THE NOVEMBER MEETING IN PASADENA

The five hundred twenty-eighth meeting of the American Mathematical Society was held at the California Institute of Technology in Pasadena, California, on Saturday, November 17, 1956. Attendance was about 115, including 86 members of the Society.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, Professor Bertram Yood delivered an address on *Semi-simple Banach algebras*. He was introduced by Professor Edwin Hewitt, and the sessions for contributed papers presided over by Professors Richard Arens and R. P. Dilworth.

Following are abstracts of papers presented at the meeting, those whose numbers are followed by "*t*" having been given by title. On joint papers, the presenter's name is followed by "(p)". Mr. Bryant was introduced by Professor Roy Dubisch and Dr. Montague by Professor Alfred Tarski.

Algebra and Theory of Numbers

70*t*. B. W. Brewer: *On certain character sums and related congruences.*

The number of solutions (u, w) of the congruence $u^k+Q\equiv w^2 \bmod p$, p a prime, is given by $p+(Q|p)+\psi_k(Q)$, where $\psi_k(Q)=\sum_{u=1}^{p-1}(u^k+Q|p)$, and has been expressed in terms of certain quadratic partitions of p for $k=3$, 4, 5, 6, and 8. For $p=a^2+b^2$ $(a\equiv 1 \bmod 4)$, $\psi_4(Q)=-2a(Q^{1/2}|p)-2$ if $(Q|p)=1$, and $\psi_4(Q)=\pm 2b$ if $(Q|p)=-1$ (Jacobsthal). The ambiguity in sign in the latter case has been removed by E. Lehmer (Pacific Journal of Mathematics vol. 5 (1955) pp. 103–118) if 2 is a quartic nonresidue of p. Theorem 1: If 3 is a quadratic nonresidue of the prime $p=a^2+b^2$ $(a\equiv 1 \bmod 4$, $b\equiv a \bmod 3)$, and $(Q|p)=-1$, then $\psi_4(Q)=3b(\alpha|p)$, where $3\alpha^2+Q\equiv 0 \bmod p$. Theorem 2: If the prime $p=12k+1=a^2+b^2=s^2+3t^2$ $(a\equiv 1 \bmod 4$, $s\equiv 1 \bmod 3)$, then (1) $\psi_{12}(Q)=-2a[1+2([-3]^{1/2}|p)](\alpha|p)-4s-2$ if $Q\equiv\alpha^6 \bmod p$, (2) $\psi_{12}(Q)=-2a[1-([-3]^{1/2}|p)](\alpha|p)+2s-2$ if $Q\equiv\alpha^2\not\equiv\alpha_1^3 \bmod p$, (3) $\psi_{12}(Q)=\pm 2b[1-2([-3]^{1/2}|p)]$ if $Q\equiv\alpha^3\not\equiv\alpha^2 \bmod p$, (4) $\psi_{12}(Q)=\pm 2b[1+([-3]^{1/2}|p)]\pm 6t$ if $Q\not\equiv\alpha^2$, $Q\not\equiv\alpha_1^3 \bmod p$. Theorem 3: If the prime $p=c^2+2d^2$ (hence $p=8n+1$ or $8n+3$), then $\sum_{x=0}^{p-1}((x+2)(x^2-2)|p)=2c$, where $c\equiv(-1)^{n+1} \bmod 4$; and if $p\neq c^2+2d^2$ (hence $p=8n-1$ or $8n-3$), then $\sum_{x=0}^{p-1}((x+2)(x^2-2)|p)=0$. Theorem 3 is an exact analog of the known results for primes of the form a^2+b^2 and s^2+3t^2. (Received September 24, 1956.)

71. S. J. Bryant: *Isomorphism order for abelian groups.*

A group G is said to have isomorphism order k if G has the following property: If H is a group such that every subgroup of H which can be generated by k or fewer elements is isomorphic to a subgroup of G then H is isomorphic to a subgroup of G.

Theorem: An abelian group G has isomorphism order k if and only if G is a direct sum of two groups, one torsion the other torsion free. The torsion free summand is a vector space over the rationals of dimension less than k while the torsion summand can be written as a direct sum of fewer than k subgroups of the rationals mod one. (Received October 2, 1956.)

72. R. C. Lyndon: *Solutions of equations in free groups and semigroups.*

Problem. Given words $f(x_1, \cdots, x_n)$, $g(x_1, \cdots, x_n)$, what can be said about the set of n-tuples of words $w_1, \cdots, w_n$ such that $f(w_1, \cdots, w_n)=g(w_1, \cdots, w_n)$? *Results.* (1) $x^2y^2=z^2$ in a free group implies x, y, z all in a common cyclic subgroup; proof by ad hoc manipulation. (Problem of Vaught) (2) It is more or less obvious that, unless $f=g$, the set $w_1, \cdots, w_n$ must lie in a free subgroup (subsemigroup) of rank less than n. (3) $x^2y^2z^2=t^2$ in a free semigroup implies x, y, z, t all lie in a free subsemigroup of rank no greater than 2; for any quadratic equation (each x_i appearing at most twice), method extends to decide if a similar result holds. (4) Given $f(x_1, x_2, \cdots, x_n)$ in a free group, there is an effective method for obtaining a finite set of words $w_i(x_2, \cdots, x_n; t_1, \cdots, t_m)$, depending on integer parameters t_j, such that the set of values w_i for all values of the t_j is the set of words $w(x_2, \cdots, x_n)$ such that $f(w, x_2, \cdots, x_n)=1$. (5) Proof of (4) involves the study of R-groups G, for R a ring of operators, characterized by the axioms for (noncommutative) groups together with the following: $1g=g$; $ag\cdot bg=(a+b)g$ $a(bg)=(ab)g$; $g\cdot a(hg)=a(gh)\cdot g$. For R the ring of polynomials over the integers Z, the decision what retractions ϕ, of R onto Z, map w into 1, is reduced to a system of diophantine equations. In the application to (4) these equations may be taken as linear. (Received October 25, 1956.)

73. R. S. Pierce: *A generalization of atomic Boolean algebras.* Preliminary report.

Let α be an infinite cardinal number. A partially ordered set P is called α-finitary if P is closed under meets, has a zero and enjoys the property: (*) if M is a subset of P of cardinality $\leqq\alpha$ with the finite meet property, then M has a nonzero lower bound in P. A Boolean algebra B is called α-atomic if B contains a dense subset which is α-finitary (with respect to the induced ordering). Theorems: 1. If B is α-atomic for all α, then B is atomic in the usual sense and conversely. 2. A free σ-complete B.A. is $\aleph_0$-atomic. 3. If a Boolean algebra B contains a dense subalgebra which is α-atomic, then B is α-atomic. In particular, the normal completion of an α-atomic B.A. is α-atomic. 4. If B is an α-atomic B.A., then B contains a dense subalgebra B_0 which is isomorphic to an α-atomic, α-field. 5. An α-atomic B.A. is α-distributive. 6. An $\aleph_\xi$-atomic, $\aleph_{\xi+1}$-complete B.A. is an $\aleph_{\xi+1}$-homomorph of an $\aleph_{\xi+1}$-field. 7. There exists a σ-field F (necessarily not $\aleph_0$-atomic) with the property that its normal completion is not $\aleph_0$-distributive. (Received October 4, 1956.)

Analysis

74t. G. E. Cross: *On the uniqueness of multiple trigonometric series.*

Consider the multiple trigonometric series $\sum c_m e^{i(m,x)}$, where $m=(m_1, \cdots, m_n)$, $x=(x_1, \cdots, x_n)$, $(m, x)=m_1x_1+\cdots+m_nx_n$, and the summation is over all integers $m_j\geqq 0$. The series is said to be summable (T, k) if the series $\sum C_p(x)$ is summable (C, k). Here $C_p(x)$ denotes the "triangular" sum $\sum c_m e^{i(m,x)}$, $m_1+\cdots+m_n=p$. It is shown that if, if a multiple trigonometric series is summable (T, k) for all values of x, then the coefficients c_m are given by repeated integrals of dimension $n+1$, the inner integral being a $\mathcal{P}^{k+2}$-integral defined by James (Trans. Amer. Math. Soc. vol. 76 (1954) pp. 149–176, §8) extended in the obvious way for a complex function of a real variable. In the case of double trigonometric series $\sum c_{mn}e^{i(mx+ny)}$, $m\geqq 0$, $n\geqq 0$, it is shown that, if there is a countable set x_j, y_j with $x_j\neq y_j$, such that the series is summable (T, k) to zero for all $x=x_j+t$, $y=y_j+t$, $0\leqq t\leqq 2\pi$, then the series vanishes identically. (Received October 4, 1956.)

75. Edwin Hewitt: *The asymmetry of certain measure algebras.*

Let G be a locally compact Abelian group, and let $\mathfrak{M}(G)$ be the algebra of all complex-valued, bounded Radon measures on G, where addition and scalar multiplication are defined set-wise, and the product $\lambda*\mu$ of λ, $\mu\in\mathfrak{M}(G)$ is given by $\lambda*\mu(A) =\int_G\mu(x^{-1}A)d\lambda(x)$ for all Borel sets $A\subset G$. With $\|\lambda\|$ = the total variation of λ and $\tilde{\lambda}(A)=\lambda(A^{-1})$, $\mathfrak{M}(G)$ is a commutative Banach algebra with the involution $\lambda\rightarrow\tilde{\lambda}$. Theorem. If every neighborhood of the identity in G contains an element of infinite order, then $\mathfrak{M}(G)$ is asymmetric. Specifically, there exists a measure $\nu\in\mathfrak{M}(G)$ and a multiplicative linear functional M on $\mathfrak{M}(G)$ such that $M(\gamma)=1$ and $M(\tilde{\gamma})=0$. (For the case G = the additive real numbers, this was proved by Yu. A. Šreĭder [Mat. Sbornik N.S. vol. 27 (69) (1950) pp. 297–318].) It follows that the usual Fourier-Stieltjes transforms are not dense in the space of all multiplicative linear functionals on $\mathfrak{M}(G)$ and that there exists a measure $\mu\in\mathfrak{M}(G)$ such that μ^{-1} does not exist, and yet the Fourier-Stieltjes transform of μ is bounded in absolute value away from zero. (Received September 19, 1956.)

76*t*. V. L. Klee, Jr. *An example in the theory of topological linear spaces.*

If E is a locally convex Hausdorff linear space, then (a) for each $x\in E\sim\{\phi\}$, E admits a continuous linear functional f such that $fx\neq 0$. In sharp contrast, there are Hausdorff linear spaces S such that (b) the only continuous linear functional on S is that which is identically zero. Henriksen has asked whether the property (a) of a Hausdorff linear space E must be inherited by its quotient space E/M, where M is a closed subspace of E. (Note that the answer is affirmative if E is locally convex, or if E is complete and metrizable and M admits in E a closed supplementary subspace.) The present paper describes a metric linear space E with property (a) and closed supplementary subspaces M and N of E such that E/M and E/N both have property (b). Some other examples are also constructed—in particular, a linear space L and total linear subspaces F and G of L^* such that L is separable under the F-topology and under the G-topology, but not under the $(F+G)$-topology. (Received October 1, 1956.)

77. A. V. Martin: *On derivatives and neighborly functions.*

W. W. Bledsoe defined a function f: $S\rightarrow T$ (where S and T are spaces having metrics d and d') to be *neighborly* at the point x of S if, for every $\epsilon>0$, there is a nonempty open sphere U of S such that, for every y in U, $d(x, y)+d'(f(x), f(y))<\epsilon$ (see Bledsoe, *Neighborly functions*, Proc. Amer. Math. Soc. vol. 3 (1952) pp. 114–115). Since U is not required to contain x, this is a generalization of a continuous function. Theorem: If f is any real-valued function of a real variable whose derivative f' is defined everywhere and is Riemann-integrable over every finite interval, then f' is neighborly. This theorem offers some prospect of being useful in connection with the unsolved problem of finding an intrinsic characterization of those functions which are derivatives. Theorem: Let h map the complete metric space S into the metric space T, and let G be the graph of h in the topological product space $S\times T$. Then h is neighborly at every point of S if and only if the set of points of $S\times T$ of the form $(x, h(x))$, where h is continuous at x, is dense in G. (Received October 2, 1956.)

78. J. C. C. Nitsche: *Remark on harmonic mappings.*

Let $x=\text{Re } F(\zeta)$, $y=\text{Re } G(\zeta)$, $z=x+iy$, $\zeta=\xi+i\eta$ be a one-to-one harmonic mapping of the unit disc $\Gamma[|\zeta|<1]$ onto the unit disc $C[|z|<1]$ leaving the origin fixed. (Without loss of generality one can assume $F(0)=G(0)=0$.) In C there exists a function $\phi(x, y)$ such that $\phi_{xx}=|G'|^2\cdot[\text{Im } F'G']^{-1}$, $\phi_{xy}=-[\text{Re } F'G']\cdot[\text{Im } F'G']^{-1}$, $\phi_{yy}=|F'|^2\cdot[\text{Im } F'\bar{G}']^{-1}$ and $\phi_{xx}\phi_{yy}-\phi_{xy}^2=1$ (cf. a preceding abstract and K. Joergens, Math. Ann. vol. 129, Lemma 4). According to a theorem of H. Lewy (Bull. Amer. Math. Soc. vol. 42) $\text{Im } (F'\bar{G}')=x_\xi y_\eta-x_\eta y_\xi\neq 0$ in Γ. The transformation $w=u+iv$, $u=x+\phi_x$, $v=y+\phi_y$ yields a one-to-one mapping of C onto a domain of the w-plane containing at least the interior of a circle with radius 1 (cf. Techn. Rep. No. 54, 1956, Stanford Univ. and H. Lewy, Trans. Amer. Math. Soc. vol. 41 (1937) pp. 365–374). Now observe that w is an analytic function of ζ and $dw/d\zeta=F'(\zeta)+iG'(\zeta)$. Thus because of Schwarz's lemma, at $\zeta=0$, $|dw/d\zeta|\geqq 1$ and $|F'|^2+|G'|^2\geqq|F'+iG'|^2/2\geqq 1/2$. Since $|F'|^2+|G'|^2=x_\xi^2+x_\eta^2+y_\xi^2+y_\eta^2$, one therefore gets the inequality $(x_\xi^2+x_\eta^2+y_\xi^2+y_\eta^2)_{\xi=\eta=0}\geqq 1/2$. The bound 1/2 is an improvement of the bound $2-8(\sum_{n=2}^{\infty} n^{-2})/\pi$ given by E. Heinz (see E. Heinz, Goettinger Nachr. Math.-Phys. Kl., IIa, 1952). Prof. H. Hopf kindly has communicated to the author another very simple proof of the bound 1/2. (Received October 3, 1956.)

79. Mishael Zedek: *Infra- (n, s)-polynomials on a real set.* Preliminary report.

The terminology and notation of Walsh and Zedek's *On generalized Tchebycheff polynomials*, Proc. Nat. Acad. Sci. U.S.A. vol. 42 (1956) is used. If $I_n^s(z)\equiv z^n+A_1z^{n-1}+\cdots+A_jz^{n-s}+a_{s+1}z^{n-s-1}+\cdots+a_n$ is an infra-(n, s)-polynomial belonging to a real compact set E in the complex plane consisting of at least $n-s$ points, then for any set of $s+1$ zeros of $I_n^s(z)$ not on the convex hull $C(E)$ of E, either the sum of angles subtended by E at the subset of zeros which are above the real axis, or the sum of the angles corresponding to the zeros below the real axis, or both, are at least π. If the prescribed coefficient A_1 in $I_n^1(z)$ is nonreal, then all the zeros of $I_n(z)$ lie on $C(E)$ or on the same side of the real axis as $-A_1/n$. In fact, $n-1$ of them lie on a semi-circle having the convex hull $C(E)$ as its diameter. For $I_n^2(z)$ it may be shown that if A_1 is real then $I_n^2(z)$ has no real zeros outside $C(E)$, unless A_2 is also real, in which case $I_n^2(z)$ may have a real zero there only when all its zeros are real. If A_1 is not real and has, say, a negative imaginary part, then $n-2$ of the zeros of $I_n^2(z)$ are in the union of two discs. $C(E)$ is the diameter of one of them and is also a chord of the other disc subtending an angle of $2\pi/3$ at its center. The center of the larger disc is above the real axis. (Received October 4, 1956.)

Applied Mathematics

80. J. B. Rosen: *Nonlinear programming. The gradient projection method.*

Let R be a bounded n-dimensional convex region defined by the constraint inequalities $\lambda_i\equiv\sum_{j=1}^{n} n_{ij}x_j-b_i\geqq 0$, $i=1, 2, \cdots, k$ and $\lambda_{k+j}\equiv x_j\geqq 0$, $j=1, 2, \cdots, n$.

Let $F(x)$ be a real-valued function of class $C^1 \in R$. Given an arbitrary starting point $x^0 \in R$, the Gradient Projection method will find a point $x_{max} \in R$ (interior or on the boundary B) for which $F(x)$ has a maximum (or local maximum). The method follows the gradient $g(x) \equiv \nabla F(x)$ by a specified stepwise procedure and will converge to x_{max}. For $x_{max} \in B$ it is necessary to choose at each step a linearly independent subset of $l<n$ of the hyperplanes $\lambda_i=0$, $i=1, 2, \cdots, k+n$. The projection, ${}_lg$, of g on the intersection of these l hyperplanes is then followed a specified distance. This stepwise procedure is continued until x_{max} is reached where either $|{}_lg|=0$, or for every choice of $l<n$ hyperplanes, a step in the direction of ${}_lg$ violates at least one constraint. The latter applies if x_{max} is at a vertex of R. The method is based on an algorithm for choosing the subset of l hyperplanes and a recursion formula for an $n\times n$ projection matrix ${}_lP$ such that ${}_lg={}_lPg$. The recursion formula is ${}_{l+1}P={}_lP-({}_lu_{l+1})\cdot({}_lu'_{l+1})$ where ${}_lu_{l+1}={}_lPn_{l+1}/|{}_lPn_{l+1}|$, ${}_lu'_{l+1}$ is its transpose and n_{l+1} is the normal to the hyperplane $\lambda_{l+1}=0$. It is proved that the method will find x_{max}, and an estimate is given showing that computation time is proportional to $n^2(k+n)$. A procedure is given for finding a starting point $x^0 \in B$ if no starting point $\in R$ is known. (Received July 27, 1956.)

Logic and Foundations

81. Leon Henkin: *Cylindrical algebras of dimension* 2. Preliminary report.

Tarski and Thompson have defined the notion of an *α-dimensional cyclindrical algebra with diagonal elements* (CA_α), Bull. Amer. Math. Soc. vol. 58 (1952) p. 65, and raised the question of representing these abstract structures as *proper* algebras. For dimension 2, CA_2's are now found which are not isomorphic to any proper CA_2. For a CA_2 to be representable it is necessary and sufficient that it satisfy the following two identities: $C_\eta - C_\xi(C_\eta x\cdot -d_{\xi\eta})\cdot C_\xi(x\cdot y)\cdot C_\xi(x\cdot -y)=0$, for $\xi, \eta=0, 1, \xi\neq\eta$. These identities are satisfied in every CA_3, so that a CA_2 is representable if and only if it can be neatly imbedded in a CA_3. The notion of a *neat* imbedding is defined on p. 40 of the author's monograph *La structure algébrique des théories mathématiques* (Paris, 1956). On p. 36 of this work it is erroneously stated that Tarski and Chin have shown every CA_2 to be representable. The Tarski-Chin result holds only for algebras without diagonal elements (Bull. Amer. Math. Soc. vol. 54 (1948) pp. 80–81). (Received October 2, 1956.)

82. Richard Montague: *Independently axiomatizable theories.*

See Abstract 83 which follows and Bull. Amer. Math. Soc. Abstract 62-6-783. *Definition.* Let (Con u) be the standard formula of Peano's arithmetic expressing the consistency of the formula u. Then the theory T is *uniformly reflexive* if Peano's arithmetic, together with all the sentences (Con Δ), where Δ is the Gödel-numeral of a valid sentence of T, is relatively interpretable in T. *Theorem* 1. If T is recursively axiomatizable and uniformly reflexive, then T is independently axiomatizable. *Theorem* 2. The following theories satisfy the hypothesis of Theorem 1: (a) general set theory; (b) Peano's arithmetic; (c) Zermelo-Fraenkel set theory; (d) all recursively axiomatizable extensions without new constants of (b) and (c). *Theorem* 3. Let T be a theory with a recursive axiom system B; if T is independently (but not finitely) axiomatizable, then there is a recursive function F such that whenever ϕ is a conjunction of finitely many axioms of B, $F(\phi)$ is a valid sentence of T which is not derivable from ϕ. Theorem 1 uses the "general theorem" of Abstract 83 of which Theorem 3 is the converse. Theorem 2 (b) is due to Mostowski. (Received October 24, 1956.)

83. Alfred Tarski: *The existence of independent axiom systems for Peano's arithmetic.*

For terminology see Tarski, Mostowski and Robinson, *Undecidable theories.* The following question was communicated to the author by Kreisel: *Is theory* $\mathfrak{P}$ *(Peano's arithmetic) independently axiomatizable,* i.e., is there a recursive axiom system A for $\mathfrak{P}$ such that no axiom in A is derivable from the remaining axioms? *The answer to this question is affirmative.* It can be obtained with the help of the following general theorem: *Let* $\mathfrak{T}$ *be a theory with a recursive axiom system* B; *assume that there is a recursive function* F *(on and to expressions of* $\mathfrak{T}$*) such that whenever* Φ *is a conjunction of finitely many axioms of* B, $F(\Phi)$ *is a valid sentence of* $\mathfrak{T}$ *which is not derivable from* Φ. *Then* $\mathfrak{T}$ *is independently (but not finitely) axiomatizable.* If $\mathfrak{P}$ is taken for $\mathfrak{T}$, the existence of a function F satisfying the hypothesis of this theorem is assured, e.g. by a construction which Ryll-Nardzewski (Fundamenta Mathematicae, vol. 39) applied in proving that $\mathfrak{P}$ is not finitely axiomatizable. For related results see the Abstract 82 of Montague. (Received October 24, 1956.)

Statistics and Probability

84. Herman Rubin and H. G. Tucker (p): *Estimating the parameters of a differential process.*

Let the value of a differential process at time t be $X(t)$. It is desired to estimate the parameters of the process from observation of a sample function over a fixed interval. One method of estimation is to estimate the expected number of jumps in any size range by the actual number, and if the sizes of the jumps are $y_1, \cdots, y_n, \cdots$, the variance of the normal component is $\int (dX(t))^2 - y_i^2$. The trend term is estimated by $X(t) - X(0) - \sum y_i^3/(1 - y_i^2)$. Another method of estimation, which with probability one yields the same estimate, and which is adaptable to discrete observations, is based on the general solution of the central limit problem. The estimate is sufficient, and the estimate of the jump density is unbiased. (Received October 5, 1956.)

Topology

85. J. M. Gary: *Higher order cyclic elements.*

Some results of G. T. Whyburn (Amer. J. Math. vol. 56 (1935) pp. 133–146) concerning higher order cyclic elements are generalized to arbitrary compact spaces. Let M be a compact Hausdorff space and E a closed set in M containing all the $(r-1)$-dimensional cyclic elements. Then the main theorem shows the mapping $i^*: H_r(E) \to H_r(M)$, induced by inclusion, to be an isomorphism onto. Some results and examples concerning the relation between r-monotone mappings and r-dimensional cyclic elements are obtained. (Received October 3, 1956.)

86. L. E. Ward, Jr.: *Mobs, trees, and fixed points.*

Definitions of terms and symbols are given in the author's papers, Proc. Amer. Math. Soc. (1954) pp. 144–161 and pp. 992–994. A compact Hausdorff space X is a *generalized tree* if and only if X admits a continuous order-dense partial, order such that $L(x) \cap L(y)$ is a nonempty chain for each $x, y \in X$ and such that each continuum Y contained in X contains a unique minimal element. A number of characterizations of trees and generalized trees are given in terms of mobs, partially ordered spaces, and purely topological notions. It is proved that the generalized trees have the fixed point property; this is a generalization of a recent result of K. Borsuk (Bull. Acad. Polon. Sci. (1954) pp. 17–20.) (Received September 20, 1956.)

87. R. L. Wilder: *Some consequences of a method of proof of J. H. C. Whitehead.*

By abstraction and extension of a method of proof used by J. H. C. Whitehead [*Note on the condition n-colc*, to appear soon in Mich. Math. J.], two lemmas concerning dual inverse and direct systems of vector spaces are obtained which lead to (1) an affirmative solution of Problem 2.1, p. 381 of my Colloquium book [Amer. Math. Soc. Colloquium Publications vol. 32 (1949)], under quite general conditions, (2) a solution of a problem left unsettled by S. Kaplan in his dissertation [Trans. Amer. Math. Soc. vol. 62 (1947) pp. 248–271], and (3) a proof that for any point x of a locally compact space S such that x has a well-ordered (by inclusion) cofinal set of neighborhoods in S, the relation $p^n(x, S) = p_x^n(S)$ holds for all n, in contradiction of certain conclusions of Alexandroff concerning these numbers [Annals of Math. vol. 36 (1935) pp. 1–35; see Examples 1, 2, 3 and 5, p. 23]. (Received October 2, 1956.)

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