

*Abstract set theory.* By A. A. Fraenkel. Amsterdam, North-Holland, 1953. 12+479 pp. \$10.00.

This book is essentially a translated, revised, and expanded edition of the first three chapters of the author's well-known German treatise (*Einleitung in die Mengenlehre*, Berlin, 1st ed. 1919, 3d ed. 1928). In the preface the author mentions that he is preparing another book (*Foundations of set theory*). The list of topics to be discussed in that book ("antinomies of the transfinite, axiomatic methods of basing set theory, logistic attitudes from *Principia Mathematica* to the present day, intuitionism and neo-intuitionism, axiomatics in general and metamathematics") indicates that *Foundations* will bear the same relation to the last two chapters of *Einleitung* as does the present book to the first three.

Reviewing the first edition of *Einleitung*, G. A. Pfeiffer (Bull. Amer. Math. Soc. vol. 27 (1921) pp. 333-334) registered a mild complaint against the author's prolixity. In his review of the third edition, T. C. Benton (Bull. Amer. Math. Soc. vol. 36 (1930) pp. 27-28) chides the author for relegating the discussion of the axioms of set theory to the end of the book (i.e., to the part not contained in the version under review now).

If *Einleitung* is prolix, the present book is much more so. The introduction and first three chapters of *Einleitung* cover 209 pages; the same material now covers 331. In every discussion the author's aim is completeness; he mentions all the examples, all the philosophical interpretations and misinterpretations, and all the references that could conceivably be called relevant. While this might decrease the usefulness of the book as a text, it can only add to its value as a reference.

The treatment is intentionally non-axiomatic; the author wants to steer a course between naive carelessness and axiomatic formalism. He chooses to do this by the formulation of certain "principles." The principles are, in fact, highly informal statements of well-known set-theoretic axioms. There are seven of them: extension, subset, pairing, union, infinity, power, and choice. The principle of subsets, for instance, reads as follows: "given a set  $S$  and a property  $\pi$  meaningful for the elements of  $S$ , there exists the set containing those elements of  $S$ , and only those, which possess the property  $\pi$ ."

In most respects there is no significant difference between the third edition of *Einleitung* and the present book. The subject matter, the style, the terminology, and the notation are all the same. (Sometimes this rigidity is mildly regrettable. Thus, for instance, union and intersection are still denoted by  $\Sigma$  and  $\Pi$ , and the power set of a set  $S$  is

denoted by  $US$ ; the  $U$ , presumably, stands for *Untermenge*.) Just as in the earlier edition, each section concludes with a useful collection of exercises.

A novel and laudable feature is the bibliography. This impressive list covers 137 pages; it is practically a separate publication in the same binding. (Possibly the publishers intend to make the bibliography and the main text available separately. The pages of the bibliography bear two numbers—one in continuation of the number system of the text and one independent of it.)

Readers not acquainted with *Einleitung* may gain an idea of the contents of the present book from the titles of its eleven sections; they are as follows. 1. Concept of set. Examples of sets. 2. The fundamental concepts. Finite and infinite. 3. Denumerable sets. 4. The Continuum. Transfinite cardinal numbers. 5. Ordering of cardinals. 6. Addition and multiplication of sets and cardinals. 7. Exponentiation of cardinals. The problem of infinitesimals. 8. Ordered sets. Similarity and order-types. 9. Linear sets of points. 10. General theory of well-ordered sets. Finite sets. 11. Ordinals and alephs. The well-ordering theorem.

PAUL R. HALMOS

*Introduction to the theory of functions of a complex variable.* By W. J. Thron. New York, Wiley, 1953. 10+230 pp. \$6.50.

Here at last is a rigorous book on complex variables in the English language. There has long been a need for such a text and the author set for himself the task of filling this gap, and without doubt he has succeeded to a remarkable degree. The book is written in the Satz-Beweis style of Edmund Landau, each item being carefully labeled: Axiom, Theorem, Proof, Remark, etc., and in the reviewer's opinion this is of course the only possible style, where a rigorous exposition is the object in view. The book is divided into 31 short sections, instead of the more conventional longer chapters, and these shorter sections induce in the reader a comfortable feeling that not too much sustained effort will be required to read the book.

As every young instructor learns when he first strides into a class room resolved to make everything crystal clear to his pupils, the number system must first be put on a solid foundation, and this is not easy. Thron comes directly to grips with problem, and the allied problem of the foundations of geometry, and the first fourteen sections (more than one-third of the book) are devoted to these and related topics, and lie outside the domain proper of the theory of functions of a complex variable. These section headings are as follows: