BOOK REVIEWS

Leçons sur le calcul des coefficients d'une séries trigonométrique. Quatrieme Partie. *Les totalisations. Solution du problême de Fourier.* Premier Fascicule: *Les totalisations.* Deuxième Fascicule: *Appendices et tables générales.* By Arnaud Denjoy. Paris, Gauthier-Villars, 1949. Fasc. I, pp. 327–481, 1500 fr.; Fasc. II, pp. 483–715, 2200 fr.

These two books constitute the concluding part of a treatise, in four parts, devoted to the solution of one of the fundamental problems of trigonometrical series. The problem is to express trigonometrical series, that are everywhere convergent, in the Fourier form. This problem was successfully attacked in 1921 by Denjoy in five notes published in the Comptes Rendus de l'Academie des Sciences de Paris. In 1938, he gave a course of lectures on this topic at Harvard University, and these lectures are written up now in the form of a book.

The background to the problem is furnished by the uniqueness theory of trigonometrical series on the one hand and by the theory of integration of functions of a real variable on the other. If two trigonometrical series converge everywhere to the same sum, the series are identical; that is, the corresponding coefficients in the two series are equal. This is a theorem of Cantor (1872) which guarantees that if a function $f(x)$ has a trigonometric development

$$\frac{1}{2} a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

which converges everywhere, then this development is unique. If, in addition, the function $f(x)$ is Lebesgue integrable, then it is a Fourier development; in other words, the coefficients a_n, b_n are Fourier coefficients, related to $f(x)$ by means of the classical formulas of Fourier. This is a theorem of de la Vallée-Poussin (1912). The problem is to uphold this in the general case, without the restrictive assumption of Lebesgue integrability on $f(x)$. Denjoy solved this problem by evolving an elaborate process of *totalisation symmetrique à deux degrês* for the calculation of the coefficients. This process may be looked upon as a generalization of the notion of integration, and termed "trigonometric integration." It is more powerful than what is commonly known as the ordinary Denjoy integral which Denjoy had invented for the analogous, but somewhat less complicated, problem of proving that a function which is differentiable everywhere is the "integral" of its derivative. Interesting contributions have been made to the theory of trigonometric integration, such as Verblunsky's approximate Denjoy integral and Burkill's Cesaro-Perron integral, in the intervening years between the author's original researches (1921) and the publication of this book, though there is no mention of them here.

Starting from de la Vallée-Poussin's result, the treatise covers all the ground that is necessary to reach the final result of the author. The fourth part, under review, comprises Chapters VII to IX, with a few Appendices at the end. In Chapter VII the author develops the theory of the Denjoy integral (*totalisation simple*) with the aid of a new notion of totalisation of series, presented here for the first time. In Chapter VIII he treats Stieltjes integrals relative to general measures. In Chapter IX he presents a complete solution of the main problem, explained above, and illustrates with examples the impossibility of relaxing any of the conditions formulated in his definition of the "trigonometric integral." The Appendices deal mainly with the special Denjoy integral using majorants and minorants, besides containing a rather severe criticism of Perron's definition of integral on the ground that it is nonconstructive.

In contrast with the earlier notes of the author which were brief, the present work is very elaborate and even diffuse. It bears witness to the highly ingenious and original mind of the author. To appreciate it, one has to read the book in full; no part of it can be detached from the rest. This, however, is not an unmixed blessing. Though the title of the book sounds very special, its content is not narrow; it is really a survey, in the singular fashion of the author, of the various sectors of the theory of functions of a real variable that surround the very difficult problem of the calculation of coefficients of trigonometrical series. The work that is embodied in this book has already had considerable influence on that of other mathematicians; in this sense, one regrets that the book did not appear sooner. Anyone interested in the theory of non-absolutely convergent integrals would consider the book valuable.

K. CHANDRASEKHARAN

Lectures on classical differential geometry. By D. J. Struik. Cambridge, Addison-Wesley, 1950. 8+221 pp. $6.00.

There is many a good reason to welcome this new book on differential geometry.

First of all, there is the very fact that it is devoted to *classical* differential geometry, that is, to the wealth of ideas from which all further developments have been derived. The comprehensive his-