BOOK REVIEWS

Linear analysis. By A. C. Zaanen. New York, Interscience; Amsterdam, North-Holland; Groningen, Noordhoff, 1953. 7+600 pp. $11.00.

This book goes a long way (but not all the way) toward filling the gap in the mathematical literature caused by the fact that Banach's book has been out of date for several years. The book is divided into three parts: measure theory, operator theory, and integral equations. The overlap with the material in Banach's book is, of course, in the second part, which is the longest and most important one.

The writing is clear and well organized; the author is an excellent expositor. A conspicuous and pleasing feature is the quality and quantity of examples. Not only is there an adequate supply of exercises at the end of each chapter, but throughout the body of the book there are many detailed discussions of standard and non-standard examples: sequence spaces, the Orlicz generalization of L_p spaces, sequential transformations, integral kernels, etc. There is some emphasis, but not a disproportionate amount, on the author's own work on symmetrisable transformations and kernels.

Hilbert spaces receive much more attention here than in Banach, but the treatment is more from the point of view of Banach spaces than would be the case in a book devoted to Hilbert space. The spectral theorem is not proved.

An unusual aspect of the book is the author's explicitly stated desire to avoid use of the well-ordering theorem, because of its "controversial" nature. This, apparently, is not done in the intuitionistic spirit; the author's proofs are like all ordinary mathematical proofs, and, in particular, they make free use of the principle of excluded middle. The axiom of choice, at least for countably many choices from arbitrarily large sets, is used, more or less explicitly, several times.

As a consequence of the avoidance of transfinite methods, many standard theorems appear in the book either for separable spaces only, or else accompanied by the *assumption* of the validity of the Hahn-Banach extension theorem. Presumably as a further consequence of the same philosophy, weak *convergence* is systematically preferred to weak *topology*, and, for instance, the Tychonoff-Alaoglu theorem on the weak compactness of the unit sphere in a conjugate space appears in its sequential form only.

A fair idea of the material covered can be obtained from the titles of the seventeen chapters; they are as follows. (1) Point sets. Euclidean space. (2) Measure. Measurable functions. (3) Integration. (4) Additive set functions. (5) The Lebesgue spaces L_p and the Orlicz spaces L_ϕ. (6) Banach space and Hilbert space. (7) Bounded linear transformations. (8) Banach spaces of finite dimension. (9) Bounded linear transformations in Hilbert space. (10) Range, null space and spectral properties of bounded linear transformations. (11) Compact linear transformations. (12) Compact symmetrisable, self-adjoint and normal transformations in Hilbert space. (13) General theory of non-singular linear integral equations. (14) Integral equation with normal kernel. (15) Integral equation with a symmetrisable kernel, expressible as the product of a kernel of finite double-norm and a bounded non-negative function. (16) Integral equation with Marty kernel. (17) Integral equation with Garbe kernel or Pell kernel.

Despite its self-imposed limitations, the book contains so much material, and treats its topics so thoroughly, that it is a welcome addition to the literature of functional analysis; it is recommended as both a reference for the expert and a text for the student.

PAUL R. HALMOS

Principles of numerical analysis. By A. S. Householder. New York, McGraw-Hill, 1953. 10+274 pp. $6.00.

With the age of supersonic aircraft, hydrogen bombs, large automatic control systems, and so on, has come a large increase in the volume and importance of scientific computation. The procession of automatically sequenced digital computers following J. W. Mauchly's construction of the ENIAC during the war is providing an enormous capacity to solve these computing problems and others. But many users and programmers of these machines know relatively little mathematics, while mathematicians are often quite unaware of the mathematical literature on computing methods. Much of the newer literature is found only in journals of diverse fields, or in reports of various research projects.

To cope with the situation, here and there serious mathematicians have been formed into groups with numerically inclined physicists and others, in order to study computing methods and devise new ones. The mathematicians on such a team are likely to call themselves *numerical analysts.* But there has been no agreed definition of numerical analysis as these people use the words, and no standard reference work. Books by Milne, Scarborough, Hartree, and others are primarily oriented toward desk calculating machinery, and most are written for mathematical amateurs or undergraduate students.