RESEARCH PROBLEMS

21. Richard Bellman: *Integral equations.*

Solve the integral equation $\lambda f(x) = \int_0^1 e^{-xy} f(y) dy$. (Received May 21, 1954.)

22. Richard Bellman: *Dynamic programming.*

Solve the functional equation $f(x) = \text{Max } (g(x) + f(ax), h(x) + f(bx))$, given that $0 < a, b < 1$; $h(x), g(x) > 0$; $h(0) = g(0) = 0$; $h'(x), g'(x) > 0$; $h''(x), g''(x) > 0$. (Received May 21, 1954.)

23. Richard Bellman: *Calculus of variations.*

Given the problem of determining y so as to maximize $J(y) = \int_0^T F(x, y) dt$, subject to the relation $dx/dt = G(x, y)$, $x(0) = c$, and the constraint $0 \leqq y \leqq x$; determine the conditions upon F and G which will ensure that the solution has the form $y = x$ in $[0, t_1)$, $0 < y < x$ in (t_1, t_2), $y = 0$ in $[t_2, T]$, where t_1 may be zero, t_2 may be t_1, and t_2 may be T. (Received May 21, 1954.)

24. Richard Bellman: *Stability theory.*

Necessary and sufficient conditions that a polynomial possess only roots with negative real parts were given by Hurwitz in the form of determinantal inequalities where the elements of the determinants are the coefficients in the polynomial. The problem of determining necessary and sufficient conditions that a matrix A possess only characteristic roots with negative real parts may be reduced to the above by calculating the characteristic polynomial of A. Can one avoid this complicated computation and obtain determinantal criteria directly in terms of the elements of A? (Received May 21, 1954.)