After a short introduction on set theory and the system of rational integers the author begins with a preliminary treatment of semi-groups and groups. Non-associative binary compositions are discussed briefly. This is followed by a fairly orthodox treatment of rings, integral domains, and fields. The third chapter deals with various types of extensions of rings and fields, such as the field of fractions of an integral domain, polynomial rings, and simple field extensions.

The next chapter contains a discussion of factorization theory for commutative semi-groups and integral domains. It is shown that the unique factorization theorem holds for principal ideal domains and for Euclidean domains. Unique factorization is also discussed for polynomial rings over rings which have unique factorization.

In Chapter V the author discusses groups with operators, generalizing many of the earlier results on groups and group homomorphisms. The isomorphism theorems are proved and this leads up to a proof of the Jordan-Hölder theorem. This is followed by a discussion of the concept of direct product and a proof of the Krull-Schmidt theorem. Infinite direct products are discussed briefly.

The first part of the sixth chapter deals with the theory of modules. Ascending and descending chain conditions are discussed and a proof is given of the Hilbert basis theorem. The second half of this chapter contains a discussion of ideal theory in Noetherian rings. It is shown that every ideal can be represented as the intersection of primary ideals and two uniqueness theorems are proved about this intersection.

In the final chapter an introduction to the theory of lattices is given. Modular lattices, complemented lattices, and Boolean algebras are treated briefly. A number of results proved earlier in the book are discussed here from a lattice-theoretic point of view—the Jordan-Hölder theorem being proved for modular lattices.

With this volume the author has made an excellent beginning. The completed work should be one of the best general treatments of abstract algebra available.

W. H. MILLS

Introduction to number theory. By T. Nagell. New York, Wiley, 1951. 309 pp. $5.00.

This is essentially a revised edition of a book published in Swedish in 1950. The present edition contains more problems and an additional chapter on the prime number theorem but unfortunately replaces the useful two line biographies of some 63 mathematicians who have contributed to number theory by a corresponding "name index." Some idea of the contents can be obtained from the chapter titles: I, Divisibility; II, On the distribution of primes; III, Theory of congruences; IV, Theory of quadratic residues; V, Arithmetical properties of the roots of unity; VI, Diophantine equations of the second degree; VII, Diophantine equations of higher degree; VIII, The prime number theorem.

The first chapter is fairly standard and contains the unique factorization theorem, a treatment of the arithmetic functions $\phi(n)$, $\mu(n)$, $\tau(n)$, the linear diophantine equation and proofs of the irrationality of e and π.

Chapter II deals with the sieve of Eratosthenes, the Euler product for the zeta function, and Tchebycheff's result that the true order of $\pi(x)$ is $x/\log x$. In addition, the author gives a worthwhile general discussion of the results of modern research in the theory of primes including the Goldbach problem, the twin prime problem, and Brun's method. However, he neglects to mention A. Selberg's work on sieves, and his statements concerning $\pi(x)-\mathrm{Li}(x)$ and the gaps between consecutive primes can both be strengthened. In addition, his remark that $2^{127}-1$ is the largest known prime is no longer true as the result of recent work. (At the moment of writing, the largest prime known is $2^{1279}-1$, as communicated to the reviewer by D. H. Lehmer.)

Chapter III gives an unusually complete account of the theory of congruences including the identical congruence, divisibility of polynomials with respect to a modulus, the Chinese remainder theorem, polynomial congruences to prime power moduli, primitive roots and indices, power residues, integer representing polynomials, and a remainder theorem first appearing in a little known paper of A. Thue (Christiania Videnskabs Selskabs Forhandlinger 1902, No. 7, pp. 7–31) and subsequently rediscovered or generalized by Aubry, Scholz, Vinogradov, Brauer and Reynolds, Rédei, and others. The exposition of identical congruences might have been clarified by a suitable notational device which would distinguish between the two possible meanings of $f(x)\equiv g(x) \pmod{n}$. Also, the author's treatment of congruences to prime power moduli assumes a good knowledge of discriminants and will therefore need amplification for most elementary number theory classes in this country; the final result, due originally to the author and giving the number of solutions of a polynomial congruence to a prime power modulus, is not to be found in many other books. This proof and a number of others in this chapter are somewhat lacking in clarity and they are likely to prove difficult for beginning students; in a few places, the proofs can be simplified by appealing to previous results.

Included in the fourth chapter are the Jacobi symbol, Gauss' lemma, the quadratic reciprocity law, and the infinity of primes in various special arithmetic progressions. The chapter is marred by a number of small flaws.

Chapter V deals with the cyclotomic polynomial, its irreducibility over the rational field, and its prime divisors; the basic properties of Gaussian sums are proved with the aid of suitable trigonometric and polynomial identities.

Chapter VI contains results on the representation of integers as weighted sums of squares, the four square (Bachet-Lagrange) theorem, various results on the Pell and similar equations, and a discussion of lattice points on conics.

Chapter VII deals with some of the usual diophantine equations like $x^4+y^4=z^2$, proves the Euclidean algorithm and unique factorization in $K((-1)^{1/2})$, $K((-2)^{1/2})$, and $K((-3)^{1/2})$, proves the impossibility of the Fermat equation $x^n+y^n=z^n$ for $n=3$ and 7, and proves a result of Kummer and some properties of cubic curves. A general description of the present state of knowledge regarding lattice points on plane algebraic curves is given and the work of Thue, Siegel, Mordell, and Weil is mentioned.

The last chapter is devoted to the elementary proof of the prime number theorem recently discovered by A. Selberg and P. Erdös. This outstanding work of Selberg is well known and was mentioned on his receiving the Fields medal at the International Congress of 1950; that of Erdös received official recognition at the end of 1951 in the following citation on the occasion of the award to him of the Frank Nelson Cole Prize in the Theory of Numbers:

"To Paul Erdös, for his many papers in the Theory of Numbers, and in particular for his paper *On a new method in elementary number theory which leads to an elementary proof of the prime number theorem*, Proceedings of the National Academy, vol. 35, pp. 374–385, July 1949, in which he makes important contributions to the new elementary theory of primes inaugurated by A. Selberg."

Despite the generally recognized importance of Erdös' work, the author has so completely neglected to mention it that the only impression one can gather from the book is that Erdös is a mathematician who gave some lectures attended by van der Corput who developed the exposition in the form given by the author.

The book contains 180 exercises which are to be found at the end of five of the eight chapters. The author's statement that these are not routine is an understatement; many will prove quite difficult for students.

The reviewer feels that the appeal of this book will be to the more advanced and mature student and that it will be valued chiefly for the special topics of Chapters V–VIII now made conveniently available. Most beginning students will probably find the book too difficult and it may therefore prove unsuitable as a classroom text for a first course.

LOWELL SCHOENFELD

Ordinary non-linear differential equations in engineering and physical sciences. By N. W. McLachlan. Oxford University Press, 1950. 6+201 pp. $4.25.

In the preface to this work the author states: "owing to the absence of a concise theoretical background, and the need to limit the size of this book for economical reasons, the text is confined chiefly to the presentation of various analytical methods employed in the solution of important technical problems." It is therefore with forethought, and perhaps with malice aforethought, that the author has presented the mathematical theory in a form which is highly disorganized and which reveals ever so tellingly the inadequacy of present mathematics to explain what are now common experiences of the engineer and physicist.

The state of disorganization of the mathematics is such that one might conclude that there is no theory of differential equations and that all one can hope for in practice is that the differential equation encountered has been solved in the literature. Thus Chapter II, on *Equations readily integrable*, consists solely of the following examples: $y' = -x/y$; $y' = (x+y)/(x-y)$; $y' = 2 - x/y$; Bernoulli's equation; certain Riccati equations; the simultaneous equations: $y' = z + y[(y^2+z^2)^{1/2} - 2a]$, $z' = -y + z[(y^2+z^2)^{1/2} - 2a]$; $y'' = y'^2(2y-1) \cdot (y^2+1)^{-1}$; $y'' + ay'^2y^{-1} = 0$; $ax^3y'' = (xy'-y)^2$; $ay''' + yy'' + y'^2 = 0$; the Lane-Emden equation. Chapter III concerns *Equations integrable by elliptic functions.* Again the choice of examples is arbitrary. The other chapters are restricted to a very few special equations and add little towards any general point of view.

Granted the deficiencies of the existing mathematical theory of differential equations, the reviewer believes that the author has exaggerated the situation. Even his list of "equations readily integrable" is a very inadequate presentation of what is known. One gains the impression that solution of a differential equation is to mean only solution in terms of elementary (or elliptic!) functions, and that only a simple analytical expression for the solution can be of use. The fact that the differential equation itself defines functions is ignored, although it is implicit in the approximate methods of solution de-