

rived from points  $O$ ,  $U$ ,  $O+U$ ,  $O+U\alpha$  and  $O+U\beta$ , and it is shown that Pappus' Theorem is equivalent to the commutative law  $\alpha\beta=\beta\alpha$ . In all such work, great care is taken to deal with all the special cases that may arise when points with different names coincide.

Chapter VI is an interlude on synthetic geometry, where linear spaces and incidence are primitive concepts, and the Propositions of Incidence are axioms. After constructing F. R. Moulton's non-Desarguesian plane geometry, the authors introduce Desargues' Theorem as an extra axiom to be used when the number of dimensions is only 2. A projectivity is defined as a product of perspectivities, and it is proved that any such chain of perspectivities can be reduced to as few as three. This is always an awkward piece of work, and the present version is as neat as any. (§§4 and 5 could perhaps have been shortened by first proving the uniqueness of the sixth point of the general quadrangular set.) The algebra of points on the projective line is developed in the manner of Veblen and Young, but with some improvements of detail. Homogeneous coordinates in  $n$  dimensions are introduced in a way that most ingeniously avoids any appeal to the Fundamental Theorem. Then various restrictions are considered. Pappus' Theorem is taken as a ninth axiom, and Desargues' Theorem is deduced from it as in Baker's *Introduction to plane geometry* (Cambridge, 1942, p. 26). Finite geometries are ruled out by a tenth axiom to the effect that a parabolic projectivity cannot be periodic.

Chapter VII contains an elegant exposition of Grassmann coordinates, generalizing the familiar properties of line-coordinates in 3-space. In the two final chapters, collineations and correlations are defined as linear transformations and are classified with characteristic thoroughness. The chapter on correlations is particularly valuable, as the authors have not shirked the formidable task of enumerating the various canonical forms for  $n$  dimensions. This last chapter also brings into prominence the skill of the compositors of the Cambridge University Press, which is to be congratulated on producing such a fine book in these difficult times.

H. S. M. COXETER

*Set functions.* By Hans Hahn and Arthur Rosenthal. The University of New Mexico Press, 1948. 9+324 pp. \$12.50.

When Hans Hahn died in 1934, he left manuscripts for the second volume of his treatise on real function theory. Arthur Rosenthal has now completed and edited that part of the work which deals with the theory of measure. This book gives a scholarly presentation of the foundations of the subject, taking account both of the theory of

completely additive functions on  $\sigma$ -fields and of the theory of Carathéodory.

The first chapter deals with the basic properties of additive and totally additive set functions, zero sets and complete fields, and the decomposition into regular and singular parts. Chapter II is devoted to the Carathéodory theory of measure. Special attention is paid to regular measures, and to  $n$ -dimensional Lebesgue measure. In the third chapter the author discusses the properties of measurable functions and sequences of such functions. The theory of integration is developed in Chapter IV by characterizing the indefinite integral as a new measure satisfying certain inequalities. A discussion of the approximation of integrals by sums, some mean value theorems, and convergence theorems, is followed by a section on product measures and the Fubini theorem. The last chapter deals with the Vitali covering theorem, the differentiation of measures and interval functions, and some applications to density and approximate continuity.

The book is carefully written and systematic. The proofs are given in great detail, a fact which may help many who wish to become acquainted with the fundamentals of measure theory.

HERBERT FEDERER

*Sur les groupes classiques.* By Jean Dieudonné. (Actualités scientifiques et industrielles, no. 1040; Publications de l'Institut de Mathématiques de l'Université de Strasbourg. VI.) Paris, Hermann, 1948. 82 pp.

The main purpose of this little book is to obtain the structural properties of the classical groups which can at present be obtained by purely algebraic methods. By skillful organization, complete mastery of his subject, and constant adherence to the "conceptual" point of view so fruitfully introduced into linear algebra in modern times, the author achieves this purpose with simplicity, efficiency, and elegance. The results are, with some exceptions, either old ones (to be found in the pioneering works of L. E. Dickson), or extensions of old ones to more general situations. But the long complicated matrix computations of the older literature, in which the ideas are frequently buried beyond recall, are here almost entirely replaced by conceptual arguments expressed in geometric language which brings out for inspection the intuitive geometric motivation in the proofs.

The classical groups are the full linear groups  $GL_n(K)$ , the symplectic groups  $Sp_n(K)$ , the orthogonal groups  $O_n(K, f)$ , and the unitary groups  $U_n(K, f)$ . The full linear groups over an arbitrary skew