance and treatment of the fifth chapter which is even physically half the book.

RICHARD ARENS

Plane waves and spherical means applied to partial differential equations. By Fritz John. New York, Interscience Publishers, Inc., 1955. 8+172 pp. $4.50.

This book is dedicated, as the title indicates, to an exposition of the author's results on various problems concerning partial differential equations, results obtained by using relations between a function in an n-dimensional Euclidean space and its integrals over planes and spheres. While treatises and encyclopedia articles supposedly strive for completeness either in the extent of subject-matter treated or in its detailed presentation, a tract has no such pretension and may present a slice of the subject-matter of its field organized to the author's taste. The present example, the second of a new series of Interscience Tracts, has been fitted to a very elegant taste indeed. Yet while concentrating on techniques and ideas which he has personally developed and finds congenial, Fritz John has touched upon many of the most important topics of the present-day field of partial differential equations.

It would be somewhat difficult to conceive of this book being used in any general way as a textbook on partial differential equations. The very stylistic economy which makes reading it so pleasant for anyone acquainted with the field has as its compensating disadvantages an allusiveness and a lack of discursive explanations and pedagogic emphases which would make the book difficult for students. An unfortunate consequence of the book's ultra-clear but Spartan style could be that it may be passed up by analysts outside the circle of workers in partial differential equations who might profit from becoming familiar with its elegant analytical technique. As the author shows by example, there are many analytical problems in which simplifications can be achieved and convergence difficulties avoided by substituting his Radon transform for the more familiar Fourier-Laplace transformation.

By a plane wave in E^n is meant a function $g(y\cdot x)$ of x, where $g(s)$ is a function of the scalar variable s, y is a fixed vector in E^n, and $y\cdot x$ the Euclidean scalar product in E^n. Chapter I discusses the expansion of a C^1 function f in E^n as a superposition of plane waves of the form $|(y-z)\cdot x|^k$ or $|(y-z)\cdot x|^k \log |(y-z)\cdot x|$. This expansion is equivalent to formulae of Radon expressing f in terms of its integrals over planes. These formulae are applied in Chapter II to obtain a solution of the Cauchy problem on a half-space for homogeneous hyperbolic equations with constant coefficients. After the geometry of the normal surface has been studied in a projective representation, an explicit integral formula is obtained for the solution of the problem in which the kernel of the integral is given in terms of an integral over the normal surface. In this and further results on domains of dependence, the author re-derives results previously obtained by Herglotz, Petrovsky, Gårding, Leray, and Bureau using variously Fourier methods, Riesz integrals, and the finite and logarithmic parts of divergent integrals.

Chapter III is dedicated to the author's construction, first published in 1950, of a fundamental solution in the small for general linear elliptic equations with analytic coefficients. Here the Radon formulae are applied in combination with the Cauchy-Kowalewski theorem and a method of Holmgren first used to prove the uniqueness of solutions of the Cauchy problem for equations with analytic coefficients. The results are extended to the case of systems and explicit formulae given for a fundamental solution in the large for the constant coefficient case.

Means over the surface of spheres of various radii enter the discussion in Chapter IV which is concerned with obtaining a representation of a rather complicated kind for a function f in terms of its iterated means over spheres with radii bounded away from zero. By an elementary argument, iterated spherical means are used to obtain solutions of the Euler-Poisson-Darboux equation. Chapter V is dedicated to Asgeirsson's mean-value theorem with applications to the Darboux and wave equations. A generalization to ellipsoidal means due to A. S. Howard is discussed. Chapter VI contains a study of the problem of determining a function from its integrals over spheres of a fixed radius, prefaced by a formal discussion of a more general class of convolution equations.

The differentiability theorems for solutions of various classes of partial differential equations by the use of spherical means which are presented in the last two chapters are the culmination of the techniques expounded in this book. In Chapter VII differentiability theorems are established for solutions of linear and nonlinear elliptic systems by means of the identities of Chapter IV. In Chapter VIII, similar regularity properties are established for the integrals over a family of time-like curves of solutions of nonelliptic partial differential equations. The proofs are carried through in a strikingly explicit and concrete analytical fashion. In the elliptic case, no fundamental solutions are utilized. The results include the differentiability of weak solutions of linear elliptic equations, a fact which plays a crucial role in the L^2-theory of elliptic boundary value problems.

It seems certain that this work by Professor John will occupy an important and permanent position in the literature on partial differential equations.

F. E. BROWDER

Topological transformation groups. By D. Montgomery and L. Zippin. New York, Interscience Publishers, Inc., 1955. 11+282 pp. $5.50.

In writing this book the authors have had two purposes: (1) to present, in connected form, the recent solution by the authors and A. Gleason, based on the work of many mathematicians, of the famous 5th problem of Hilbert, stating that a topological group which is locally homeomorphic to Euclidean n-space E^n is a Lie group; and (2) to report on the work done during the past twenty years on transformation groups, the emphasis being on the way a group acts on a space as a group of transformations.

The first topic represents the final step in a long development to which many outstanding mathematicians have contributed: Introduction of the idea of Lie groups by Lie, via sets of transformations