Introduction to the foundations of mathematics. By R. L. Wilder. New York, Wiley; London, Chapman and Hall, 1952. 14+305 pp. $5.75.

For a number of years R. L. Wilder has given a very successful course in Foundations of Mathematics at the University of Michigan. This course has come to be well known, and the present book has grown out of it. The book promises to be as successful as the course. It is sound, thorough, modern, and readable, and well suited as a textbook for a course in foundations. It will probably be required reading for many other courses, and in addition should be valuable as a reference work for anyone interested in the philosophy or foundations of mathematics.

The book is divided into two parts. Part I, on Fundamental Concepts and Methods of Mathematics, presents the reader with actual instances of materials and methods of modern mathematics related to the foundations. For example, here is a list of some of the topics included in the seven chapters of Part I: The axiomatic method, independence, completeness, and consistency of axiom systems, axioms for simple order and equivalence, the theory of sets, paradoxes, the axiom of choice, cardinal and ordinal numbers, transfinite induction, the Hamel basis, Zorn's lemma, the real number system, Dedekind cuts, the Peano axioms for the integers, the complex number system, groups, semigroups, rings, ideals, integral domains, fields, vector spaces, group theory applied to algebra and geometry, and topology.

Part II of the book, called Development of Various Viewpoints on Foundations, takes up the broad questions that have arisen naturally from a consideration of the sort of situation met with in the first part. The beginning chapter of Part II traces the history of foundational questions up to about the year 1908. The next three chapters are devoted to three of the principal schools of thought on foundations: the Frege-Russell or logistic school, the intuitionist or Brouwer school, and the formalist or Hilbert school. In the last chapter, on the cultural setting of mathematics, the author presents some of his own views.

A feature of the book that will add greatly to its usefulness as a text is the inclusion, at the end of each chapter of Part I, of a list of suggested readings, followed by an extensive list of good problems. The eleven pages of bibliography, and the index of topics and of names, are other convenient features. Practically every topic the beginning student of foundations should be familiar with appears somewhere.

The chapter headings are: Chapter I, The Axiomatic Method;

Chapter II, Analysis of the Axiomatic Method; Chapter III, Theory of Sets; Chapter IV, Infinite Sets; Chapter V, Well-Ordered Sets; Ordinal Numbers; Chapter VI, The Linear Continuum and the Real Number System; Chapter VII, Groups and Their Significance for the Foundations; Chapter VIII, The Early Developments; Chapter IX, The Frege-Russell Thesis: Mathematics an Extension of Logic; Chapter X, Intuitionism; Chapter XI, Formalism; Chapter XII, The Cultural Setting of Mathematics.

More than enough material has been presented for a one-semester course, with the idea that some of it may be omitted. For this reason it would be unfair to call attention to other important topics that might have been included, but were not. As a matter of fact, this reviewer can think of but few such. There is but little emphasis on the distinction between axioms and rules of procedure. It is not surprising that the Church calculi of lambda-conversion and the related combinatory logic of Curry, both studied also by Rosser and Kleene, do not appear, although other works of these authors are treated. There is very little about semantics. There also seems to be lacking any sense of crisis in the foundations. One can deduce from the statements appearing in the book that we have no logical formulations which are certainly adequate for analysis and also provably consistent, although we do have formulations adequate for analysis and possibly consistent, and others adequate for parts of arithmetic and analysis and provably consistent. Whether this situation is inevitable, or may be improved, is one of the great challenges of the present state of the subject. This aspect does not seem to be emphasized.

The author's presentation of the three chief schools of logistics, intuitionism, and formalism is very fair and complete. He does not take sides, and tries to make the best possible case for each viewpoint. His own opinions, expressed in the last chapter, do not seem to touch on controversial issues. Rather, he presents a sort of sociological view of the development of mathematics, as determined by the culture of the age. This viewpoint is so broad and tolerant that each reader can deduce from it what he wishes. It seems unlikely that it will stir up any controversy or recriminations.

The style throughout is light and easy. One exception is the statement on page 278: "What great man, if honest with himself, has not observed the passing chimney-sweep without remarking to himself, 'There, but for the "concatenation of events," go I.' " If this sentence had been expressed in the notation of symbolic logic and then analyzed, it would probably have been amended, or perhaps omitted.

Altogether we have here a very fine addition to mathematical literature.

ORRIN FRINK

On the metamathematics of algebra. By A. Robinson. (Studies in Logic and the Foundations of Mathematics.) Amsterdam, North-Holland, 1951. 9+195 pp. 18.00 fl.

The purpose of this book is to show how the methods of symbolic logic may be applied to derive new results in algebra. This is a somewhat novel idea, since usually symbolic logic has been used to strengthen the foundations, rather than to add to the superstructure, of mathematical systems. The results obtained are in the nature of metatheorems rather than theorems; they deal with entire classes of algebraic systems rather than with one system at a time.

Some typical results are the following: 1. Any theorem formulated in the restricted calculus of predicates (in terms of equality, addition, and multiplication) which is true for all commutative fields of characteristic 0, is true for all commutative fields of characteristic $p > p_0$, where p_0 is a constant depending on the theorem. 2. Any theorem of the restricted calculus of predicates which is true for all non-Archimedean ordered fields is true for all ordered fields. 3. Any theorem of the restricted calculus of predicates which is true for the field of all algebraic numbers is true for any other algebraically closed field of characteristic 0.

A feature of the method is the following: since the results require only that the algebraic systems dealt with have certain broad, general properties, they naturally suggest significant generalizations of various algebraic notions. Examples of concepts generalized are: that of algebraic number, or a number algebraic with respect to a given commutative field; the notion of the polynomial ring obtained by adjoining n indeterminates to a commutative ring; and the concept of ideal. In general, the notions that may be handled by the author's method are those capable of being formulated within the restricted predicate calculus.

A number of the results obtained are significant for symbolic logic as well as for algebra. For example, it is shown that every model of an axiomatic system formulated in the restricted calculus of predicates can be extended. Consequently such a model cannot satisfy an axiom of completeness in Hilbert's sense. Since the concept of an ordered field can be formalized within the restricted calculus of predicates while the concept of an Archimedean field does in fact possess a model which is complete in Hilbert's sense (namely the field of all