fields, but with occasional excursions into abstract fields), contains considerable material on irreducibility criteria and introduces the concepts of the root (or splitting) field of a polynomial and normal field extension. Chapter III (91 pages) concerns the galois group of a polynomial (defined as a permutation group), gives the fundamental theorem of galois theory, and contains an appendix on Loewy's treatment of non-normal field extensions. Chapter IV (90 pages) discusses the connection between solvable groups and solvability by radicals, with numerous classical applications, and contains an appendix giving a table of the cyclotomic polynomials for $2<m\leqq 60$. Chapter V treats finite fields and the question of equations with prescribed galois group. A final Appendix is devoted to the elements of the theory of rational numbers.

The book proceeds at a leisurely pace and is readable. In the reviewer's opinion, the author has eschewed the modern approach with excessive zeal; this at times results in a loss of clarity. On the positive side one finds a great wealth of illustrative examples and exercises.

E. R. KOLCHIN

An introduction to probability theory and its applications. Vol. I. By William Feller. New York, Wiley, 1950. 12+419 pp. $6.00.

This is the first volume of a projected two-volume work. In order to avoid questions of measurability and analytic difficulties, this volume is restricted to consideration of discrete sample spaces. This does not prevent the inclusion of an enormous amount of material, all of it interesting, much of it not available in any existing books, and some of it original. The effect is to make the book highly readable even for that part of the mathematical public which has no prior knowledge of probability. Thus the book amply justifies the first part of its title in that it takes a reader with some mathematical maturity and no prior knowledge of probability, and gives him a considerable knowledge of probability with the necessary background for going further. The proofs are in the spirit of probability theory and should help give the student a feeling for the subject.

Probability theory is now a rigorous and flourishing branch of analysis, distinguished from, say, measure theory, by the character and interest of its problems. It is true that probability theory, like geometry, had its origin in certain practical problems. However, like geometry, the theory now concerns itself with problems of interest per se, many of which are very idealized, and have only a remote connection or no presently visible connection, with practical problems.

At the same time the development of the science is continually stimulated by challenging problems arising in the various fields of application. This book contains a huge number of examples illustrating almost every aspect of the theory developed. These examples are very interesting and not at all of the ad hoc variety. It is no mean feat to present so many interesting examples between two covers. They enhance the interest of the theory even for the pure mathematician, except perhaps for the extreme diehard of the "God save mathematics from its applications" school.

The introductory chapter consists of a brief philosophical introduction. The author declares that the modern theory of probability, whose successes it is his intention to depict, is limited to what he says may roughly be called "physical or statistical probability." The intuitive notion of probability, connected with general inductive reasoning and the correctness of judgments, does not, he says, come within the scope of this theory. In the first chapter he introduces his axiom system, with the proviso that in the present book only denumerable spaces are to be considered. Chapter 2 treats the classical elements of combinatorial analysis. Chapter 3 is called *The simplest occupancy and ordering problems*, and Chapter 4 is called *Combination of events*. Both treat various combinatorial problems. In these chapters occur one or two problems on testing hypotheses, in which a hypothesis is rejected because under it the probability of the event observed is very small. This reviewer has misgivings about such arguments. What about other events of small probability which are not held to contradict the hypothesis being tested? It is only when one takes into account the admissible alternative hypotheses that this procedure will make sense. Naturally this question of admissible alternatives is not treated in the present book, since it is not within its scope and would take the author too far afield.

Chapters 5, 6, and 7 treat usual and classical topics. In Chapter 5 the author introduces the notion of conditional probability. In chapters 6 and 7 he discusses the binomial, multinomial, and Poisson distributions, and the normal approximation to the binomial distribution. It is amazing what a fresh and well-informed approach can do even to these standard topics. As an example we cite the discussion of large deviations (that is, the probability that the reduced number of successes will exceed x, where $x\rightarrow\infty$ in a suitable manner as $n\rightarrow\infty$, where n is the number of Bernoulli trials) on page 144, and problems 12 to 17 on pages 147 and 148, where results of Smirnov and Khintchine are developed.

Chapter 8 is entitled *Unlimited sequences of Bernoulli trials*. The proof by Doob (Ann. of Math. vol. 37 (1936) pp. 363–367) of the impossibility of a gambling system is given. The strong law of large numbers (for Bernoulli trials) is proved by using one of the above-mentioned results on large deviations. Then follows a proof of the law of the iterated logarithm. This is carried further in two exercises which introduce the reader to Feller's own results (Trans. Amer. Math. Soc. vol. 54 (1943) pp. 373–402). In Chapter 9 the author introduces the notions of chance variable and of expectation. In addition to the usual results (and many results not usual, such as introducing the reader to stratified sampling by means of two exercises), he proves Kolmogorov's inequality, which is used in the next chapter to prove that $\sum\sigma_i^2/i^2 < \infty$ is a sufficient condition for the strong law of large numbers to hold for independent chance variables. The author does not employ spaces whose elements are infinite sequences, because this would raise measure theoretic questions which he is determined to avoid in the present volume; strong convergence and similar properties are defined by means of passages to the limit. In Chapter 10 the author proves the (weak) law of large numbers for identically distributed, independent chance variables with a finite expected value (Khintchine's theorem). Since the author does not, in this volume, make use of theorems on the characteristic function (generating functions of integral-valued variables are used elsewhere), with the aid of which this theorem can be proved very expeditiously, he makes use of the device of truncating the chance variables appropriately so that the main parts have, of course, finite variances. In subsequent sections and numerous exercises he discusses the strong law of large numbers under varying conditions, various implications of the central limit theorem (proof deferred until the second volume), and his own theory of fair games (Ann. Math. Statist. vol. 16 (1945) pp. 301–304) with application to the Petersburg paradox. In Chapter 11 integral-valued variables are discussed. Generating functions and the notion of convolution are introduced. Numerous applications are made; a typically interesting one is to obtain the probability of the termination of a chain reaction.

The remaining chapters are, in many respects, the richest in the book, replete with recent results of the author and others, and it is useless to do more than name their topics. Two chapters are devoted to recurrent events (roughly, the stochastic process starts from scratch each time a certain pattern has occurred), and two chapters treat Markov chains. One chapter treats problems in gambler's ruin, random walk, and diffusion processes, the latter two in several dimensions. The final chapter is a brief introduction to the simplest time-dependent stochastic processes.

The outline given above of the book's contents is very bare and grossly inadequate. Since the book is not a unified treatment of just a few topics it is, however, difficult to do otherwise in a reasonable compass. To sum up, this is a superb book, and a delight to read. The gathering together of so much material in so brilliant a manner represents a prodigious amount of labor for which the mathematical public is greatly indebted. The reviewer congratulates the author; he has set a lofty standard for would-be writers of similar books to attain.

J. WOLFOWITZ

Theoretische Mechanik. Eine einheitliche Einführung in die gesamte Mechanik. By G. Hamel. (Die Grundlehren der mathematischen Wissenschaften, vol. 57.) Berlin, Springer, 1949. 16+796 pp. 161 figs.

In writing this book, the author had the following aims: to give a unified treatment avoiding the usual separation into mechanics of particles and mechanics of continua, and, following Lagrange, to present a deductive treatment based on the principle of virtual work, d'Alembert's principle, and Lagrange's "liberation principle" (Befreiungsprinzip). According to the author, Lagrange has used this last principle consistently even though he never formulated it explicitly. The author states this principle as follows: "if a constraint imposed on a mechanical system is relaxed, the corresponding reaction becomes an applied force which depends primarily on the deformation previously prevented by the constraint." For example, in the transition from an incompressible to a compressible perfect fluid, the Lagrangian multiplier of the condition of incompressibility becomes the pressure and this depends on the density variations which were originally excluded by the condition of incompressibility.

In this reviewer's opinion, the author has been entirely successful in carrying out his intentions. It goes almost without saying, that the resulting treatise is not suitable for beginners in spite of the fact that the subject is developed from first principles. In fact, the author's remark concerning Hertz' *Mechanics* ("geistreich, aber schwer zu lesen") applies equally well to his own book which is also rich in ideas but hard to read. To the reader who has already achieved mastery of the field along conventional lines, however, the work will open new horizons.

An unusual feature for a book written at this level is an extensive collection of Problems and Solutions (pp. 527–789).

The space available for this review does not permit detailed comments on the contents; the following list of chapter headings (with particularly significant section headings added in parentheses) will