# RESEARCH PROBLEMS

## 1. Richard Bellman: *Dynamic programming.*

Solve

$$f(x, y) = \text{Max} \begin{bmatrix} p_1[r_1x + f((1 - r_1)x, y)], \\ q_1[s_1y + f(x, (1 - s_1)y)], \\ p_2[r_2x + s_2y + f((1 - r_2)x, (1 - s_2)y)] \end{bmatrix}, \qquad x, y \geqq 0,$$

where $0 < p_1, p_2, q_1, r_1, r_2, s_1, s_2 < 1$; see Proc. Nat. Acad. Sci. U.S.A. vol. 39 (1953) pp. 1077–1082; vol. 38 (1952) pp. 716–719. (Received September 23, 1954.)

## 2. Richard Bellman: *Probability theory.*

Let $\{Z_k\}$ be a sequence of random matrices having a common probability distribution, say $p$, of being $A$ and $(1-p)$ of being $B$, where $A$ and $B$ are two matrices having all positive elements. Let $X_N = \prod_{k=1}^{N} Z_k$, and $x_{ij}(N)$ be the $ij$th element in $X_N$. Determine the limiting distribution of $\log x_{ij}(N)$, suitably normalized. See Proc. Nat. Acad. Sci. U.S.A. vol. 39 (1953) and Rand Paper 398, to appear in a forthcoming issue of Duke Math. J. (Received September 23, 1954.)

## 3. Richard Bellman: *Analysis.*

Let $\prod_{k,l=0}^{\infty} (1 - x^k y^l t)^{-1} = \sum_{n=0}^{\infty} q_n(x, y)t^n$. Obtain a formula for $q_n(x, y)$. (Received September 23, 1954.)

## 4. Richard Bellman: *Number theory.*

Let $f(x)$ be an irreducible polynomial with integer coefficients and the property that $f(x) > x$ for $x \geqq a$. Prove that the sequence $\{x_n\}$ defined by the recurrence relation $x_{n+1} = f(x_n)$, $x_0 = a$, an integer, cannot represent primes for all large $n$. See Bull. Amer. Math. Soc. vol. 53 (1947) pp. 778–779. (Received October 2, 1954.)

## 5. Richard Bellman: *Number theory.*

Let $x$ be a rational number greater than one, and let $[y]$ denote, as customary, the greatest integer contained in $y$. Prove that $[x^n]$ cannot be prime for all large $n$. (Received October 2, 1954.)

## 6. Richard Bellman: *Number theory.*

Prove that $\sum_{n=1}^{N} d(n^3+2) \sim cN \log N$ as $N \to \infty$. See Duke Math. J. vol. 17 (1950) pp. 159–168. (Received October 2, 1954.)

## 7. A. D. Wallace: *Manifolds with multiplication.*

Let $M$ be a compact connected manifold without boundary and provided with a continuous associative multiplication such that $MM = M$. Does the following hold: either (i) $M$ is a group or (ii) $xy = y$ for each $x$, $y$ or $xy = x$ for each $x$, $y$? It is known that (i) holds if we replace "$MM = M$" by "there is a two-sided unit"; see Summa Brasil. Math. vol. 3 (1953) pp. 43–55. (Received October 4, 1954.)

## 8. A. D. Wallace: *Fixed points for topological lattices.*

A topological lattice is a Hausdorff space $X$ together with two maps (continuous functions) $\wedge$: $X\times X\to X$ and $\vee$: $X\times X\to X$ satisfying the usual conditions. Let $X$ be a compact connected topological lattice. (1) Does $X$ have the fixed point property? (2) If $X$ is also metric, is $X$ an absolute retract in the sense of Borsuk? It is clear that (2) implies (1) when $X$ is metric. Easy examples show that (2) fails if $X$ is not metric. These conjectures are supported by numerous special cases (unpublished) as well as by the fact that $X$ has to be trivial in the sense of cohomology for any coefficient group; see Summa Brasil. Math. vol. 3 (1953) pp. 43–55. (Received October 4, 1954.)

## 9. A. D. Wallace: *Two problems on topological semi-groups.*

Let $S$ be a clan. That is, $S$ is a compact connected Hausdorff space together with a continuous associative multiplication with two-sided unit. (1) If $S$ is finite-dimensional and a homogeneous space, is $S$ a group? This is known to hold if also $S$ is a manifold (Summa Brasil. Math. vol. 3 (1953) pp. 43–55) or if $S$ is indecomposable (Math. J. Okayama Univ. vol. 3 (1953) pp. 1–3). (2) Let $K$ be the smallest nonvoid subset of $S$ such that $SK\subset K\supset KS$. If $S$ is an AR (ANR), is $K$ an AR (ANR)? It is easy to affirm this if $K$ meets the center of $S$ (using an unpublished result of R. J. Kock) since $K$ is then a retract of $S$. In this case $K$ is a group and hence is a zero for $S$ if $S$ is an AR. (Received October 4, 1954.)

## 10. A. D. Wallace: *Differentiability of continuous multiplications.*

Let $S$ be a clan, that is, let $S$ be a compact connected Hausdorff space together with a continuous associative multiplication with two-sided unit. Let $S$ be topologically contained in $R^n$, let $u$ be the unit of $S$, let $H(u)$ be the maximal sub*group* of $S$ containing $u$ (Anais Acad. Brasil. Ci. vol. 25 (1953) pp. 335–336), and let $F$ be the boundary of $S$ relative to $R^n$. It is known (Math. J. Okayama Univ. vol. 3 (1953) pp. 23–28) that $H(u)\subset F$. If $H(u)=F$, it follows from an unpublished result of R. H. Bing, together with well-known results on topological groups, that $H(u)$ is a Lie group. Assume that this is so. (1) Can the multiplication be assumed differentiable on all of $S$? (2) If $T$ is the quotient space of $S$ mod $H(u)$, is $T$ a differentiable manifold with boundary? (Received October 4, 1954.)

## 11. A. D. Wallace: *An addition theorem for cohomology.*

Let $X$ be a compact Hausdorff space, let $X=X_1\cup X_2$ with $X_1$ and $X_2$ closed, and let $H^n$ be the $n$-dimensional cohomology group over a fixed abelian group. Suppose that $h\in H^n(X_1\cap X_2)$ is such that $i^*h=0$ if $i$: $A\to X_1\cap X_2$ is the inclusion map and $A$ is any closed proper subset of $X_1\cap X_2$. Assume also that $h$ is not extendable to $H^n(X_1)$ but is extendable to $H^n(B)$ if $B$ is a closed proper subset of $X_1$ which includes $X_1\cap X_2$. If, finally, a similar condition holds for $X_2$, is it true that $H^{n+1}(X_1\cap X_2)\neq 0$? An analogue of this result is known to hold for homology and is easy to prove. (Received October 4, 1954.)

## 12. Dieter Gaier: *A problem on entire functions.*

Let $f(z)$ be an entire function with $|f(z)|\leqq Ae^{B|z|}$, and assume that $\lim f(z)=0$ for $z\to\infty$ along a path $C$. Is it true that then also $\lim f'(z)=0$ for $z\to\infty$ along $C$? The case that $C$ is a straight line was considered in Math. Zeit. vol. 58 (1953) p. 454. (Received October 18, 1954.)