The reviewer feels that the appeal of this book will be to the more advanced and mature student and that it will be valued chiefly for the special topics of Chapters V–VIII now made conveniently available. Most beginning students will probably find the book too difficult and it may therefore prove unsuitable as a classroom text for a first course.

LOWELL SCHOENFELD

Ordinary non-linear differential equations in engineering and physical sciences. By N. W. McLachlan. Oxford University Press, 1950. 6+201 pp. $4.25.

In the preface to this work the author states: "owing to the absence of a concise theoretical background, and the need to limit the size of this book for economical reasons, the text is confined chiefly to the presentation of various analytical methods employed in the solution of important technical problems." It is therefore with forethought, and perhaps with malice aforethought, that the author has presented the mathematical theory in a form which is highly disorganized and which reveals ever so tellingly the inadequacy of present mathematics to explain what are now common experiences of the engineer and physicist.

The state of disorganization of the mathematics is such that one might conclude that there is no theory of differential equations and that all one can hope for in practice is that the differential equation encountered has been solved in the literature. Thus Chapter II, on *Equations readily integrable*, consists solely of the following examples: $y'=-x/y$; $y'=(x+y)/(x-y)$; $y'=2-x/y$; Bernoulli's equation; certain Riccati equations; the simultaneous equations: $y'=z+y[(y^2+z^2)^{1/2}-2a]$, $z'=-y+z[(y^2+z^2)^{1/2}-2a]$; $y''=y'^2(2y-1)\cdot(y^2+1)^{-1}$; $y''+ay'^2y^{-1}=0$; $ax^3y''=(xy'-y)^2$; $ay'''+yy''+y'^2=0$; the Lane-Emden equation. Chapter III concerns *Equations integrable by elliptic functions*. Again the choice of examples is arbitrary. The other chapters are restricted to a very few special equations and add little towards any general point of view.

Granted the deficiencies of the existing mathematical theory of differential equations, the reviewer believes that the author has exaggerated the situation. Even his list of "equations readily integrable" is a very inadequate presentation of what is known. One gains the impression that solution of a differential equation is to mean only solution in terms of elementary (or elliptic!) functions, and that only a simple analytical expression for the solution can be of use. The fact that the differential equation itself defines functions is ignored, although it is implicit in the approximate methods of solution described. It is often clear that the practical application does not demand a formula for solutions but only a knowledge of certain qualitative properties: especially, existence and stability of periodic solutions. The author ignores the contributions to this aspect by Poincaré, Birkhoff, and many others in recent times.

Despite the defects mentioned, the book has a number of good points which should render it of value to engineers. In the later chapters a number of important examples are treated in great detail and, where the mathematics fails, physical experiments are called upon to solve the equations. Any method based on experiments, even with such precise instruments as the modern electronic differential analyzer, does of course beg the question, for one can never be certain as to exactly what equation has been solved. However, in wisely guided hands, the instruments can be used to obtain clues as to the form of solutions and a more precise mathematical analysis may then be able to establish the agreement of equation with experiment. This process has been carried through completely in very few cases and in the present work the mathematics is carried only to the point of a first or second approximation by a perturbation method. In the fourth chapter (*Equations having periodic solutions*) this is done for the van der Pol equation, the physical experiments concerning a thermionic valve circuit and an electric generator-motor combination. Equations of form $\ddot{y}+a\dot{y}+f(y)=A \cos bt$ are handled similarly, with a beam-spring system and a hydro-electric surge chamber as illustrations.

Chapters V and VI concern the representation of approximate solutions in the form $A(t) \sin \phi(t)$. The approximations are always obtained by some variation on the perturbation method and the only justifications for their accuracy are physical plausibility arguments or actual experiments. One is continually impressed with the agreement of this crude theory and experiment. In Chapter VII certain non-linear equations are approximated by equations of Mathieu type and known properties of the Mathieu functions are used to discuss stability; a variety of physical illustrations are described. Of special interest is the occurrence of subharmonic resonance. Chapter VIII concerns the method of isoclines and (quite briefly) numerical methods. The book concludes with a large bibliography.

Although intentionally weak on the theoretical side, this book offers much valuable information to the engineer and to the mathematician presents a definite challenge which is hard to ignore.

W. KAPLAN