THE NOVEMBER MEETING IN LOS ANGELES

The five hundred ninth meeting of the American Mathematical Society was held at the University of California, Los Angeles, on Saturday, November 27, 1954. Attendance at the meeting was approximately 95, including 80 members of the Society.

By invitation of the Committee to Select Hour Speakers for Far Western Sectional Meetings, Professor Edmund Pinney, of the University of California, Berkeley, delivered an address entitled *Nonlinear differential equations*. Professor Pinney was introduced by Professor H. F. Bohnenblust.

The sessions for contributed papers were presided over by Professor Robert Steinberg and Dr. G. E. Forsythe.

Abstracts of papers presented at the meeting follow. Abstracts whose titles are followed by "*t*" were presented by title. In the case of joint authorship, the name of that author who presented the paper is followed by "(p)." Mr. Rosenblum was introduced by Professor F. Wolf and Mr. Moore by Dr. G. W. Evans II.

ALGEBRA AND THEORY OF NUMBERS

143. Harvey Cohn: *Approach to Markoff's minimal forms through modular functions*. II.

In an earlier preliminary report (Bull. Amer. Math. Soc. Abstract 59-4-443), the author outlined a formal procedure which can now be referred entirely to the fundamental domain of a modular function. Consider a fundamental domain D of genus 1 bounded by four geodesic arcs in the upper half plane with vertices on the boundary and subject to the following conditions: vertices are rational, transformations of the group are integral unimodular. Then Markoff's forms are associated with the fixed points of substitutions connecting the opposite sides. The chain of generating operations of the forms is nothing else but the (modular) change of basis operations. The domain D does not correspond to any congruence subgroup (in fact the linear substitutions associated with D include "all" denominators). Chains of analogous forms can be generated, however, corresponding to the fundamental domains of genus 1 for congruence subgroups such as modulo 11, 17, etc. (Received October 8, 1954.)

144. Alfred Horn: *A characterization of unions of linearly independent sets*.

Let X be a vector space of finite or infinite dimension over any division ring. Let S be a subset of X and let k be a positive integer. If for any finite subset T of S we have $|T| \leqq k \cdot \text{rank } T$, where $|T|$ is the cardinal of T, then S can be divided into k linearly independent sets. This confirms a conjecture of K. F. Roth and R. Rado. If k is an infinite cardinal and $|T| \leqq k \cdot \text{rank } T$ for any subset T of S, then in general S cannot be divided into k linearly independent sets. (Received October 18, 1954.)

145t. Joachim Lambek: *Groups and herds.* Preliminary report.

A *herd* H is a set with a ternary operation f such that $fabb = a = fbba$ for all a, b in H; it is called *associative* if $ffabcde = fabfcde$ for all a, b, c, d, e in H. The congruence relations in any herd commute; this is known to imply the Jordan-Hölder-Schreier theorem for herds with a selected element. Every associative herd H, some element of which has been designated as 1, may be regarded as a group G with $a \cdot b = fa1b$, $a^{-1} = f1a1$; conversely H is obtained from G by defining $fabc = a \cdot b^{-1} \cdot c$; this correspondence is one-to-one. If any other element of H is chosen to serve as identity, a group isomorphic with G results. The subherds of H are the cosets of the subgroups of G; the group of automorphisms of H is the holomorph of G. The relation between associative herds and groups is analogous to that between affine spaces and vector spaces. (Received October 14, 1954.)

146. T. S. Motzkin: *Bounds relating to Hilbert's Nullstellensatz.*

Using a new proof of that theorem it is shown by simultaneous induction that if the polynomials $f_1, \cdots, f_r$ in $K[x]$ (or: in $K[x, y]$), of degree $\leqq d$, are $\neq 0, \cdots, 0$ for every $\xi = (\xi_1, \cdots, \xi_n)$, $\xi_1, \cdots, \xi_n$ in the algebraic closure of K (or: of $K(y)$), then $\sum f_i g_i = a$ for some $g_1, \cdots, g_r$ in $K[x]$ (or: $K[x, y]$) of degree $\leqq d_n \leqq (d_{n-1}+d)\delta_{n-1} - d$, $d_0 = 0$ (or: of degree $\leqq \delta_n - d$) with $a = 1$ (or: for some a in $K[y]$ of degree $\leqq \delta_n \leqq C_{n+d_n+d,n+1} - C_{n+d_n,n+1}$); here K is any given field, $x = (x_1, \cdots, x_n)$ and $y = (y_1, \cdots, y_m)$ are indeterminates, $K[\ ]$ and $K(\ )$ denote ring and field extension of K. All g_i but $r_0 = C_{n+d,d}$ may be chosen as 0; the intersectional (or Helly) dimension of the family of hypersurfaces of degree d in affine or projective n-space over an infinite field is $r_0 - 1$. (Received October 13, 1954.)

Analysis

147t. Stefan Bergman: *Stream function of a doublet in a subsonic flow.*

Using the results of [1] Trans. Amer. Math. Soc. vol. 62 (1947) p. 452 ff., the author constructs the stream function of a doublet for a subsonic flow. The modified stream function (see (2.13b) of [1]) $\psi^* = (\psi / \mathcal{H})$ satisfies in the pseudologarithmic plane the equation (1): $\psi^*_{Z\bar{Z}} + F(Z+\bar{Z})\psi^* = 0$, see (2.15b) of [1]. Here $Z = \lambda + i\theta$ where λ is a function of speed and θ the angle which the velocity vector forms with the positive x-axis (in the physical plane). By the transformation $\zeta = \zeta(Z) \equiv (Z - Z_0)^{1/2} + \zeta_0$ the Riemann surface R with the branch points at Z_0 is mapped into the ζ-plane. (1) goes over into (2): $\Psi^*_{Z\bar{Z}} + |dZ/d\zeta|^2 F[Z(\zeta) + \bar{Z}(\bar{\zeta})]\Psi^* = 0$. Let $\Psi^{*(L,1)}(\zeta, \bar{\zeta}, \zeta_0, \bar{\zeta}_0)$ be the fundamental solution of (2) (see (5.3a) of [1]) with the affix at $\zeta_0 = \alpha_0 + i\beta_0$. Differentiating $\Psi^{*(L,1)}$ with respect to α_0 one obtains a singularity $\Psi^{*(1,1)}$ of the first order at the point ζ_0. If further one replaces $(\zeta - \zeta_0)$ by $(Z - Z_0)^{1/2}$ and multiplies by H, one obtains a solution $\psi^{(1,1)}$ of (1) which becomes infinite at the branch point Z_0. It represents the stream function of a doublet. (Received November 27, 1954.)

148t. Stefan Bergman: *A formula for stream functions of subsonic flows around profiles of a certain type.*

Let p be the integral operator transforming analytic functions of a complex variable into solutions of the equation (2) of the previous abstract. The real and imaginary parts of $p[\zeta^n(Z)]$, $n=0, 1, 2, \cdots$, where $\zeta(Z)=(Z-Z_0)^{1/2}+\zeta_0$, represent a set of particular solutions of equation (1) of the previous abstract. They are defined on the Riemann surface of the function $(Z-Z_0)^{1/2}$. Orthonormalizing these functions with respect to a domain B bounded by segments of Re Z = const. and segments of Im Z = const., one determines the stream function ψ_0 so that $\psi=\psi^{(1,1)}+\psi_0$ becomes constant along the boundary of B. Here $\psi^{(1,1)}$ is the stream function representing a doublet (see the previous abstract). Then ψ represents the stream function of a flow around a body, bounded (in the physical plane) by segments of straight lines and by segments of free boundaries. (See also Proceedings of First U.S. Nat. Congress of Appl. Mechanics (1951/52) pp. 705 ff.) (Received November 27, 1954.)

149. F. H. Brownell: *Extended asymptotic eigenvalue distributions for plane domains with corners.*

Let D be a bounded, open, connected set in the plane with boundary B, and let λ_p, $0<\lambda_p\leqq\lambda_{p+1}$, be the eigenvalues of the membrane problem, $-\nabla^2u=\lambda u$ on D, $u=0$ on B. It is assumed that B is the disjoint union of $n+1$ Jordan curves (so D has n holes) and that except at a finite number of corners, where the angle of the tangent vector to B proceeding in the positive sense has a jump increase of α_j, $-\pi<\alpha_j<\pi$, the curvature $c(s)$ of B exists satisfying on the closed arcs between corners $|c(s)-c(s')|\leqq|s-s'|^\eta M$, $0<\eta\leqq 1$, s being arc length on B. Under these conditions over $\omega\geqq 1$, $\sum_{p=1}^{\infty} 1/\lambda_p(\lambda_p+\omega^2) = (\mu_2(D)/4\pi)(\ln(\omega^2)/\omega^2)+C/\omega^2+(l(B)/8)1/\omega^3)-[(1-n)/6+\sum_{\alpha_j} Q(\alpha_j)](1/\omega^4)+O(1/\omega^{4+\eta})$, which extends some results of Pleijel [Arkiv Mat. vol. 2 (1953) no. 26] applying to simply connected D ($n=0$) with infinitely differentiable boundary. The function $Q(\alpha)$ is continuous over $|\alpha|<\pi$, differentiable at $\alpha=0$ with $Q'(0)=0=Q(0)$, and is given explicitly as the sum of a series of computable integrals with convergence ratio $\leqq|\alpha|/\pi$. If B is polygonal, $O(1/\omega^{4+\eta})$ can be replaced by $O(e^{-\omega\rho})$, $\rho=r_0/16$, r_0 the minimum distance between corners. The above estimates yield [Brownell, Pacific Journal of Mathematics (1954)] the asymptotic distribution of λ_p as over $y\geqq 1$, $\sum_{\lambda_p\leqq y}; 1=N(y)=(\mu_2(D)/4\pi)y-(l(B)/4\pi)y^{1/2}+[(1-n)/6+\sum_{\alpha_j} Q(\alpha_j)]+R(y)$, $R(y)=\tilde{O}(y^{-\eta/2}\ln y)$, or $=\tilde{O}(y^{-r})$ for every $r>0$ if B is polygonal. These $\tilde{O}$ estimates on the remainder $R(y)$ have the sense that certain Gaussian averages of $R(y)$, which drop out oscillating parts, are of the indicated order. Analogous results for euclidean k-space, $k>2$, are readily obtained. (Received September 28, 1954.)

150. G. W. Evans, II, R. L. Brousseau, and Ralph Keirstead (p): *Instability considerations for various difference equations derived from the diffusion equation.*

Several methods of iterating the implicit difference equation $\alpha(U_{j+1}^{k+1}-2U_j^{k+1}+U_{j-1}^{k+1})/(\Delta x)^2=(U_j^{k+1}-U_j^k)/\Delta t$, where $U_j^k=U(j\Delta x, k\Delta t)$, for a solution of the differential equation $\alpha U_{xx}=U_t$ are described. The methods are applicable to one or more space dimensions and to cylindrical as well as Cartesian coordinates. A discussion of convergence of the iteration schemes and a comparison of solutions obtained by these methods with the analytic solution of a simple diffusion problem is included. (Received October 4, 1954.)

151. J. M. G. Fell: *A generalization of a theorem of Rellich.*

The author proves an infinite-dimensional generalization, applicable to second quantization of Rellich's Theorem (Gottingen Nachrichten, Math.-Phys. Klasse, 1946, pp. 107–115) on the uniqueness of operators satisfying the usual quantum-mechanical commutation relations. For each x in a Hilbert space H, let A_x be a (not necessarily bounded) linear operator on a second Hilbert space K, satisfying the algebraic laws: $A_{ax+by}=\bar{a}A_x+\bar{b}A_y$, $[A_x, A_y]=0$, $[A_x, A_y^*]=(y, x)$. Under convergence restrictions similar to those of Rellich, and requiring irreducibility, the author shows that to each orthonormal basis $\{x_r\}$ of H, there corresponds an orthonormal basis $\{F(n_1, n_2, \cdots)\}$ of K, where $n_1, n_2, \cdots$ ranges over all sequences of non-negative integers with $\sum n_i<\infty$; such that, for $y=x_r$, one has $A_y[F(n_1, \cdots)]=(n_r)^{1/2} \cdot F(n_1, \cdots, n_r-1, \cdots)$ and $A_y^*[F(n_1, \cdots)]=(n_r+1)^{1/2}F(n_1, \cdots, n_r+1, \cdots)$. Similar results are obtained with antisymmetric commutation rules. (Received October 13, 1954.)

152. C. J. A. Halberg, Jr. (p) and A. E. Taylor: *On the spectra of linked operators.* Preliminary report.

Let X, Y be complex linear spaces, Z a non-null complex linear space contained in both X and Y. Let X be a Banach space X_1, Y a Banach space Y_2 under the norms n_1, n_2 respectively. Let Z be a Banach space Z_N under the norm $N(z) =\max [n_1(z), n_2(z)]$. Let T_1, T_2 be bounded distributive operators on X_1, Y_2 respectively, such that $T_1z=T_2z\in Z$ when $z\in Z$. Operators satisfying these conditions will be said to be "linked." If, in addition, it is assumed that Z is dense in X_1, T_1 and T_2 will be said to be "linked densely with respect to X_1." Let T be the operator defined on Z_N by $Tz=T_1z$ and let $\rho(S)$ indicate the resolvent set of an operator S. Then the main result of this report is that, if T_1 and T_2 are linked densely relative to X_1 and $\rho(T_1)\cap\rho(T_2)\subset\rho(T)$, then any component in the spectrum of T_1 has a nonvoid intersection with the spectrum of T_2. The proof involves the integration of operator-valued functions around suitable contours in the complex plane. Special cases involving the sequence spaces l_p are considered. (Received October 13, 1954.)

153. C. H. Meng: *On ϵ-unitary operators.*

A linear bounded operator T on a Hilbert space H is called an ϵ-unitary operator if $\|T^*T-I\|\leqq\epsilon$. The adjoint of an ϵ-unitary operator can easily be shown to be an ϵ-unitary operator. For $\epsilon<1$, a unitary operator U can be constructed such that $\|T-U\|\leqq\epsilon$. The relationship between the spectrum of T and that of U is studied. In particular, a criterion that $e^{i\phi}$ lies in the spectrum of U is that there exists a sequence of unit vectors $\{x_n\}$ in H such that $\|(T-e^{i\phi}R)x_n\|\rightarrow 0$ as $n\rightarrow\infty$. (Received October 14, 1954.)

154. R. E. Moore: *Numerical approximation of multiple integrals.*

A general procedure is developed for "minimizing" the number of evaluations of the integrand (hence the time) necessary to obtain a numerical approximation of prescribed accuracy to a given multiple integral. The procedure is designed specifically for use in connection with digital computing machines. (Received October 8, 1954.)

155. Marvin Rosenblum: *Perturbation of the continuous spectrum and unitary equivalence.* Preliminary report.

Let H be a Hilbert space and let A and B be possibly unbounded self-adjoint operators in H such that $B-A=\sum_{j=1}^{\infty}\lambda_j(\cdot, \phi_j)\phi_j$, where the ϕ_j are orthonormal and

$\sum_{j=1}^{\infty} |\lambda_j| < \infty$. Suppose that A and B have resolutions of the identity E_x and F_x respectively, and for all f in H and $j=1, 2, \cdots$, $(E_x f, \phi_j)$ and $(F_x f, \phi_j)$ are absolutely continuous functions of x such that $d(E_x f, \phi_j)/dx$ and $d(F_x f, \phi_j)/dx$ belong to L_p for some fixed $p>1$. Theorem: Under the above conditions, it follows that B is unitarily equivalent to A. The theorem is proved by exhibiting a densely defined isometric operator U_0 that satisfies $BU_0 = U_0 A$ and extending U_0 to a unitary operator. The case $p=1$ is now under study. (Received October 13, 1954.)

156. Marvin Shinbrot: *A boundary value problem with a small parameter.*

The following boundary value problem is considered: (1) $\epsilon\ddot{x}+a(t)\dot{x}+b(t)x=0$, $x(0)=\alpha$, $x(1)=\beta$, where $\epsilon>0$ is a small parameter and $a(t)$ and $b(t)$ are of class C^1. Two situations are permitted: $a(t)\geqq\delta>0$ or else $a(t_0)=0$, $0\leqq t_0<1$ while $a(t)>0$, $t>t_0$, $a(t)<0$, $t<t_0$ (if $t_0=0$, this last condition is omitted, of course). If $t_0>0$, let $u_1(t)$ and $u_2(t)$ denote solutions of the "reduced equation" $a(t)\dot{u}+b(t)u=0$ having the initial values $u_1(0)=1$, $u_2(1)=1$; if $t_0=0$, let $u_2(t)$ be defined as before, but take $u_1(t)\equiv 0$. It is then shown that (1) possesses a unique solution provided that $u_1(t)$ is bounded as $t\rightarrow t_0-0$, $u_2(t)$ is bounded as $t\rightarrow t_0+0$, and ϵ is small enough. Furthermore, there is an $\eta=\eta(\epsilon)=o(1)$ as $\epsilon\rightarrow 0$ such that $x(t)\rightarrow\alpha u_1(t)$, $\dot{x}(t)\rightarrow\alpha\dot{u}_1(t)$ on $0\leqq t\leqq t_0-\eta$, $x(t)\rightarrow\beta u_2(t)$, $\dot{x}(t)\rightarrow\beta\dot{u}_2(t)$ on $t_0+\eta\leqq t\leqq 1$ and these modes of approach are uniform. It is noted that the proof admits generalization also to the case $a(0)=a(t_0)=0$, $0<t_0<1$. (Received September 7, 1954.)

157*t*. D. A. Storvick: *Extension of a theorem of Seidel to a class of meromorphic functions.*

Let $f(z)$ be meromorphic with bounded characteristic in $|z|<1$, and let the values which $f(z)$ assumes in $|z|<1$ lie in a domain G whose boundary Γ has positive capacity. Furthermore, let the radial limit values of $f(z)$ belong to Γ for almost all $e^{i\theta}$ on $|z|=1$. Such functions have been studied by O. Lehto [Ann. Acad. Sci. Fennicae, A. I., no. 160 (1953) pp. 1–15] and M. Tsuji [J. Math. Soc. Japan vol. 4 (1952) pp. 91–95]. It is shown that every point of Γ which is arcwise accessible from G is a radical limit value of such a function $f(z)$; this result extends a theorem of W. Seidel [Trans. Amer. Math. Soc. vol. 36 (1934) p. 208] for bounded analytic functions. (Received October 7, 1954.)

158*t*. František Wolf: *On scalar operators in Banach space.* Preliminary report.

Let U be a bounded operator in a reflexive Banach space, such that it satisfies, with all its polynomials and for all x, the relation $\|p(U)x\|^2\leqq M\|x\|\cdot\|p^2(U)x\|$ always with the same constant M. Then U is a scalar operator in the sense of Dunford (cf. Pacific Journal of Mathematics vol. 4 (1954) p. 332), i.e. there exists a spectral measure E, such that $f(U)=\int f(\theta)dE(\theta)$, and for every Borel set σ of the complex plane $\|E(\sigma)\|\leqq M$. By applying the above inequality successively to $p^2(U)$, $p^4(U)$, etc. and using the spectral mapping theorem one obtains $\|p(U)x\|\leqq M\|x\| \sup_{\theta\in\sigma(U)} p(\theta)$. From here, by a well-known theorem, we deduce the scalarity of U. The above condition was stated for the first time by F. V. Atkinson (Monatshefte für Mathematik vol. 53 (1949) p. 278) and used in the case of a completely continuous U to deduce in a different way what amounts to the scalarity of U. The above can be extended to unbounded operators if, for x, we use a convenient everywhere dense set of x in the domain of U. (Received October 13, 1954.)

Applied Mathematics

159t. Marvin Shinbrot: *Singular perturbations with a turning point.* Preliminary report.

Asymptotic expansions (as $\epsilon \to +0$) have been sought for the solutions of the differential equation $\epsilon\ddot{x}+a(t)\dot{x}+b(t)x=0$ in an interval $[0, T]$ in which $a(t)$, $b(t)$ are of class C^1. The function $a(t)$ is allowed to have a zero at a point $t=t_0$, $0 \leqq t_0 \leqq T$ (the "turning point"). Let $u_1(t)$ and $u_2(t)$ denote solutions of the "reduced equation" $a(t)\dot{u}+b(t)u=0$, satisfying the following conditions: if $t_0=0$, $u_1(t)\equiv 0$, $u_2(1)=1$; if $t_0=T$, $u_1(0)=1$, $u_2(t)\equiv 0$; if $0<t_0<T$, $u_1(0)=1$, $u_2(1)=1$. Then, the desired expansions have been found provided (i) $u_1(t)$ is bounded as $t \to t_0-0$ and $u_2(t)$ is bounded as $t \to t_0+0$; (ii) $|a(t)| \geqq c|t-t_0|^\gamma$, $c=$constant, $0 \leqq \gamma \leqq 1$. The generalization to the case where $a(t)$ has finitely many zeros in $[0, T]$ is trivial. Work is proceeding on more general situations; in particular, an attempt is being made to eliminate the condition $\gamma \leqq 1$. (Received September 7, 1954.)

Geometry

160t. T. K. Pan: *Centers of curvatures of a vector field.*

Let v be a vector field on a surface in an ordinary space. Along a curve C on S, v has several curvatures. In this paper, definitions of centers of various curvatures of v along C are given to include those of a curve on a surface as special cases. Relations among these centers are investigated. Most theorems on centers of curvatures of a curve on a surface are generalized; for example: (1) With respect to C at P which is neither the asymptotic line of v at P nor the indicatrix of v at P, the three centers of curvatures for P of v—center of absolute curvature, center of angular spread, and center of normal curvature—are collinear. (2) The centers of normal curvature for P of v along any two curves making equal angles with the line of curvature of v at P are coincident; the locus of centers of angular spread for P of a pencil of vector fields along C is a circle. (3) The locus of orthocenter of associate curvature of the orthogonal vector field of v with respect to a line of curvature of v is the edge of regression on the developable surface formed by straight lines on the orthogonal trajectories of v along the line of curvature of v. (Received October 11, 1954.)

161. F. A. Valentine: *Three point arcwise convexity.*

A convex arc is, by definition, an arc which is contained in the boundary of a plane convex set. A convex curve, as distinguished from a convex arc, is a closed connected portion of the boundary of a plane convex set. Definition 1. A set S is said to satisfy the three point arcwise convexity property if each triple of points $x \in S$, $y \in S$, $z \in S$ is contained in a convex arc belonging to S. *Theorem* 1. *Let S be a closed set in the plane which has at least three points. Then S has the three points arcwise convexity property if and only if it satisfies at least one of the following three conditions*: (1) *It is a closed convex set.* (2) *It is a convex curve.* (3) *It is a closed convex set except for one bounded convex hole, that is, it is obtained by deleting from a closed convex set a bounded open convex subset.* (Received October 13, 1954.)

Logic and Foundations

162t. R. M. Robinson: *Primitive recursive functions.* II.

This supplements *Primitive recursive functions,* Bull. Amer. Math. Soc. vol. 53 (1947) pp. 925–942, especially §7, Theorem 3. Let J, K, L be the Cantor pairing functions, defined by $2J(u, v) = (u+v)^2+3u+v$, $KJ(u, v) = u$, $LJ(u, v) = v$. It is now shown that all primitive recursive functions of one variable can be obtained from S and K (or from S and L) by repeated use of the formulas $Fx = Ax+Bx$, $Fx = BAx$, and $Fx = B^x 0$ to define new functions. Also, the scheme $Fx = Ax+Bx$ may be replaced by $Fx = J(Ax, Bx)$. The latter form proves useful in studying the arithmetical representation of recursively enumerable set. (Received October 4, 1954.)

Statistics and Probability

163. Donald Davidson and Patrick Suppes (p): *A finitistic axiomatization of subjective probability and utility.*

This paper is concerned with a simultaneous axiomatization of subjective probability and utility, which is based on the following six primitive notions: (1) a finite set K, ordinarily a set of things valued; (2) a binary relation P of preference whose field is K; (3) a finite set X of outcomes of a random mechanism; (4) a family $\mathcal{J}$ of subsets of X (the elements of $\mathcal{J}$ are the chance events for which a subjective probability is determined); (5) a distinguished chance event E^* whose subjective probability is determined as 1/2; (6) a quinary relation M such that for x, y, u and v in K and E in $\mathcal{J}$, $x,y\ M(E)\ u,v$ if and only if the individual in question is indifferent between (a) the prospect of x if E occurs and y if E does not occur and (b) the prospect of u if E occurs and v if E does not occur. The main result is that for every system $\langle K, P, X, \mathcal{J}, E^*, M\rangle$ satisfying the axioms there exists a unique subjective probability function s and a utility function ϕ unique up to a linear transformation such that (for x, y, u and v in K and E and F in $\mathcal{J}$): $x\,P\,y$ if and only if $\phi(x) \geqq \phi(y)$; $s(E)+s(\tilde{E}) = 1$; $s(\tilde{E}) \geqq 0$; $s(X) = 1$; if $E \subseteq F$ then $s(E) \leqq s(F)$; $x,y\ M(E)\ u,v$ if and only if $s(E)\phi(x) + s(\tilde{E})\phi(y) = s(E)\phi(u) + s(\tilde{E})\phi(v)$. For fundamental reasons unrestricted additivity of probability fails in this finitistic set-up; in fact $\mathcal{J}$ is closed under complementation but not under union. (Received October 14, 1954.)

164t. T. E. Harris: *Recurrent Markov processes.* Preliminary report.

Let X be a Borel-measurable subset of the real line and let m be a countably additive measure on the Borel subsets of X; $m(X)$ may be infinite, but m is assumed sigma-finite. Let $P(x, E) = \text{Prob}\ (x_{n+1} \in E \mid x_n = x)$ be the transition function of a Markov process defined on X. The process is called recurrent if $m(B) > 0$ implies that for all starting points x_0 the probability is 1 that $x_n \in B$ infinitely often. *Theorem.* If x_n is recurrent, there exists a sigma-finite measure π satisfying $\pi(E)$ $\int_X \pi(dx)P(x, E)$ for every Borel set E in X. If $\pi(X)$ is finite it can be normalized to give a stationary absolute probability distribution. The significance of the case $\pi(X) = \infty$ has been discussed by Harris and Robbins, Proc. Nat. Acad. Sci. U.S.A. vol. 39 (1953) pp. 860–864. (Received October 13, 1954.)

Topology

165t. C. E. Burgess: *Continua and certain types of homogeneity.*

Definitions of *nearly-homogeneous* point sets and *n-homogeneous* point sets were given in the author's paper *Some theorems on n-homogeneous continua*, Proc. Amer. Math. Soc. vol. 5 (1954) pp. 136–143. Let M be a nearly-homogeneous bounded plane continuum which separates the plane and has only a finite number of complementary domains. It is shown that (1) every indecomposable proper subcontinuum of M is a continuum of condensation of M and (2) M is a simple closed curve if it either is aposyndetic or is arcwise connected or contains a simple closed curve. (In papers cited in the author's paper mentioned above, F. B. Jones and Herman Cohen have presented similar results for homogeneous continua.) Let K be a compact metric continuum which is n-homogeneous, where $n>1$. It is shown that K is a simple closed curve if either some two of its points are not separated by any of its subcontinua or it is locally connected and separated by some set consisting of n points. (Received October 13, 1954.)

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