On the other hand, it would appear that Carathéodory wished to revive certain classical aspects of the theory of analytic functions which generally do not receive much attention nowadays.

Here is a book which will be of permanent interest not only to the specialist but to all who are inclined to graze in function-theoretic pastures.

MAURICE HEINS

Lezioni de geometria moderna. Vol. 1. *Fondamenti di geometria sopra un corpo qualsiasi.* By B. Segre. Bologna, Zanichelli, 1948. 4+195 pp. 1200L.

This admirable little book comprises a course given by the author at the University of Bologna. It will be followed by two volumes devoted, respectively, to non-linear projective geometry and invariants of birational transformations.

Since an objective is to have the basis of (projective) geometry reflect the great generality achieved in recent years by abstract algebra, almost half of the 180 pages of text (twelve of the seventeen chapters) are exclusively algebraic. In a rapid but clear manner the reader is presented with the essentials of residue classes of integers, groups, rings, corpora and fields, homomorphisms, sub-rings and ideals, zeros and decomposability of polynomials, algebraic and transcendental extensions of fields, finite corpora, and Galois fields.

The word "corpus" (plural, corpora) requires an explanation. The author avoids the contradiction in terms current in English (and other languages) that refers to an algebraic structure which has all the properties of a field except that commutativity of multiplication is not assumed (and may even be denied) as a *noncommutative field.* He calls such a structure a corpus (corpo) and reserves "campo" for a field. The reviewer feels that this terminology might well be generally adopted.

The algebraic preliminaries disposed of, the remaining five chapters proceed at a still brisk but somewhat slower pace. In Chapter 13 a *(right) linear space over a corpus* γ is defined as a set S of "points" in which certain subsets (subspaces) are distinguished, and the following two properties subsist.

I. There is a one-to-one correspondence between the points ξ of S and ordered (right) homogeneous n-tuples $(x_1, x_2, \cdots, x_n)$ of elements of γ (not all zero); that is

$$\xi \sim (x_1, x_2, \cdots, x_n) = (x_1c, x_2c, \cdots, x_nc) \neq (0, 0, \cdots, 0), \qquad c \in \gamma.$$

II. A subspace S' of S consists of all points ξ of S representable by

$$\xi = \xi^{(1)}c_1 + \xi^{(2)}c_2 + \cdots + \xi^{(h)}c_h \qquad (h \geqq 1)$$

where $c_1, c_2, \cdots, c_n$ vary over γ and $\xi^{(r)} \in S$ $(r = 1, 2, \cdots, h)$.

When γ is a finite corpus (and $n \geqq 2$) a linear space S over γ is a finite linear space.

Right linear dependence and independence are defined in the usual manner and a subspace has dimension h if it may be defined by $h+1$ linearly independent points. It is seen that if $\lambda, \mu, \nu, \rho \in S_1$, (line) $\nu = \lambda + \mu$, $\rho = \lambda a + \mu b$, and $r = ab^{-1}$, there is attached to the ordered quadruple λ, μ, ν, ρ as cross ratio $(\lambda\mu\nu\rho)$ the class $\{r\}$ of elements of γ consisting of the conjugates $c^{-1}rc$ of r by the nonzero elements c of γ. The correspondence between ρ and the cross ratio $(\lambda\mu\nu\rho)$ is thus one-to-one if and only if γ is a field.

The reviewer found Chapter 14 to be exceedingly well done. It deals with so-called graphic spaces (spazi grafici) and their relations to linear spaces. A set S forms a graphic space of n dimensions ($n \geqq 2$) provided certain proper subsets (subspaces) are defined, to each of which is attached as dimension a number $<n$ such that the following properties subsist.

I. For each $h = -1, 0, 1, \cdots, n-1$ subspaces S_h of dimension h exist. (S_{-1} is the null subspace and S_0 the subspace of one point.)

II. From $S_h \subset S_k$ follows $h \leqq k$, with the equality holding if and only if $S_h = S_k$.

III. The points common to two arbitrary subspaces S_h, S_k form a subspace S_p. It follows that there is one and only one S_s of minimum dimension containing S_h and S_k.

IV. With the above notation, the dimension formula $h+k = p+s$ holds.

Every linear space is graphic, but not conversely. It is shown that Desargues' Theorem is valid in every graphic space of dimension greater than 2 which is irreducible (that is, each line (S_1) has at least three points). An irreducible graphic space is proved linear if and only if the Desargues Theorem holds in the space (and hence every irreducible graphic space of more than two dimensions is linear). A simple example of a two-dimensional, irreducible graphic space without the Desargues property is given.

The validity of the Pascal-Pappus Theorem in a linear space is proved equivalent to commutativity of multiplication in the base corpus γ, and the Desargues Theorem is shown to be a consequence, in any graphic plane, of the first named theorem.

Collineations, homographies, involutions, correlations are discussed in graphic spaces with the Pascal property, and the book ends with a brief account (8 pages) of finite linear spaces. There are no exercises.

The book treats with clarity and precision an astonishing amount of material, and is a very welcome addition to the literature of the subject.

LEONARD M. BLUMENTHAL

The location of critical points of analytic and harmonic functions. By J. L. Walsh. (American Mathematical Society Colloquium Publications, vol. 34.) New York, American Mathematical Society, 1950. 8+384 pp. $6.00.

As the title indicates, the book is concerned with the critical points of analytic functions $f(z)$ of the single complex variable $z=x+iy$ and of harmonic functions $u(x, y)$ of the two real variables x and y. As is well known, a critical point of $f(z)$ means a zero of its derivative $f'(z)$, and a critical point of $u(x, y)$ means a point where both partial derivatives $\partial u/\partial x$ and $\partial u/\partial y$ vanish. The former are the points where the map by $w=f(z)$ fails to be conformal and are the multiple points of the curves $|f(z)|=\text{const.}$ and $\arg f(z)=\text{const.}$ The latter are the equilibrium points in the force field having $u(x, y)$ as force potential and are the stagnation points in the flow field having $u(x, y)$ as velocity potential. Thus the subject matter of the book is one of considerable importance in both pure and applied mathematics.

In this book the analytic functions considered are largely polynomials, rational functions, and certain periodic, entire, and meromorphic functions. The harmonic functions considered are largely Green's functions, harmonic measures, and various linear combinations of them. The interest in these functions centers about the approximate location of their critical points. The approximation is in the sense of determining minimal regions in which lie all the critical points or maximal regions in which lies no critical point.

This book not only has a unity of subject matter, but it also has very nearly a unity of method. The method is based upon the observation that, with $z-a=re^{i\theta}$ and thus $(\bar{z}-\bar{a})^{-1}=r^{-1}e^{i\theta}$, the vector $(\bar{z}-\bar{a})^{-1}$ has the direction of the line segment from the point a to the point z and a magnitude equal to the reciprocal of the length of this line segment. Accordingly, the vector $m(\bar{z}-\bar{a})^{-1}$ may be regarded as force with which a particle of mass m repels a unit particle at z; the sum $F(z)=\sum m_k\ (\bar{z}-\bar{a}_k)^{-1}$ may be regarded as the resultant force upon a unit particle at z due to the system of discrete masses m_k at a_k, and the integral $J(z)=\int[\bar{z}-\bar{a}(t)]^{-1}dm(t)$ may be regarded as the resultant force at z due a continuous spread of matter. Now, it turns