BOOK REVIEWS

Einführung in die Differentialgeometrie. By W. Blaschke. Berlin, Springer, 1950. 8+146 pp. 16 DM.

This new book of Blaschke's is a text covering the metric differential geometry of curves and surfaces lying in ordinary Euclidean space. It covers roughly the same material as the first volume of his *Vorlesungen über Differentialgeometrie*, Berlin, Springer, 3d ed., 1930, but the two books are remarkably different both in content and approach.

The most striking innovation in the present work is the systematic use of E. Cartan's exterior differential forms to which Blaschke is a fairly recent convert. The adoption of this technique has caused him to rewrite large portions of his older book completely. There is more on minimal surfaces here than there was earlier, but the former sections on differential geometry in the large have been condensed or omitted and the earlier treatment of line geometry is omitted entirely. On the whole these changes are for the better, and this book is an excellent introduction to the subject with appropriate emphasis on its classical and modern aspects.

The book is compactly written and contains more information than one would anticipate from so small a number of pages. To a large extent this is made possible by the author's familiar custom of presenting as "exercises" brief outlines of numerous theorems together with appropriate references. These "exercises" are one of the most valuable features of the book. It is unlikely that universities in the United States could use such a book as a classroom text (even if it were in English), but it would serve as an excellent supplement to more usable texts such as the recent outstanding one by D. J. Struik, *Differential geometry*, Addison-Wesley, 1950.

After an initial chapter on vector and matrix algebra, the author turns to a treatment of strips and curves. His emphasis on strips rather than on curves is quite unorthodox and will not find universal favor. Indeed it is surprising (to say the least) to find that the Frenet formulas for a curve are introduced as a lemma preparatory to the Four-Vertex Theorem. The exercises include references to curves of constant width, helices, and the isoperimetric property of the circle (seven proofs).

The third chapter considers the calculus of exterior differential forms. Although the treatment is clearer than those in Cartan's books, there is still an air of mystery about this beautiful technique. An ele-

mentary treatment of this subject which explains the numerous subtleties involved would be very welcome.

In the fourth chapter the author turns to the theory of surfaces and discusses their intrinsic properties. The usual topics are discussed including total curvature, the Gauss-Bonnet Theorem, parallel displacement, and special nets. This treatment continues in the fifth chapter which discusses geodesics, surfaces of constant curvature, parallel curves, Liouville nets, and conformal mapping.

The extrinsic properties of a surface such as lines of curvature, asymptotic lines, Meusnier's Theorem and Dupin's Theorem appear in chapter six. All of this is done by Cartan's methods and the exercises present the same theory in Gauss's notation and then in tensor notation. The chapter also includes a discussion of the rigidity and bending of surfaces.

The final chapter treats minimal surfaces and the problem of Plateau. Appropriately Blaschke introduces complex coordinates and complex geometric elements, so that an understanding of this chapter requires a knowledge of the theory of functions of a complex variable.

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Supersonic flow and shock waves. By R. Courant and K. O. Friedrichs. New York, Interscience, 1948. 16+464 pp. $7.00.

This book is an excellent up-to-date account of the related problems of supersonic flow and non-linear wave propagation. Its content ranges from theory of hyperbolic partial differential equations to the practical problems of flow in nozzles and jets.

The point of view with which this book is written is best described by the words of the authors: "The book has been written by mathematicians seeking to understand in a rational way a fascinating field of physical reality, and willing to accept compromise with empirical approach." This rational approach is extremely valuable in such a field, where convenient but inaccurate concepts are often found to creep into existing literature. For example, in other discussions, the concept of a Mach line or a characteristic is sometimes introduced by associating it with a small disturbance. While this is no doubt a convenient way to discuss many properties associated with the characteristic, it sometimes leads to the erroneous concept that a characteristic is a line of disturbances. Again, in other discussions, the use of characteristics is often treated so closely with the numerical method of step-by-step integration of a supersonic flow field, that there is a danger of taking the latter as an essential part in the method of characteristics. In this book, all these misleading discus-