BOOK REVIEWS

Sur la fonction noyau d'un domaine et ses applications dans la théorie du transformations pseudoconformes. By Stefan Bergman. (Mémorial des Sciences Mathématiques, no. 108.) Paris, Gauthier-Villars, 1948. 250 fr.

This memoir is the continuation of the book *Sur les fonctions orthogonales de plusieurs variables complexes avec les applications à la théorie des fonctions analytiques* (Interscience, 1941, and Mémorial des Sciences Mathématiques, no. 106, 1947), by the same author (see Bull. Amer. Math. Soc. vol. 48 (1942)).

The starting point for the considerations outlined in the following is the kernel function $K_G(z, \bar{t})$, $z=(z_1, z_2)$, $\bar{t}=(\bar{t}_1, \bar{t}_2)$, uniquely associated with any given domain G, which has been introduced by the author and used with ever growing success. $K_G(z, \bar{t}) = \sum_{\nu=1}^{\infty} \phi^{(\nu)}(z)\overline{\phi^{(\nu)}(t)}$, where $\{\phi^{(\nu)}\}$ represents an arbitrary system of functions which are analytic in G and complete and orthonormal over G in the L^2 metric. K is a relative invariant under analytic transformations of the four-dimensional (z_1, z_2)-space.

Chapter I contains an investigation of the behavior of $K_G(z, \bar{t})$ on the boundary of G. To this end: (1) K_G is identified as the solution of the following minimum problem: The normalized function K_G minimizes $\int_G |f|^2 d\omega_z$ ($d\omega_z$ is the four-dimensional volume element) under the condition $f(t_1, t_2)=1$, where the value $\lambda_G(t)$ of the minimum is

$$(1) \qquad \lambda_G(t) = 1/K_G(t, \bar{t});$$

(2) K_G is compared with the kernel functions of such domains, both inscribed in and circumscribed about G, that have the property that their kernel functions can be constructed explicitly.

The author lists a number of such domains and their respective kernel functions. Thus, one obtains bounds for K_G; these bounds are then particularly valuable as we approach the boundary of G. Boundary points R are classified according to the smallest number n, such that $[\rho(z)]^n K(z, \bar{z})$ has a limit as the point (z) approaches R. (Here, ρ denotes the euclidian distance from (z) to R.) If R is a point totally pseudo-convex in the sense of E. E. Levi, then $n=3$. Cases are also considered where $n=0$, 1, or 2.

The core of the entire presentation is in Chapter II. By means of minimum problems, two covariant mapping functions are assigned to every region in such a way that to every couple consisting of a domain G and any fixed interior point, there is associated a unique representative domain. All proper (m, p) domains, in the sense of H. Cartan, that is, domains which admit automorphisms of the form

$$z_1' = z_1 e^{im\theta}, \qquad z_2' = z_2 e^{ip\theta}, \qquad (m, p) = 1,$$

are such representative domains. Next, using the properties of the invariant Hermitian form

$$ds^2 = \sum T_{mn} dz_m d\bar{z}_n, \qquad T_{mn} = \frac{\partial^2 \log K(z, \bar{z})}{\partial z_m \partial \bar{z}_n},$$

the author demonstrates the connection between these results and the work of Elie Cartan on homogeneous domains (domains which always admit an automorphism interchanging any two given points).

In Chapter III, a given domain is mapped analytically onto domains interior to a fixed domain A. The kernel function of A then serves to prove bounds for various geometric magnitudes associated with the mapping function.

These distortion theorems are obtained by the use of the following procedure, which is denoted by the author as "the method of the minimum integral." It is shown that various quantities connected with the invariant metric, such as the expression $\sum T_{mn} u_m \bar{u}_n$, the curvature of the metric, and so on, can be expressed by the value of the minimum $\lambda_G(t)$ of the integral $\int_G |f|^2 d\omega$ under various normalization conditions. This fact permits us to obtain bounds for them, using conveniently chosen interior and exterior domains of comparison.

As a special case, choosing $f(t_1, t_2) = 1$ as the normalization condition, one obtains the relation (1), and thus, if $B \supset G$,

$$K_B(t, \bar{t}) \leqq K_G(t, \bar{t}).$$

(Since, in the case of one variable, $ds_B^2(t) = K_B(t, \bar{t}) |dt|^2 \leqq K_G(t, \bar{t}) |dt|^2 = ds_G^2(t)$, the above inequality is the lemma of Schwarz-Pick.)

The usefulness and power of the method of the kernel function is particularly evident if one specializes this approach to the case of one complex variable.

In Chapter IV, it is shown that the domain functions, such as the Green and Neumann functions, $G(z, t)$ and $N(z, t)$, the harmonic measures, as well as functions mapping simply- or multiply-connected domains into canonical domains, can be expressed in an unexpectedly simple manner by the kernel function. In particular, it is shown that $N_B(z, t) - G_B(z, t) = 2\pi k_B(z, t)$, where $k_B(z, t)$ is the kernel function for the Laplace equation.

Of especial interest to the geometer will be the frequent recourse to fundamental domains defined by the invariant metric.

Throughout this memoir, numerous examples are calculated for special domains. Extensive bibliography and detailed indications of original papers are given.

H. BEHNKE

La théorie de la relativité restrainte. By O. Costa de Beauregard. Paris, Masson, 1949. 6+174 pp. 800 fr.

In this little volume, the author presents a textbook of the special theory of relativity, with special emphasis on those aspects of the theory to which he himself has contributed. As a result, he has shown that it is possible to this day to write a textbook on the subject that is not repetitive.

The preface contributed by Professor de Broglie is most illuminating. It appears that the author's principal contribution has been the thorough investigation of three-dimensional integrals in Minkowski space, particularly as regards their transformation properties. While the results are probably well known to differential geometers, their consistent application to physically interesting questions affords the physicist an introduction to relativistically invariant three-dimensional (space-like) integrals.

Space-like integrals are frequent in physical theories. The integral over the electric charge density must be extended over a three-dimensional space-like domain to yield the total charge; the integral over the entropy density in three dimensions gives the total entropy, and so on. The corresponding four-dimensional integrals lack physical significance. In the standard physical literature Schwinger was among the first to discover that one cannot examine the properties of such integrals conveniently if one restricts oneself to plane surfaces. The reason is that in going over from one surface to a neighboring one (connected with each other by means of an infinitesimal transformation) one finds that the variation of the integral consists of terms having the form of a (three-dimensional) volume integral and additional terms that appear in a (two-dimensional) surface integral. Now if the only domains to be considered are space-like coordinate hypersurfaces in Minkowski space, the surface integrals are to be taken at infinity, and the convergence considerations that must be carried out, though feasible, are artificial. It is much more convenient to consider at first neighboring domains that coincide everywhere except in a bounded domain. And that point of view requires the consideration of curved hypersurfaces. All this has, of course, been