ABSTRACTS OF PAPERS

The abstracts below are abstracts of papers presented by title at the Fifty-Fifth Summer Meeting of the American Mathematical Society. Abstracts of papers presented in person at that meeting will be included in the report of the meeting which will be published in the November issue of this BULLETIN.

Abstracts are numbered serially throughout this volume.

ALGEBRA AND THEORY OF NUMBERS

445. W. V. Parker: *On matrices whose characteristic equations are identical.*

Two generalizations of an earlier theorem (Bull. Amer. Math. Soc. vol. 55 (1949) p. 115) are given. (1) Let A be an $n\times m$ matrix of rank $r<n$ and let C be an $m\times n$ matrix such that $ACA=kA$ (k a scalar). If B is an $m\times n$ matrix, the characteristic equation of AB is $x^{n-r}\phi(x)=0$ and the characteristic equation of $A(B+C)$ is $x^{n-r}\phi(x-k)=0$. (2) If M and N are square matrices such that MN (or NM) $=N^2=0$, then M and $M+N$ have the same characteristic equation. (Received July 5, 1949.)

446. H. J. Ryser: *A note on a combinatorial problem.*

Let v elements be arranged into v sets such that each set contains exactly k distinct elements and such that every pair of distinct sets has exactly λ elements in common ($0<\lambda<k<v$). It is shown that these hypotheses imply that $\lambda=k(k-1)/(v-1)$. It then follows readily that in the given arrangement each element must occur exactly k times and every pair of elements must occur exactly λ times. For further results concerning the above combinatorial problem see the recent abstracts of Chowla and Ryser entitled *Combinatorial problems* I and II. (Received June 12, 1949.)

ANALYSIS

447. Garrett Birkhoff: *Group theory and differential equations.*

Let an nth order system of ordinary differential equations Γ be invariant under a solvable Lie group having m-dimensional sets of transitivity. Then the integration of Γ can be reduced to the integration of an $(n-m)$th order system, and quadratures. (Received May 5, 1949.)

448. S. H. Chang: *A generalization of a theorem of Hille and Tamarkin with applications.*

Hille and Tamarkin (Acta Math. vol. 57 (1931) pp. 1–75, p. 46) proved that if $\partial^p K(x, y)/\partial x^p = K^{(p)}(x, y)$ $(p=1, 2, \cdots, s-2)$ be continuous and $K^{(s-1)}(x, y) =\int_a^x g(t, y)dt+c(y)$ where $g(x, y)\subset L^2$, so that $\|g(x, y)\|^2=\int_a^b\int_a^b |g(x, y)|^2dxdy<+\infty$, then the set of characteristic values $\{\mu_h\}$ of $K(x, y)$ satisfies $1/|\mu_h|=o(h^{-s-1/2})$. In this paper the author proves that under the same hypothesis, we also have $1/|\lambda_h| =o(h^{-s-1/2})$, where $\{\lambda_h\}$ denotes the set of singular values of $K(x, y)$, that is, E. Schmidt's characteristic values of unsymmetric kernels. From this result, the author gets a new simpler proof of Hille and Tamarkin's theorem and also proves that if a kernel satisfies Hille and Tamarkin's condition, then there is a decomposition of $K(x, y)$ into at least $2s$ L^2 factors: $K(x, y)=K_1K_2 \cdots K_{2s}(x, y)$, where $K_1K_2(x, y) = \int_a^b K_1(x, s)K_2(s, y)ds$, $K_1K_2K_3(x, y)=(K_1K_2)K_3(x, y)$, and so on. In the appendix, the author gives another very simple proof that if $\partial^\rho K(x, y)/\partial x^\rho$ $(\rho=1, 2, \cdots, s)$ is continuous in $a \leqq x, y \leqq b$, then $1/|\mu_h| = o(h^{-s-1/2})$ and $1/|\lambda_h| = o(h^{-s-1/2})$; it is a generalization of a theorem of Weyl. (Received June 7, 1949.)

449. S. H. Chang: *A relation between the characteristic values and singular values of integral equations.*

It is known that an unsymmetric L^2 kernel $K(x, y)$ may have no characteristic values while the corresponding kernel $KK^*(x, y) = \int_a^b K(x, s)K(y, s)ds$ has at least one characteristic value and may even have infinitely many characteristic values. Now in this paper, assuming that $K(x, y)$ has characteristic values, and letting $\{\mu_h\}$ be the set of characteristic values of $K(x, y)$ and $\{\lambda_h^2\}$ be the set of characteristic values of the kernel $KK^*(x, y)$ such that $|\mu_1| \leqq |\mu_2| \leqq \cdots$, $|\lambda_1| \leqq |\lambda_2| \leqq \cdots$, the author gives two proofs that $|\lambda_1| \leqq |\mu_1|$ with some other interesting deductions. A parallel theorem about matrices can be easily deduced from a theorem of A. Loewy and R. Brawer (Tôhoku Math. J. vol. 32 (1929) pp. 45–49). By a parallel theorem is meant a theorem which plays a role in the Fredholm solution of linear integral equations which is analogous to Cramer's rule for solving linear systems of algebraic equations. However, it is evident that theorems about matrices cannot cover the situations encountered in integral equations. (Received June 7, 1949.)

450. S. H. Chang: *On the distribution of the characteristics values and singular values of linear integral equations.*

In the first part, a new method is given to determine the order of magnitude of characteristic values and singular values of linear integral equations, as given by the following theorem: Let $\{\mu_h[K]\}$ and $\{\lambda_h[K]\}$ be the sequences of characteristic and singular values, respectively, of the real L^2 kernel $K(x, y)$ and let $D_{kk'}(\lambda) = \sum_{n=0}^{\infty} (-1)\, c_n\lambda^n$ be the Fredholm determinant of $KK'(x, y) = \int_a^b K(x, s)K(y, s)ds$. Then the convergence of the series $\sum_{h=1}^{\infty} 1/|\lambda_h[K]|^{t}$ $(\tau>0)$ implies the convergence of the series $\sum_{h=1}^{\infty} 1/|\mu_h[K]|^{\tau}$. The converse is not true. Also a necessary and sufficient condition for the convergence of the series $\sum_{h=1}^{\infty} 1/|\lambda_h[K]|^{\tau}$ $(0<\tau<2)$ is the convergence of the series $\sum_{h=1}^{\infty}|C_h|^{\rho/2h}$. In the second part, the author proves that for the composite kernel $K(x, y) = K_1K_2 \cdots K_m(x, y)$ where $K_1K_2(x, y) = \int_a^b K_1(x, s)K_2(s, y)ds$, $K_1K_2K_3(x, y) = (K_1K_2)K_3(x, y)$, and so on, each $K_i(x, y) \subset L^2$, the series $\sum_{h=1}^{\infty} 1/|\lambda_h[K]|^{2/m}$ and $\sum_{h=1}^{\infty} 1/|\mu_h[K]|^{2/m}$ are both convergent. Also for any Marty kernel $K(x, y) = QL(x, y)$, where $Q(x, y)$ is a semidefinite continuous symmetric kernel, the series $\sum_{h=1}^{\infty} 1/|\lambda_h[K]|^{2/3}$ is convergent. Finally in the appendix, the author proves that if $K(x, y)$ is a real symmeric kernel such that $\partial^r K(x, y)/\partial x^r$ is continuous in $a \leqq x, y \leqq b$, then $K(x, y)$ has a decomposition into at least $2r$ factors: $K(x, y) = K_1K_2 \cdots K_{2r}(x, y)$. (Received May 6, 1949.)

451. S. H. Chang: *On the integral equations with normal kernels.*

A. Loewy and R. Brawer proved that for a matrix V such that $V\bar{V}' = \bar{V}'V_3$ the characteristic roots of the matrix $W = V'\bar{V}$ are of the form $\tau_i\bar{\tau}_k$, where $\tau_1, \tau_2, \cdots, \tau_n$ are the characteristic roots of the matrix V (Tôhoku Math. J. vol. 32 (1929) pp. 45–49). In this paper, the author proves that for normal kernels of integral equations, that is, kernels satisfying the condition $KK^*(x, y) = K^*K(x, y)$ or $\int_a^b K(x, s)\overline{K(y, s)}ds = \int_a^b \overline{K(s, x)}K(s, y)ds$, the set of characteristic values $\{\lambda_h^2\}$ of $KK^*(x, y)$ and those $\{\mu_h\}$ of $K(x, y)$, arranged in the usual order $|\lambda_1| \leqq |\lambda_2| \leqq \cdots$, $|\mu_1| \leqq |\mu_2| \leqq \cdots$, must satisfy the relation $|\mu_h| = |\lambda_h|$ $(h = 1, 2, \cdots)$. The author also obtains in the paper: (i) a proof that any normal kernel has at least one characteristic value; (ii) a generalization of a theorem of Heywood and Goursat to the effect that if $K(x, y) = \text{L.I.M. } \sum_{h=1}^{n} K_h(x, y)$, where $K_iK_j(x, y) = 0$, $i \neq j$; $i, j = 1, 2, \cdots$, and $\|K(x, y)\|^2 = \sum_{h=1}^{\infty} \|K_h(x, y)\|^2 < +\infty$, then $D_K^*(\lambda) = \prod_{h=1}^{\infty} D_{K_h}^*(\lambda)$; here $D_K^*(\lambda)$ is the modified Fredholm determinant in Carleman's sense of $K(x, y)$; (iii) a general solution of integral equations with normal kernels, and related results. (Received June 7, 1949.)

452. V. F. Cowling: *On the analytic continuation of Newton series.*

Let $h > 0$ and $0 < B \leqq \pi$. Let $a(w)$ be regular in the region $|\text{Arg } (w - h| \leqq B$, and there satisfy the condition $|a(h + Re^{i\psi})| \leqq R^k \exp(-LR \sin \psi)$ for some k and some L, $0 < L < 2\pi$, and all large R. Then if $f(z) = \sum_{n=0}^{\infty} a(n)(-1)^n(C_{z-1,n})$ has an abscissa of convergence $\sigma < +\infty$, it is entire. (Received June 16, 1949.)

453. I. I. Hirschman and J. A. Jenkins: *On lacunary Dirichlet series.*

This paper considers the relation between the degree of lacunarity of a Dirichlet series and the allowable modular orders of its zeros on the abscissa of holomorphy. A special case of these results is that if f is represented as the sum of a Dirichlet series whose exponents have exponent of convergence $\nu < 1$, and if f has an exponential zero of order μ with $\mu > \nu/(1 - \nu)$, then $f \equiv 0$. (Received April 5, 1949.)

454. E. J. McShane: *Linear functionals on certain Banach spaces.*

By using the property of uniform convexity, a simple derivation is obtained for the form of the general linear functional on spaces L_p, the integrals being over any space on whose subsets a measure function is defined, and also for the linear functional on spaces $L_p(B)$ of functions $f(\cdot)$ with values in a Banach space B and having $\|f(\cdot)\|^{p-1}f(\cdot)$ integrable in the sense of Bochner. (Received March 29, 1949.)

455. Leopoldo Nachbin: *A theorem of the Hahn-Banach type for linear transformations.*

A (real) normed space E is said to have the *extension property* if, for any normed space X and any vectors subspace $S \subset X$, every continuous linear transformation f: $S \to E$ has a continuous linear extension F: $X \to E$ with $\|F\| = \|f\|$. A necessary and sufficient condition for this is that the collection of all (closed) spheres of E should have the following property: every subcollection, any two members of which intersect, has a nonvoid intersection. Every complete vector lattice E, with an order unity e, can be made into a normed space in a natural way, the norm being $\|t\| = \inf \{\lambda: -\lambda e \leqq t \leqq \lambda e\}$. Then E has the extension property and e is an extreme point of the unity sphere. Conversely, if a normed space E has the extension property and contains e as an extreme point of the unity sphere, there is one and only one way of making E into a complete vector lattice with e as an order unity such that the norm deduced from the ordering be identical to the given one. By using some results established by S. Kakutani (Ann. of Math. vol. 42 (1941)) and by M. H. Stone (Canadian Journal of Mathematics vol. 1 (1949)), a representation theorem of a normed space with the extension property as the system of all continuous real functions over certain compact Hausdorff spaces and a relation to the theory of complete Boolean algebras are given. The question, involved in these results, as to whether every normed space with the extension property must contain some extreme point in its unity sphere is left undecided. (Received May 17, 1949.)

456. Jane C. Rothe: *The existence of multiple solutions of elliptic differential equations.*

By using topological methods, existence theorems are obtained for the differential equation (1) $F(x, y, z, p, q, r, s, t) = \psi_0(x, y)$ having in a region K an initial solution $z_0(x, y)$ which is elliptic relative to F. More precisely, if ϕ_0 is the boundary value of z_0, it is proved that for ϕ_1 and ψ_1 sufficiently close to ϕ_0 and ψ_0, equation (1) has a solution z_1 in region K when ψ_1 is substituted for ψ_0 in (1); and z_1 has boundary value ϕ_1. The theorems are new in that it is not necessary to assume that the Jacobi differential equation associated with (1) has only the zero solution in K. That is, the main assumption used in the classical-treatment is dropped. Also the existence of several distinct solutions is demonstrated for certain quasi-linear elliptic equations: $A(x, y)\partial^2 z/\partial x^2 + B(x, y)\partial^2 z/\partial x \partial y + C(x, y)\partial^2 z/\partial y^2 + f(x, y, z, p, q) = \psi(x, y)$. (Received June 23, 1949.)

457. C. F. Stephens: *Nonlinear q-difference equations.*

The author applies to q-difference systems a method used for difference systems in an earlier paper (Trans. Amer. Math. Soc. vol. 64 (1948) pp. 268–282), with suitable modifications, and obtains results similar to those found for difference systems. (Received June 21, 1949.)

458. J. L. Walsh and H. Margaret Elliott: *Polynomial approximation to harmonic and analytic functions: generalized continuity conditions.*

Zygmund has shown that the uniform condition (*) $|f(s+h)+f(s-h)-2f(s)| \leqq L|h|$ for a continuous periodic function of the real variable s is significant in the study of approximation of $f(s)$ by trigonometric polynomials. The authors investigate this condition in the complex domain and show that if $f(z)$ is analytic interior to an analytic Jordan curve C, continuous in the closed interior $\overline{C}$, and if $f^{(k)}(z)$ exists on C and satisfies there a condition (*), then $f(z)$ can be approximated in $\overline{C}$ by polynomials in z with degree of approximation $1/n^{k+1}$, and conversely. Let $g(x, y)$ denote Green's function for the exterior of C with pole at infinity, and let C_R denote the level curve $g(x, y) = \log R$ (>0); if $f^{(k)}(z)$ exists on C_R and satisfies there a condition (*), then $f(z)$ can be approximated in $\overline{C}$ by polynomials in z with degree of approximation $1/R^n n^{k+1}$; if $f(z)$ can be approximated in C with degree of approximation $1/R^n n^{k+2}$, then $f^{(k)}(z)$ exists on C_R and satisfies (*). If a Jordan curve C is sufficiently smooth, if $f(z)$ is analytic interior to C and continuous in $\overline{C}$, and if (*) is satisfied on C, then the condition $|f(z+\Delta z)+f(z-\Delta z)-2f(z)| \leqq L'|\Delta z|$ is valid uniformly for $z, z-\Delta z, z+\Delta z$ in $\overline{C}$. (Received May 20, 1949.)

459. D. V. Widder: *An inversion of the Lambert transform.*

The Lambert transform is defined by the equation $F(x) = \int_0^\infty a(t)/[e^{xt}-1]dt$. It is shown that the transform can be inverted by the symbolic operator $(1/\zeta(D)(1/\Gamma(D))$ applied to the function $F(e^{-x})$. Here $\zeta(s)$ is the zeta function of Riemann, $\Gamma(s)$ is the familiar function of Euler, and D stands for the operation of differentiation. The symbolic inversion operator is realized by the combination of the operation $\sum_0^\infty \mu(n)F(nx)$, which converts the Lambert transform into a Laplace transform, and the Post-Widder formula, which is known to invert the latter. Here $\mu(n)$ is the Möbius function. (Received May 28, 1949.)

Logic and Foundations

460. J. B. Rosser and Hao Wang: *Nonstandard models for formal logics.*

A model for a logic is called nonstandard if it fails to have certain properties which are apparently called for by the axioms of the logic. Thus, if a logic L contains the ordinal numbers and in a model M of L the ordinal numbers of L are represented by a subset of M which is not well-ordered, then M is a nonstandard model of L. For the system of logic known as Quine's New Foundations, it is shown that there are no standard models. For a certain logic L with a simple theory of types, the existence of a standard model is shown, and it is further shown that within various other formal logics (including one with a simple theory of types) one can prove that L has a standard model. It is further shown for logics in general that if they are ω-consistent, then they cannot prove about themselves that if they are consistent, then they must have a standard model. Since the usual theorems about modelling can be proved within such logics (a proof of this is sketched), it follows that the property of having a standard model is quite exceptional, and so may fail for other logics besides Quine's New Foundations. (Received April 26, 1949.)

Topology

461. Mary E. Estill: *Concerning abstract spaces.*

Let Axiom 1_3 denote the axiom resulting from the omission of condition (4) in the statement of Axiom 1 of R. L. Moore's *Foundations of point set theory*. Let Axiom $1''$ denote the axiom obtained by replacing condition (4) of Axiom 1 by the statement: if $g_1, g_2, \cdots$ is a sequence such that, for each n, g_n is a region of G_n containing g_{n-1}, then $g_1, g_2, \cdots$ have a point in common. In his paper *Concerning separability* Moore has shown that if Axioms 0 and 1 hold true and there do not exist uncountably many mutually exclusive domains, then space is separable. In the present paper it is shown that this proposition does not remain true if Axiom 1 is replaced by Axiom 1_3. The relationship between several modifications of Axiom 1 are considered. In particular it is shown that there is a space satisfying Axioms 0 and 1_3 which is not a subspace of any space satisfying Axioms 0 and $1''$ and also that there is a space satisfying Axioms 0 and $1''$ which is not a subspace of any space satisfying Axioms 0 and 1. (Received May 31, 1949.)