the question of the existence of periodic solutions is considered in itself and also in connection with the Sturm-Liouville theory. In addition the problem of the existence of unbounded solutions is treated.

One chapter of the book is devoted to the finite analogue of the calculus of variations; a necessary condition analogous to the Euler equation is derived, and sufficient conditions are obtained also.

The final chapter provides a very brief summary (totalling only 14 pages) of some topics in the theory of linear difference equations with *continuous* independent variable. Among these topics are the gamma, digamma, trigamma functions, and so on, the Nörlund sum, and the adjoint equation.

This book is a welcome addition to the literature. In the first place, its presentation of the elements of the calculus of finite differences, being compact and clean-cut, and augmented by numerous exercises, makes it a serviceable and attractive text for a course in finite differences on about the first-year graduate level. In the second place, its treatment of the theory of linear difference equations emphasizes various interesting aspects of which there has been hitherto no unified account.

WALTER STRODT

The differential geometry of ruled surfaces. By Ram Behari. (Lucknow University Studies, no. 18.) Lucknow University, 1946. 94 pp.

This book is the basis of a series of Extension Lectures on *The differential geometry of ruled surfaces in euclidean space of three dimensions*, delivered by the author at the Lucknow University during 1942.

It is a concise and comprehensive survey of the differential geometry of ruled surfaces including ample references to the literature of the subject. The notation adopted is that of ordinary cartesian coordinates as used by Forsyth in his *Lectures on the differential geometry of curves and surfaces*, and by L. P. Eisenhart in his *Differential geometry*, Boston, 1909.

The book consists of six chapters. The first three form an introduction to the subject and the last three contain the investigations of the author.

The first three chapters are concerned with the standard theory of ruled surfaces. Chapter IV is devoted in the first part to the determination of ruled surfaces whose curved asymptotic lines can be found by quadratures. This problem has also been studied by Picard, Buhl, Goursat, Srinivasiengar, Hayashi. A systematic study of the osculating quadrics of a ruled surface is developed. The following new geometric meaning of Laguerre's function L' (see Weatherburn, *Differential geometry of three dimensions*, vol. 2, p. 139) is given: The vanishing of L' along a curve on a surface is the condition that the osculating quadrics Q of the ruled surface formed by drawing normals to the given surface along the curve be equilateral (three generators of Q are mutually orthogonal). This fourth chapter is concluded with the discussion of four new theorems concerning ruled surfaces with equilateral osculating quadrics.

Chapter V begins with the theory of applicability and continuous deformations of general surfaces. The famous theorem of Gauss and the problem of Minding are discussed. The general problem of infinitesimal deformations of surfaces, which has been developed by Goursat, Darboux, Haag, is considered together with applications to ruled surfaces. Some theorems of Bonnet and Beltrami relating to the applicability of ruled surfaces are proved. A new invariant is obtained under any deformation of a ruled surface into a ruled surface.

Chapter VI deals in the first part with the general theory of a rectilinear congruence. The differential geometry of the ruled surfaces of the congruence through a fixed ray L is developed. Various proofs are given of the theorem of K. Ogura that repeated applications of forming the jacobian with respect to the differentials, starting with Sannia's two quadratic forms f and ϕ, lead only to five distinct families of ruled surfaces through a line L of the congruence.

In the second part of Chapter VI applications are made to normal congruences of lines. In a general rectilinear congruence there are ∞^2 ruled surfaces with equilateral osculating quadrics, but only ∞^1 in a normal congruence.

In the third and final part is studied the pitch p at a ray of a pencil of the congruence. This is the curvilinear integral $p=\int_C R dr$, where R is the unit vector parallel to the ray through the point of a simple closed curve C whose position vector is r and where C is on the director surface S through the ray L. This was first studied by E. Cartan. The following three theorems which reduce to the corresponding theorems of Malus-Dupin, Beltrami, Ribaucour by letting $p=0$, so that the congruence becomes normal, are established.

I. The pitch p at a ray of a pencil of the congruence formed by the incident rays remains unaltered by refraction except for a factor which is equal to the ratio of the refracting indices of the two media. The pitch associated with the congruence formed by incident rays remains unaltered by reflection.

II. If the surface of reference S be deformed in such a way that the direction of the lines of the congruence with respect to S be unchanged, then the pitch p is unaltered.

III. If tangent planes be drawn through the rays of a congruence of any surface, the pitch p remains unaltered if the surface be deformed in any manner carrying the rays in its tangent planes.

Also is discussed the limiting value of the pitch, $dp/d\sigma$, which has interesting applications, in particular to the Anormalita of Levi-Civita.

This book is very readable and can be easily understood by any student who has had a first year course in Differential Geometry. In the opinion of the reviewer, this is a worthwhile addition to the library of any one who is interested in the classical theory of surfaces.

John DeCicco

Methods of algebraic geometry. By W. V. D. Hodge and Daniel Pedoe. Cambridge University Press, 1947. 8+440 pp. $6.50.

This work by two disciples of H. F. Baker naturally retains some of the flavor of the latter's *Principles of geometry*; but in keeping with the modern trend it is more algebraic and less geometrical. The spirit of the book is indicated by the fact that there is no mention of order or continuity. The first four of the nine chapters are concerned with algebraic preliminaries, chiefly in preparation for vol. II, and are so clear and concise that they would serve very well as an introduction to modern algebra, quite apart from their application to geometry. The topics treated in this part include groups, rings, integral domains, fields, matrices, determinants, algebraic extensions, and resultant-forms. The theory of linear dependence is developed without assuming commutativity of multiplication, and there is a neat algebraic treatment of partial derivatives and Jacobians.

Analytic geometry of projective n-space is taken up in Chapter V. A point of right-hand projective number space is defined as a set of right-hand equivalent $(n+1)$-tuples of elements of a given field, not necessarily commutative; and right-hand projective space is defined as a set of elements which can be put in one-to-one correspondence with the points of such a number space by means of any one of a certain set of "allowable" coordinate systems. A linear subspace is defined as the set of points which are linearly dependent on $k+1$ linearly independent points; and the Propositions of Incidence follow readily. The notation of Möbius' barycentric calculus, as developed by Baker, arises naturally at this stage, and is used in proving Desargues' Theorem for coplanar triangles. Quadrangular constructions are given for the points $O+U(\alpha+\beta)$ and $O+U\alpha\beta$ as de-