mentary treatment of this subject which explains the numerous subtleties involved would be very welcome.

In the fourth chapter the author turns to the theory of surfaces and discusses their intrinsic properties. The usual topics are discussed including total curvature, the Gauss-Bonnet Theorem, parallel displacement, and special nets. This treatment continues in the fifth chapter which discusses geodesics, surfaces of constant curvature, parallel curves, Liouville nets, and conformal mapping.

The extrinsic properties of a surface such as lines of curvature, asymptotic lines, Meusnier's Theorem and Dupin's Theorem appear in chapter six. All of this is done by Cartan's methods and the exercises present the same theory in Gauss's notation and then in tensor notation. The chapter also includes a discussion of the rigidity and bending of surfaces.

The final chapter treats minimal surfaces and the problem of Plateau. Appropriately Blaschke introduces complex coordinates and complex geometric elements, so that an understanding of this chapter requires a knowledge of the theory of functions of a complex variable.

CARL B. ALLENDOERFER

Supersonic flow and shock waves. By R. Courant and K. O. Friedrichs. New York, Interscience, 1948. 16+464 pp. $7.00.

This book is an excellent up-to-date account of the related problems of supersonic flow and non-linear wave propagation. Its content ranges from theory of hyperbolic partial differential equations to the practical problems of flow in nozzles and jets.

The point of view with which this book is written is best described by the words of the authors: "The book has been written by mathematicians seeking to understand in a rational way a fascinating field of physical reality, and willing to accept compromise with empirical approach." This rational approach is extremely valuable in such a field, where convenient but inaccurate concepts are often found to creep into existing literature. For example, in other discussions, the concept of a Mach line or a characteristic is sometimes introduced by associating it with a small disturbance. While this is no doubt a convenient way to discuss many properties associated with the characteristic, it sometimes leads to the erroneous concept that a characteristic is a line of disturbances. Again, in other discussions, the use of characteristics is often treated so closely with the numerical method of step-by-step integration of a supersonic flow field, that there is a danger of taking the latter as an essential part in the method of characteristics. In this book, all these misleading discussions are absent. The role of characteristics as convenient coordinate curves, the propagation of discontinuities (of higher order) along characteristics, and the Mach line as the limiting position of the weak shock line are treated separately and logically. The numerical method of step-by-step integration of supersonic flow field is discussed briefly toward the end of Chapter II, but it contains the essence of the method. The practical advantage of the present treatment can be seen from the conciseness of the discussion of the problem of designing a perfect nozzle (§113 and pp. 85–86).

Since this book treats the more interesting case of non-linear phenomena, rather than the classical problems of linear wave propagation, it is very much limited to the case of two independent variables. The limited usefulness of the notion of characteristics in the case of more than two independent variables is discussed at the end of Chapter II. [Perhaps it would not have been out of place here to include a brief discussion of the connection between characteristic surfaces and the wave front, such as that in §3, Chapter VI of vol. II of Courant-Hilbert's *Methoden der Mathematischen Physik.*]

The book begins with a discussion of the basic physical notions and the mathematical formulation of basic physical laws. Thus, the book is readable by any mathematician who is familiar with mechanics in general. The second chapter gives the main body of the mathematical basis of later developments in the book. Here are discussed the theory of characteristics and in particular the theory of simple waves. The simple wave theory includes the one-dimensional progressive waves and the Prandtl-Meyer flow in the two-dimensional steady case. It shows more clearly why such simple flows are possible than conventional ad-hoc treatments.

Beginning with the third chapter, this book deals with specific problems. The discussions of the basic aspects are carried out in remarkable detail. "On the other hand, no attempt has been made . . . to provide summaries of results which could be used as recipes for attacking specific engineering problems" (from Authors' Preface).

One-dimensional problems of gas dynamics, including simple waves of rarefaction and compression, shock waves of varying strength, interaction of shock waves, are treated in the third chapter. The discussion of wave interaction is very extensive and systematic, and is perhaps the first time that such work is published in an organized presentation. This chapter also includes a discussion of detonation and deflagration waves and a brief discussion of wave propagation in elastic-plastic material.

Chapter IV treats isentropic steady plane flow. The authors have made the development in this part parallel to but independent of those in Chapter III. This brings out very clearly the analogy and differences between the problems of one-dimensional waves and those of steady plane flows. Besides such topics as the hodograph transformation, limiting lines, simple waves, there is an extensive discussion of shock interaction and reflection. The last part of this chapter deals with the exact nature of problems of steady flow past an obstacle, including an extensive and systematic discussion of the perturbation theory. This part should be highly recommended to the inquisitive workers in this field.

Two more short chapters follow. Chapter V deals with flow in nozzles and jets. Chapter VI deals with three-dimensional flows having suitable symmetry properties, so that there are still two independent variables. The book ends with an extensive and valuable list of references to books and other publications.

In conclusion, the reviewer wishes to recommend this book to every worker in the field of dynamics of a compressible fluid, whether he is studying it from the point of view of a mathematician, a physicist, or an engineer.

C. C. Lin

Moderne algebraische Geometrie. Die idealtheoretischen Grundlagen. By W. Gröbner. Vienna, Springer, 1949. 12+212 pp. $5.70.

Dr. Gröbner's book is a textbook giving the fundamentals of the ideal theory needed in algebraic geometry. The exposition is clear, elegant, and easy to read.

About one-half of the material—basic field theory, the ideal theory of polynomial rings, and the more general commutative ideal theory—can be found in van der Waerden's textbook on algebra; the remainder—material on Hilbert's function, the ring of formal power series, integral algebraic quantities, and the syzygy theory of homogeneous polynomial ideals—can be found without too much difficulty in the literature, but appears here for the first time in textbook form.

Dr. Gröbner has organized all this material excellently, occasionally improving known proofs—especially in the resultant theory and the syzygy theory—and has at all times kept the geometry in the foreground; the motivation of the development of each subject is thus at all times quite clear.

As is obvious, different ideals in a polynomial ring can have the same locus of zeros—in Dr. Gröbner's terminology, the same "Nullstellengebilde" (NG). The author attempts to establish a 1-1 correspondence between ideals and geometric objects, associating with