

given and a singular point for a first order differential equation is investigated. Most of the discussion is concerned with second order equations or the equivalent systems. For these the linear dependence of solutions, the Wronskian theory, the variation of parameters method, the circuit of a singularity in the complex plane, Fuchs theorem, solution by power series, the Sturm-Liouville theory and the asymptotic behaviour of characteristic functions and characteristic values are given. The last chapter also contains a Cauchy type existence theorem, based on majorants.

There is a valuable emphasis on individual functions, whose properties are derived from the fact that they are solutions of a differential equation. For instance the circular and elliptic functions are treated in this way. The asymptotic behaviour of the Laguerre and Legendre polynomials and the Bessel functions are used to illustrate the characteristic function theory. The hypergeometric series is developed in the last chapter. On the other hand as a matter of policy the usual methods for the integration of first order equations are omitted.

The style is clear and the book should prove a valuable reference. The author claims, quite justly, that this corresponds to a "modern course" in differential equations and there is quite a contrast with the American courses on "methods of solution" and "theory." The pressure from applications and the results of theoretical developments are clearly present. However the existence theory is not the most general possible and the elementary methods and the constant coefficient linear equations are worth considering. The need of two courses seems clear but they should be carefully organized for maximum usefulness.

F. J. MURRAY

Tables of generalized sine- and cosine-integral functions. Parts I and II. (Annals of the Computation Laboratory of Harvard University, vols. 18, 19.) Harvard University Press, 1949. Part I, 38+462 pp. \$10.00. Part II, 8+560 pp. \$10.00.

DEFINITIONS:

$$\begin{aligned}
 S(a, x) &= \int_0^x \frac{\sin u}{u} dt; & C(a, x) &= \int_0^x \frac{1 - \cos u}{u} dt; \\
 Ss(a, x) &= \int_0^x \frac{[\sin u] \sin t}{u} dt; & Sc(a, x) &= \int_0^x \frac{[\sin u] \cos t}{u} dt;
 \end{aligned}$$

$$Cs(a, x) = \int_0^x \frac{[\cos u] \sin t}{u} dt; \quad Cc(a, x) = \int_0^x \frac{[\cos u](1 - \cos t)}{u} dt$$

where

$$u = (a^2 + t^2)^{1/2}.$$

The above six functions are tabulated to six decimals over the range of both a and x from 0 to 25. The intervals in the arguments vary from 0.01 in both a and x when they are less than unity, to 0.2 in both arguments for $a \geq 10$. In addition to the main table, there are a few pages of coefficients which occur in the various formulas that were employed for computing the functions.

The introduction by J. Orten Gadd, Jr., and Theodore Singer gives the more familiar properties of the tabulated functions, the method used to compute the entries on the automatic sequence controlled calculator, and methods of interpolation in the tables. Some of the derivations in this introduction are correct for positive values of a and x only, although this limitation is not stated. For example, it is implied that

$$S(a, \infty) = \int_0^{\infty} \{ [\sin a(1 + t^2)^{1/2}] / (1 + t^2)^{1/2} \} dt;$$

this is of course true only for positive values of a . Moreover, the authors derived their integration formula from a system of linear algebraic equations. It should be pointed out that the formula is well known, and can be derived more simply from the integration of the interpolation polynomial. In fact, the coefficients of this formula have been extensively tabulated. These may be minor flaws, but the definitions of the functions deserve improvement in any future edition of the tables. For instance, the authors write

$$Cc(a, x) = \int_0^x \frac{\cos u(1 - \cos x)dx}{u},$$

with $u = (x^2 + a^2)^{1/2}$. All the other functions are defined in the same spirit, and at best these definitions are ambiguous.

The method of computation described in the introduction emphasizes rigorous accuracy, and in the light of past performances of the Harvard Computation Laboratory, there is every reason to believe that the entries are trustworthy. The format of the tables is pleasing; reproduction is by a photo offset process.

GERTRUDE BLANCH