## BOOK REVIEWS

*Analytic theory of continued fractions.* By H. S. Wall. New York, Van Nostrand, 1948. 13+433 pp. $6.50.

This is the first volume to appear in "The University Series in Higher Mathematics" which is planned to be a collection of "advanced text and reference books in pure and applied mathematics."

In order to make the book suitable as a text book the author gives detailed proofs and includes material which might be unfamiliar to "a student of rather modest preparation." Into this category fall such topics as: the Stieltjes-Vitali theorem, Schwarz' inequalities, matrix calculus, elementary properties of the Stieltjes integral, and basic concepts and formulae of the theory of continued fractions. To increase its usefulness as a text the book is provided with 131 exercises, grouped together at the end of each chapter. The material covered in this book is of such a nature that it or parts of it would make very attractive subject matter for graduate seminars. It is to be hoped that the book will thus contribute to spreading knowledge of and interest in the analytic theory of continued fractions among a larger group of people.

Numerous and important additions have been made to the analytic theory of continued fractions since the appearance of Perron's *Die Lehre von den Kettenbrüchen* in 1913. Only a few of these results were incorporated into the second edition of Perron's book in 1929. It is thus clear that the publication of the present book which brings so much, though unfortunately not all, of this material together in a single volume was welcomed by workers in this and in related fields.

The book consists of twenty chapters. Chapters I–VIII form the first part entitled "Convergence theory." The second part is called "Function theory." Classical as well as more modern results are treated in this book. That Wall himself has been during the last twenty years the foremost contributor to the advancement of the theory is apparent throughout this book. Not only are there many new theorems due to him and his collaborators but there are also numerous new proofs of previously known results.

Wall's most significant original additions to the theory are to be found in the first part. The outstanding result is probably the parabola theorem (§14) due to Scott and Wall. This in turn led Paydon and Wall and independently Leighton and the reviewer (a fact which is not mentioned in this book) to the discovery of a family of parabolic convergence regions (§32).

In the second part of the book Wall's original contributions consist largely of generalizations and extensions of previously known results and methods. This is not the case in Chap. X in which a new approach due to Wall and Frank to the problem of locating the roots of polynomial equations is presented. With every polynomial a "test fraction" is associated. The desired information is then obtained from the expansion of the test fraction into a $J$-fraction. A $J$-fraction (the term is due to Wall who has investigated $J$-fractions extensively) is a continued fraction of the form $K(-c_n^2/(b_n+z))$. The arrangement in tabular form of the computation for the expansions of functions into $J$-fractions, a device which is used a number of times in the book, seems to be due to Frank. This is not acknowledged. One also misses a reference to Frank's paper *The location of the zeros of polynomials with complex coefficients* (Bull. Amer. Math. Soc. vol. 52 (1946) pp. 890–898) in §49 as well as in the bibliography. Chap. XV is devoted to a characterization of continued fraction expansions of holomorphic functions with certain prescribed domains of definition and of values. It contains among others results of Schur on functions bounded in the unit circle. Chapters XVIII and XIX contain a large collection of expansions of functions into continued fractions. The book ends with a bibliography of 150 entries which lists all papers cited in the text as well as a number of other recent papers dealing with continued fractions.

It is the reviewer's opinion that an author who writes a book which has the appearance of being a reference book for a certain field has the obligation to include all significant results clearly belonging to this field. If complete coverage is not intended then that fact should be explicitly stated in the title of the book. The title of this book, together with the fact that it appears in a series which proclaims to be devoted to reference books, would lead a reader unfamiliar with the subject to the erroneous conclusion that this is indeed such a reference book. However, a comparison with the second part of Perron's book shows that this is not the case. In view of the fact that Perron's book is at present unavailable these omissions are regrettable. Not only classical material has been omitted however. In addition to the approaches to the convergence problem discussed in this book there is another one due to Leighton and the reviewer which led to a number of interesting results (see 7a, 56, 101a, 102, 103, 103a; the numbers refer to the bibliography at the end of Wall's book).

The author points out that a continued fraction can be considered as a sequence of linear fractional transformations and demonstrates in many places the usefulness and importance of this approach. In view of this it is surprising that he is reluctant to introduce and use iterated fractions (that is, sequences of general linear fractional transformations). It is true that in most cases iterated fractions can be transformed into continued fractions. However, even when this is possible, the resulting continued fraction may be considerably more involved than the iterated fraction from which it was derived. This is the case, for example, in Schur's expansion of functions bounded in the unit circle into iterated fractions. Wall uses continued fractions instead. Another instance where use of iterated fractions would have led not only to a somewhat more general result but also to a more elegant proof is in the discussion of the convergence of periodic continued fractions (§8). This could have been accomplished by the use of Schwerdtfeger's [84a] proof instead of Lane's. Finally, use of iterated fractions would have made inclusion of a discussion of the Pick-Nevanlinna interpolation problem extremely natural.

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*Substitutional analysis.* By D. E. Rutherford. (Edinburgh University Publications, Science and Mathematics, No. 1.) Edinburgh, University Press, 1948. 11+103 pp. 25 s.

The subject matter of this book, except for the last chapter, is Young's representation theory of the symmetric group. As Young developed the methods described here, he was always thinking of the elements of the symmetric group as substitutional operators applicable, in particular, to the theory of invariants. This fact explains the title.

In the introduction the author gives a brief account of Young's life as a country clergyman whose avocation remained the development of the mathematical ideas which interested him as a student at Cambridge. Those who knew him were always impressed by his sincerity and his modesty but above all by the originality and power with which he manipulated his own complicated machinery. The present book gives a connected account of Young's methods which has long been needed. The material was scattered throughout a long series of papers and, as is not surprising, the original presentation was sometimes involved. D. E. Rutherford has simplified it materially in places, and the reader can see the significance of the various steps taken.

As indicated above the theory here described was almost incidental in Young's work; it appeared as part of a larger plan, and in this light he always considered it. Young's originality was to some