coverage of the classical theory of Riemann surfaces, the last chapter turns to the subject of open Riemann surfaces. Since this is an extremely active subject at the present time, it is impossible to give any sort of definitive survey. However Nevanlinna does manage to introduce the reader to the principal problems in the field and to give an idea of some of the methods available.

This book arose out of the author's lectures at the University of Helsinki and is admirably suited for anyone who wishes to learn about Riemann surfaces. The author always has the present work in open Riemann surfaces in mind, so that the reader will find that he is prepared for the literature in this field.

H. L. ROYDEN

Lezioni sulle equazioni a derivate parziali. By F. G. Tricomi. Editrice Gheroni Torino, 1954. 4+484 pp.

This book furnishes an excellent introduction to the rapidly expanding theory of partial differential equations, written in the author's usual lucid and interesting style.

The work is divided into five parts. The first part, consisting of one hundred and four pages, presents a rapid but thorough summary of classical analytic tools required in the remainder of the book. The theory of integral equations, the gamma function, the hypergeometric function, the Legendre and Bessel functions are all treated. This part is well worth reading on its own.

The second part, consisting of seventy-five pages, is devoted to a discussion of the theory of characteristics for equations of the first and second order. It includes a section devoted to the Hamilton-Jacobi theory and its connection with the calculus of variations.

The third part, one hundred pages, is devoted to equations of hyperbolic type. Various classical approaches, such as those of Laplace and Riemann, are presented, and there is large section on the movement of a compressible fluid.

The fourth part, ninety-five pages, treats the equations of elliptic type. The classical techniques are given, together with a discussion of more modern methods based upon difference equations, and numerical methods such as the "relaxation" method of Southwell. A section on incompressible fluids is included.

The fifth and concluding part is devoted to equations of parabolic type and equations of mixed type. The greater part of this section is concerned with equations of mixed type, a topic investigated in great detail by Tricomi in 1923, and which in recent years has become of great importance in the study of transonic flow. The applications to this theory are given in several sections.

Two very useful features are the ample references to both quite recent and classical papers and the eighty or so representative problems gathered at the ends of the various sections.

The book is particularly to be recommended to anyone intending to work in the mathematical theory of hydrodynamics or aerodynamics.

RICHARD BELLMAN

Infinite abelian groups. By I. Kaplansky. (University of Michigan Publications in Mathematics, no. 2.) Ann Arbor, University of Michigan Press, 1954. 5+91 pp. $2.00.

The theory of finite and infinite abelian groups is comparatively rich in structure theorems and these form the central theme of Kaplansky's excellent monograph. A satisfactory characterization is available, and presented here, for the groups in the following classes of abelian groups: finite groups [Frobenius-Stickelberger], torsion groups with bounded order, finitely generated groups, countable torsion groups [Ulm], groups with division, countably generated torsionfree modules over complete discrete valuation rings [Kaplansky]. Direct sums of cyclic groups are naturally prominent in this discussion, since the groups in the classes mentioned are either direct sums of cyclic groups or direct sums of cyclic groups and groups with division or else contain direct sums of cyclic groups as essential building blocks. Thus one finds here the theorems of Prüfer and Kulikoff, characterizing certain direct sums of cyclic groups, and their application proving that every subgroup of a direct sum of cyclic groups is itself a direct sum of cyclic groups.

Various structures may be derived from an abelian group. The ring of endomorphisms is known to reflect particularly faithfully the properties of the original group; and the author proves for a large class of groups that they are completely determined by their endomorphism rings—this is one of many instances where results in this work go beyond what had been known before. Similarly the theory of characteristic subgroups has been developed considerably further than had been done by the author's predecessors.

The first eleven chapters lead up to Ulm's theorem which is proved with admirable simplicity. This is no mean task, considering the difficulty of Ulm's original proof and of Zippin's simplified version thereof. Operators make their first appearance in the 12th chapter, quite rightly in our opinion, since in a large part of the theory of