

“vector space” (although “algebra” is defined) or “quotient group” (although the phrase and the concept are ultimately used, without warning); definitions of abstract concepts that are given are almost never linked to one another (e.g., the underlying additive group of a ring or field is not mentioned); in fact, the only groups that are treated as such are groups of permutations or matrices.

Lastly, we should mention a third contrast between the present text and the one above. A fatal carelessness pervades the whole book, even when abstract concepts are only in the background. To give two examples, the reader is hard put to it to discover in any particular section whether “number” means real number or complex number (see especially the section on orthogonal matrices); linear dependence of a set of vectors is defined incorrectly (the phrase “scalars not all zero” is omitted) and so is linear dependence of one vector on a given set of vectors (the phrase “scalars not all zero” is included). This kind of carelessness finally leads the author into at least two real errors: (1) A gap in the proof of Wedderburn’s theorems, where the author uses without proof the existence of a unit element in a semi-simple algebra, the “justification” presumably being this blanket statement a few sections back: “If an algebra does not possess a modulus [unit element] then a modulus may be *adjoined*. . . . It will be assumed henceforth that if necessary a modulus has been adjoined.” (2) Theorem II, Chapter X, p. 156 is false. It reads: “If $F(\xi_1, \xi_2)$ is the extension field obtained by adjoining to F two roots ξ_1, ξ_2 of an irreducible equation in F , then the correspondence $\xi_1 \rightarrow \xi_2, \xi_2 \rightarrow \xi_1$ constitutes an automorphism of the extension field [over F].” A counter-example is provided by any cyclic extension field of degree > 2 .

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Linear transformations in n -dimensional vector space. An introduction to the theory of Hilbert space. By H. L. Hamburger and M. E. Grimshaw. Cambridge University Press, 1951. 10+195 pp. \$4.50.

Since the purpose of this book is clearly indicated by its title and subtitle, this review can consider first the order (quite reasonable) and clarity (good) with which the authors present their material.

The vector space V_n considered is always the space of sequences of n complex numbers. The scalar product, linear dependence, linear subspaces, orthogonal complements, and the algebra of linear transformations are discussed in Chapter I. Chapter II describes the elementary properties of Hermitian, normal, unitary, and projection operators and their eigenmanifolds. Chapter III derives the spectral

representation of a Hermitean H , locating the successive eigenvalues as maxima of (Hx, x) on sections of the unit sphere; this argument has a Hilbert space analogue only for completely continuous operators. After some study of resolvents and other functions of Hermitian operators, the chapter closes with extension of the preceding results to unitary and normal operators.

Chapter IV discusses principal manifolds, the minimal polynomial, and elementary divisors, gives the Jordan canonical form and Segre characteristic for a general linear transformation, and closes with a study of commutativity. Chapter V discusses the effect of introducing a new scalar product (Gx, y) , where G is positive definite. This study is needed to make up for the original concrete choice of V_n ; it is then possible to characterize the transformations with simple elementary divisors as those which are normal relative to some such scalar product.

Interesting historical notes, some referring to work of a century ago, show the authors' knowledge of the deep roots of their subject in the structure of classical mathematics. That their presentation is faithful to that same classical tradition may make the book easier for a student to begin, but it seems to this reviewer to make the secondary goal, the introduction of the reader to Hilbert space, so much the more difficult to reach in a small book.

This reviewer can (as the pre-publication reviewer for *Mathematical Reviews* could not), and therefore should, attempt to compare this book with *Finite dimensional vector spaces* by P. R. Halmos. The books overlap much more in subject matter than in attitude; Halmos acknowledges great indebtedness to von Neumann, whose name does not appear in bibliography or index of the book under review. A student unaccustomed, as so many of our undergraduates are, to axiomatic methods might profit more from this concrete and detailed study than from a surfeit of abstractions. On the other hand, Halmos's book, with its racy style and its steady slant toward Hilbert space, when contrasted with this formal, workmanlike, and detailed discussion, seems to offer one of the few examples of a paper-bound book suited better than its slick-paper competitor to the education of any student who has been bent to the appropriate axiomatic attitude.

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BRIEF MENTION

Mecanique des milieux continus et deformables. By M. Roy. Paris, Gauthier-Villars, 1950. Vol. 1, 22+198+166 pp.; vol. 2, 212+126+12 pp.