THE OCTOBER MEETING IN NEW YORK

The four hundred thirty-ninth meeting of the American Mathematical Society was held at New York University on Saturday, October 30. The attendance was approximately three hundred, including the following two hundred fifty-seven members of the Society:

Milton Abramowitz, C. R. Adams, C. F. Adler, E. J. Akutowicz, R. G. Albert, E. B. Allen, C. B. Allendoerfer, R. L. Anderson, T. W. Anderson, R. G. Archibald, L. A. Aroian, Helmut Aulbach, Frederick Bagemihl, F. E. Baker, Joshua Barlaz, A. F. Bartholomay, F. D. Bateman, P. T. Bateman, F. P. Beer, Stefan Bergman, R. R. Bernard, D. L. Bernstein, Lipman Bers, Garrett Birkhoff, H. W. Bode, A. M. F. Borgers, D. G. Bourgin, H. W. Brinkmann, Paul Brock, Azelle Brown, A. B. Brown, J. D. Campbell, P. G. Carlson, P. W. Carruth, K. Chandrasekharan, Y. W. Chen, Alonzo Church, Randolph Church, L. W. Cohen, I. S. Cohn, H. R. Cooley, T. F. Cope, Natalie Coplan, Richard Courant, L. M. Court, V. F. Cowling, J. S. Cronin, D. A. Darling, Martin Davis, M. M. Day, B. V. Dean, J. B. Diaz, R. L. Dietzold, M. P. Dolciani, J. L. Doob, Avron Douglis, C. H. Dowker, Y. N. Dowker, R. H. Downing, Arnold Dresden, Nelson Dunford, Aryeh Dvoretzky, J. J. Eachus, Samuel Eilenberg, H. M. Elliott, Bernard Epstein, R. M. Exner, W. H. Fagerstrom, A. B. Farnell, J. M. Feld, C. D. Firestone, D. A. Flanders, Edward Fleisher, W. W. Flexner, R. M. Foster, R. H. Fox, D. A. S. Fraser, Bernard Friedman, G. N. Garrison, B. P. Gill, Sidney Glusman, V. D. Gokhale, Adolph Goodman, R. E. Goodman, M. J. Gottlieb, G. E. Gourrich, P. H. Graham, Harriet Griffin, George Grossman, Emil Grosswald, E. J. Gumbel, C. B. Hailperin, Carl Hammer, R. W. Hamming, E. E. Hammond, L. B. Hedge, G. A. Hedlund, M. H. Heins, Alex Heller, L. H. Herbach, H. L. Herrick, Einar Hille, A. J. Hoffman, Banesh Hoffmann, T. R. Hollcroft, G. B. Huff, E. M. Hull, T. R. Humphreys, Witold Hurewicz, L. C. Hutchinson, Eugene Isaacson, Nathan Jacobson, Fritz John, L. G. F. Jones, Irving Kaplansky, M. E. Kellar, D. E. Kibbey, S. A. Kiss, M. S. Klamkin, J. R. Kline, Morris Kline, E. G. Kogbetliantz, Horace Komm, B. O. Koopman, Saul Kravetz, Wouter van der Kulk, H. K. Kutman, H. G. Landau, A. W. Landers, J. R. Lee, Solomon Lefschetz, Joseph Lehner, R. A. Leibler, Max LeLeiko, D. C. Lewis, Samuel Linial, M. A. Lipschutz, Charles Loewner, A. J. Lohwater, E. R. Lorch, Lee Lorch, L. A. MacColl, H. M. MacNeille, A. J. Maria, M. H. Maria, W. T. Martin, Imanuel Marx, F. I. Mautner, F. M. Mautner, D. G. Mead, Paul Meier, K. S. Miller, G. C. Miloslavsky, Deane Montgomery, C. R. Morris, F. J. Murray, D. S. Nathan, C. A. Nelson, David Nelson, J. D. Newburgh, Morris Newman, Louis Nirenberg, P. B. Norman, I. L. Novak, T. E. Oberbeck, L. F. Ollmann, Paul Olum, Morris Ostrofsky, E. R. Ott, J. C. Oxtoby, J. S. Oxtoby, T. E. Peacock, A. M. Peiser, C. R. Phelps, Everett Pitcher, E. L. Post, Walter Prenowitz, M. H. Protter, W. T. Puckett, A. L. Putnam, Hans Rademacher, Arthur Radin, J. F. Randolph, G. N. Raney, Eric Reissner, Daniel Resch, Helene Reschovsky, Moses Richardson, C. E. Rickart, J. F. Ritt, J. H. Roberts, S. L. Robinson, V. N. Robinson, R. E. Root, I. H. Rose, P. C. Rosenbloom, Bernard Sachs, Raphael Salem, H. E. Salzer, L. A. Santaló, Arthur Sard, Samuel Schecter, Henry Scheffé, I. J. Schoenberg, Pincus Schub, Abraham Schwartz, H. M. Schwartz, J. T. Schwartz, G. E. Schweigert, Abraham Seidenberg, D. B. Shaffer, C. E. Shannon, H. N. Shapiro, I. M. Sheffer, Seymour Sherman, R. A. Siegel, D. N. Silver, R. C.

Simpson, James Singer, M. M. Slotnick, N. E. Steenrod, S. K. B. Stein, Fritz Steinhardt, Wolfgang Sternberg, J. J. Stoker, R. C. Strodt, Walter Strodt, Fred Supnick, Otto Szasz, Feodor Theilheimer, J. M. Thomas, D. L. Thomsen, G. L. Tiller, A. W. Tucker, J. W. Tukey, P. M. Treuenfels, Annita Tuller, J. L. Ullman, A. H. Van Tuyl, Oswald Veblen, J. L. Walsh, W. G. Warnock, Wolfgang Wasow, G. C. Webber, Alexander Weinstein, Louis Weisner, J. G. Wendel, M. E. White, P. M. Whitman, Hassler Whitney, Albert Wilansky, Jacob Wolfowitz, Arthur Zeichner, J. J. Zeig, J. A. Zilber, H. J. Zimmerberg, Leo Zippin.

On Saturday morning there were two sections for contributed papers: one in Analysis, in which Professor J. F. Randolph presided, and one in Algebra, Topology, Logic, and Applied Mathematics, in which Professor Eric Reissner presided.

At 2:00 P.M. Professor Raphael Salem of Massachusetts Institute of Technology gave an address on *Convexity theorems*. President Einar Hille presided.

Abstracts of the papers presented follow below. Abstracts whose numbers are followed by the letter "*t*" were presented by title. Paper number 582 was read by Dr. Elliott, paper number 591 by Mr. Linial, and paper number 596 by Professor Cohen. Mr. Leutert was introduced by Dr. Stefan Bergman and Dr. M. M. Schiffer, and Mrs. Macintyre was introduced by Professor I. E. Segal.

Algebra and Theory of Numbers

1. Mary P. Dolciani: *On a problem in the additive theory of numbers.*

Using the Hardy-Littlewood circle method, the author has proved that an indefinite diagonal, ternary quadratic form $f(x)$ represents almost all integers consistent with the generic character of the form. More precisely, if $E(n)$ denotes the number of integers c less than n such that the equation $f(x)=c$ is solvable in the ring of p-adic integers for all primes p, but is not solvable in rational integers, then $E(n)=o(n)$. Similarly, the corresponding result can be proved for the case in which the variables are prime. (Received September 18, 1948.)

2*t*. A. L. Foster: *On the permutational representation of general sets of operations by partition lattices.*

Let $U=\{\cdots, x, \cdots\}$ be a "ground" class (with or without operational structure), and $\Omega=\{\cdots, \omega, \cdots\}$ an arbitrary set of unary mappings (= "monotations") of U onto $U: x\rightarrow \epsilon U$. An "$\Omega$-partition" of U is a partition whose "cells" remain intact under all the cellular monotations induced by the various ground monotations $\omega\in\Omega$ (and also by the structure operations, if any). This paper studies the partition lattice structure of the class of all (a) "permutational-," respectively (b) "univoque"-Ω-partitions, in which the above cellular "factor monotations" are exclusively (a) permutations, respectively (b) univoque cellular mappings. This theory considerably generalizes the classical imprimitivity set theory of groups of transformations. As one typical result a simple lattice condition is found (which is always satisfied if, for instance, U is finite) insuring that all permutational Ω-partitions be "derivable" from a unique "atomic" one. One is led to a representative theory, a theory of "permutations-in-the-large," and, by use of previously introduced notions [Alfred L. Foster, *Maximal idempotent sets in a ring with unit*, Duke Math. J. vol. 13 (1946)] to an invariant theory of such general sets of transformations, similar to the homomorphism theory of rings and groups. (Received August 30, 1948.)

3t. Joseph Lehner: *Proof of Ramanujan's partition congruence for the modulus* 11^3.

The author proves Ramanujan's congruence: $p(11^3n+721)\equiv 0 \pmod{11^3}$, $n=0, 1, 2, \cdots$, where $p(n)$ is the number of partitions of n. The proof, in which he makes use of the theory of modular functions, is an extension of the method employed in a previous paper (*Ramanujan identities involving the partition function for the moduli* 11^α, Amer. J. Math. vol. 65 (1943)), in which the analogous congruence was established for 11 and 11^2. (Received September 7, 1948.)

4. F. I. Mautner: *On the principal irreducible representations of a discrete group.* Preliminary report.

The author studies in more detail the direct integral decomposition of the regular representation $U:g\rightarrow U(g)$ of a countably infinite discrete group G into irreducible representations. Let G_0 be the union of the finite classes of conjugate elements of G. It is shown that if G_0 is of infinite index in G then almost all the factors (in the sense of Murray and von Neumann, Ann. of Math. vol. 37 (1936) pp. 116–229) into which the weakly closed operator algebra generated by the $U(g)$ decomposes are of class II_1. From this it follows that almost all the irreducible (unitary) representations which occur in any direct integral decomposition of U into irreducible representations must be infinite-dimensional. In the proofs are used von Neumann's theory of generalized direct sums of Hilbert spaces and also results announced by the author in the Proc. Nat. Acad. Sci. U.S.A. vol. 34 (1948) pp. 52–54. (Received September 13, 1948.)

5t. T. S. Motzkin: *The Euclidean algorithm.*

A constructive criterion for the existence of a Euclidean algorithm within a given integral domain is derived, and from among the different possible Euclidean algorithms in an integral domain one is singled out. The same is done for "transfinite" Euclidean algorithms. The criterion obtained is applied to some special rings, in particular rings of quadratic integers. Different sets of axioms for the Euclidean algorithm and related notions are compared, and the possible implications for the classification of principal ideal rings, and other integral domains, indicated. (Received October 11, 1948.)

6t. Leopoldo Nachbin: *On a characterization of the lattice of all ideals of a Boolean ring.*

A nonempty lattice L is isomorphic to the lattice of all ideals of a Boolean ring R, that is, a ring in which every element is idempotent, if and only if L satisfies the following conditions: (1) L is complete; (2) every element in L is the supremum of all smaller compact elements (x being *compact* if $x\leqq\bigvee_\lambda x_\lambda$ implies $x\leqq\bigvee_i x_{\lambda_i}$ for some finite subfamily); (3) the infimum of two compact elements is compact; (4) L is distributive; and (5) every inf-irreducible element x in L (that is, an element having no representation $x=x_1\wedge x_2$ with $x_1\neq x$ and $x_2\neq x$) distinct from its last element I is a dual atom (that is, $x<y$ implies $y=I$). Then R is essentially unique. R has a unity if and only if L satisfies the following equivalent conditions: (6) the last element of L is compact; (6′) every nonempty chain of elements of L distinct from I has a supremum distinct from I. The lattice of all open sets of a Hausdorff space satisfies (1), $\cdots$, (5) if and only if the space is locally compact and totally disconnected. Therefore, by Stone's topological representation of Boolean rings, these conditions characterize the lattice of all open sets of such a space. The compact case corresponds to adding (6), or (6′). (Received September 9, 1948.)

7*t*. Sam Perlis and G. L. Walker: *Finite abelian group algebras. II.*

Earlier results of the authors (see Bull. Amer. Math. Soc. Abstract 54-7-298) for group algebras G_F of abelian groups G of prime power order n over any nonmodular field F are extended to the case in which n is arbitrary and F is any field of characteristic not dividing n. (Received September 2, 1948.)

8*t*. H. E. Salzer: *Verification of the first twenty thousand cases of an empirical theorem with the aid of a device for mass computation.*

This investigation established that the first 20,000 multiples of 5, that is $5m$ up to $5m=100,000$, were a sum of four nonnegative tetrahedral numbers $n(n+1)(n+2)/6$ (incidentally, checking H. W. Richmond's statement about the first 4,000 multiples). Use was made of the author's elementary theorem that every multiple of 5 which is a sum of four tetrahedral numbers is a sum of two numbers of a set S, the members of S consisting of multiples of 5 that are the sum of two tetrahedral numbers. All numbers of $S\leqq 100,000$ were indicated by the position of a blacked box on coordinate paper. Simultaneous mass additions of many numbers of S to any particular number of S was performed by superposing translucent coordinate paper bearing a replica of the box-configuration for S, and marking the spaces over black boxes. This computation, being lengthy, was done only once. Although, without checking, this result cannot be absolutely guaranteed, an exception is extremely improbable, since a tiny part of all conceivable combinations of two members of S sufficed for every $5m$. A photographic analogue of this process could handle similar computations of 1000 times the range, and much more rapidly. (Received September 1, 1948.)

ANALYSIS

9*t*. E. F. Beckenbach: *Convexity theorems for Legendre polynomials.*

It is shown that for Legendre polynomials $P_n(x)$ the function $\Delta_{n,1}(x)\equiv P_n^2-P_{n-1}P_{n+1}$ is a concave function of x for all real x. The Turán-Szegö theorem that $\Delta_{n,1}\geqq 0$ for $|x|\leqq 1$ is an immediate consequence, as are other results including the following: for $|x|\leqq 1$ we have $\Delta_{n,1}(0)\geqq\Delta_{n,1}(x)\geqq(1-|x|)\Delta_{n,1}(0)$; for $|x|\geqq 1$ we have $\Delta_{n,k}(x)\equiv P_n^2-P_{n-k}P_{n+k}\leqq 0$ and a fortiori $|P_n|\leqq(|P_{n-k}|+|P_{n+k}|)/2$, $k=1, 2, \cdots$. (Received August 30, 1948.)

10*t*. Garrett Birkhoff: *Functionals on spaces of continuous functions.*

Let R be any bounded open set in Euclidean m-space; let S be the boundary of R, and K the compact set $R \cup S$. Let $C(K)$ be the Banach space of continuous functions $f(\vec{x})$ on K, with $\|f\| = \sup |f(\vec{x})|$. It is shown that every linear functional λ on $C(K)$ has the form $\lambda(f) = \int f(\vec{x}) d\mu$, where $d\mu = dg(\vec{x})$ is the additive function of intervals corresponding to a Stieltjes integral for a left-continuous function $g(\vec{x})$ of bounded variation. This gives a useful compactness principle for measure functions; the case $m=1$, K an interval, has been treated by F. Riesz. An extension to any compact metrizible compact topological space K is possible. (Received August 10, 1948.)

11. Garrett Birkhoff: *Existence of generalized Poisson integrals.*

Let $U(\vec{x})$ be any continuous function not assuming its maximum on the boundary S of a compact domain $K = R \cup S$ of definition. Then there exists a point $\vec{b}$ in R and an $\epsilon > 0$ such that for every mass-function with center of gravity $\vec{b}$, $\int U(\vec{x}) d\mu \leqq U(\vec{b}) - \epsilon \int |\vec{x} - \vec{b}|^2 d\mu$. Let $\mathfrak{S}$ be any family of continuous functions U, satisfying a fixed integro-differential equation of elliptic type: that is, such that for each point $\vec{a}$ of R there exists a family of measure functions μ_h (with total measure one) such that $\lim_{h\to 0} |\int U(\vec{x}) d\mu_h - U(\vec{a})| / \int |\vec{x} - \vec{a}|^2 d\mu_h \to 0$. Then there exists for each $\vec{a}$ in R a measure function μ_a such that $\int (U(\vec{x}) d\mu_a = U(\vec{a})$ for all $U \in \mathfrak{S}$, and $\int \vec{x} d\mu_a = \vec{a}$. A maximum principle and uniqueness theorem are corollaries. It also shows that any continuous function whose generalized Laplacian (in the sense of Petrini or Privaloff-Saks) vanishes identically in R is harmonic. (Received August 10, 1948.)

12. P. W. Carruth: *Roots and factors of ordinals.*

An ordinal is termed a *root* of a second ordinal if the latter is a power of the former. Use is made of results of S. Sherman (Bull. Amer. Math. Soc. vol. 47 (1941) pp. 111–116) to determine necessary and sufficient conditions for one ordinal to be a root of another. These conditions lead to a proof of the fact that the set of roots of an ordinal is closed. Results of E. Jacobsthal (Math. Ann. vol. 67 (1909) pp. 130–144) are used to give a short alternative proof of this last fact and of the known fact that the set of left factors of an ordinal is closed. Results also are obtained concerning the decomposition of an ordinal into the product of prime factors. (Received September 17, 1948.)

13. Aryeh Dvoretzky: *On sections of power series.* II.

The results announced in an earlier abstract (Bull. Amer. Math. Soc. Abstract 54-3-130) about the behavior of the partial sums, and so on, of a power series near a point on its circle of convergence can be extended and made more precise. Thus (1) can be strengthened into the statement that for infinitely many n each value of $|w| < \rho_n$ is assumed more than cn times, $c = c(\epsilon)$ being a positive constant. A similar remark applies to (2) and (3). The conclusion of (4) holds under the much weaker assumption $\limsup \log |a_n| / \log n \neq \pm \infty$. Also when this assumption fails, definite results may be given. Thus if $\lim \log |a_n| / \log n = -\infty$ and $\sum_0^\infty a_n z_0^n \neq 0$ we have $\liminf d_n/\gamma_n \leqq 1$ where $\{\gamma_n\}$ is a monotone sequence decreasing to zero defined as follows: Let n_0 be such that $|a_n| < 1$ for $n > n_0$, then for $n > n_0$ the points (n, γ_n) are obtained through Newton polygon construction from the sequence $\{(n, -n^{-1}) \log |a_n|)\}$. More precisely: for every $\epsilon > 0$ there exist infinitely many n for which the nth partial sum has a root in C_n: $|z - z_0 \exp \gamma_n| < (1+\epsilon) n^{-1} \log n$. Statements are also made about the domains covered by $w = s_n(z)$ when z is in C_n. Similar results hold also in the case lim sup log $|a_n|/\log n=\infty$. It is sufficient for the validity of these results that $f(z)$ be bounded away from zero and infinity as $z\to z_0$ through some sector containing the origin. When z_0 is a regular point and $f(z_0)\neq 0$, $n^{-1}\log n$ can be strengthened to $O(n^{-1})$. Finally, similar results hold for remainders and general sections. (Received September 20, 1948.)

14*t*. H. Margaret Elliott: *On approximation to functions satisfying a generalized continuity condition.*

Let C be an analytic Jordan curve. Let $u(z)$ be harmonic interior to C, continuous on the corresponding closed region $\overline{C}$, and let $\partial^k u/\partial s^k$, s arc-length measured along C, have modulus of continuity $\omega(\delta)$ on C. It is shown that there exist harmonic polynomials $p_n(z)$ of respective degrees n, $n=1, 2, \cdots$, such that $|u(z)-p_n(z)| \leqq M\omega(1/n)/n^k$, z in $\overline{C}$. Given the existence of a sequence of harmonic polynomials $p_{n_j}(z)$ of respective degree n_j, $j=1, 2, \cdots$, which satisfy $|u(z)-p_{n_j}(z)| \leqq \Omega(n_j)/n_j^k$, z in $\overline{C}$, it is shown under suitable restrictions on C, $\Omega(x)$, and the sequence n_j that $\partial^k u/\partial s^k$ exists on C and has modulus of continuity (1) $\omega(\delta) \leqq L[\delta\int_{n_1}^{1/\delta}\Omega(x)dx + \int_{\eta_0/\delta}^{\infty}\Omega(x)/x)dx]$, $0<\delta\leqq 1/n_1$. Furthermore, if $\partial^k u/\partial s^k$ has modulus of continuity $\omega(\delta)$ on C_R, $R>1$, then there exist harmonic polynomials $p_n(z)$ such that $|u(z)-p_n(z)| \leqq M_1\omega(1/n)/n^k R^n$, z in $\overline{C}$. Conversely if there exist harmonic polynomials $p_{n_j}(z)$, $j=1, 2, \cdots$, such that $|u(z)-p_{n_j}(z)| \leqq \Omega(n_j)/n_j^k R^{n_j+1}$, z on $\overline{C}$, $R>1$, then $\partial^k u/\partial s^k$ has modulus of continuity on C_R satisfying a condition of the form (1). Analogous results are obtained for analytic functions. (Received September 17, 1948.)

15*t*. H. Margaret Elliott: *On approximation to functions satisfying a generalized continuity condition, in the sense of least pth powers.*

This paper is a sequel to one on Tchebycheff approximation to functions satisfying a generalized continuity condition. Let $u(z)$ be defined on C. It is shown that if there exist harmonic polynomials $p_{n_j}(z)$ of respective degrees n_j, $j=1, 2, \cdots$, such that $\int_C\Delta(z)|u-p_{n_j}(z)|^p ds \leqq [\Omega(n_j)]^p/n_j^{kp+1}$, $p>0$, then under suitable restrictions on C (it is sufficient for C to be an analytic Jordan curve), $\Delta(z)$, $\Omega(x)$, and the sequence n_j, we have $u(z)=u_1(z)$ almost everywhere on C, where $u_1(z)=\lim_{j\to\infty} p_{n_j}(z)$, z on $\overline{C}$, and $\partial^k u_1/\partial s^k$, s arc-length measured along C, has on C modulus of continuity (1) $\omega(\delta) \leqq L[\delta\int_{n_1}^{1/\delta}\Omega(x)dx+\int_{\eta_0/\delta}^{\infty}(\Omega(x)/x)dx]$, $0<\delta\leqq 1/n_1$. Suppose the rectifiable Jordan curve C is such that $x'(w)$ is continuous and different from zero, $|w|=1$, where $z=x(w)$ maps the interior of $|w|=1$ onto the interior of C; if there exist harmonic polynomials $p_{n_j}(z)$, $j=1, 2, \cdots$, such that $\int_C\Delta(z)|u-p_{n_j}|^p ds \leqq [\Omega(n_j)/n_j^k R^{n_j+1}]^p$, $p>1$, $R>1$, then under suitable restrictions on $\Delta(z)$, $\Omega(x)$, and n_j, it is proved that $f_1^{(k)}(z)$ has modulus of continuity in $\overline{C}_R$ which satisfies a condition of the form (1), where $f_1(z)$ is the analytic function such that $\mathfrak{R}[f_1(z)]=u_1(z)$. The corresponding problems are treated for approximation measured by a surface integral. (Received September 17, 1948.)

16*t*. A. S. Galbraith, Wladimir Seidel, and J. L. Walsh: *On the growth of derivatives of functions omitting two values.*

Results of two of the authors (W. Seidel and J. L. Walsh, *On the derivatives of functions analytic in the unit circle* $\cdots$, Trans. Amer. Math. Soc. vol. 52 (1942) pp. 128–216; in particular Chap. IV) on relations between the radius of p-valence and the rate of growth of the derivatives of functions analytic in the unit circle are extended. The principal result is: If $f(z)$ omits two values, if $\{z_n\}$ is an infinite sequence of points within the unit circle for which $w_n = f(z_n) \to \infty$, and if the radius of p-valence $D_p(w_n)$, for some integer $p > 0$, satisfies $\lim_{n\to\infty}(|w_n| \log |w_n|)^p D_p(w_n) = 0$, then $\lim_{n\to\infty}(1-|z_n|^2)^k |f^{(k)}(z_n)| = 0$, $k = 1, 2, \cdots, p$. (Received October 15, 1948.)

17*t*. V. L. Klee. *On a theorem of Mazur.*

A well known theorem of Mazur states that a convex body in a linear normed space is supported at each of its boundary points. The usual proofs lean heavily on the norm, so are not applicable in more general spaces. The author gives a simple proof (using Zorn's lemma) which is valid in an arbitrary linear topological group. (Received August 9, 1948.)

18*t*. V. L. Klee: *The convex hull of a Borel set.*

Suppose that X is a Borel set of type G_u in the Banach space E. Then the convex hull $C(X)$ is also a Borel set, of type $G_{u\sigma}$. If E is finite-dimensional and X is a Borel set of a certain type in E, then $C(X)$ is a Borel set of the same type, unless X is closed and unbounded, in which case $C(X)$ is a G_δ set and F_σ set, but is not necessarily closed. The proof uses Schauder's theorem on the interiority of linear continuous transformations. (Received August 31, 1948.)

19*t*. V. L. Klee: *The Tietze-Matsumura theorem in a linear topological space.*

Suppose that E is a linear topological group in which each line is a continuous image (under the natural mapping) of the real line. Then each closed, connected, locally convex subset of E is convex. It is believed that the proof is simpler, and the restriction on the space is less than those in any preceding paper on this theorem. (Received September 2, 1948.)

20*t*. Werner Leutert: *On the solution of boundary value problems in the theory of elasticity.*

Analogous to the theory of kernel functions for partial differential equations [Bergman, Duke Math. J. vol. 14 (1947) pp. 349–366; Bergman and Schiffer, Duke Math. J. vol. 14 (1947) pp. 609–638; Bull. Amer. Math. Soc. vol. 53 (1947) pp. 1141–1151; Duke Math. J. vol. 15 (1948) pp. 535–566; Aronszajn, Proc. Cambridge Philos. Soc. vol. 39 (1943) pp. 133–153; Work performed under Contract N5ori 76-16 NR 043–046, Technical Report 5, Preliminary Note] a reproducing transformation is introduced for the system of three partial differential equations of second order for the components of the displacement vector in the theory of elasticity. The boundary value problem is stated and solved in a general form of which the three usually considered boundary value problems are limit cases. Formulas are given for the actual computation of the analogs of Green's, Neumann's, and Robin's functions. The results are a generalization of those obtained by Bergman [Math. Ann. vol. 98 (1927) pp. 248–263] for the solution of the first and second boundary value problems. (Received September 16, 1948.)

21*t*. Sheila S. Macintyre: *On the zeros of successive derivatives of integral functions.*

It is shown that if $f(z)$ is an integral function satisfying $\limsup_{r\to\infty} (\log M(r)/r)$

$<.7259$ and if $f^{(n-1)}(z_n)=0$, the sequence $\{z_r\}$ having all its limit points in the unit circle, then $f(z)\equiv 0$. Previous results are thereby improved and extended, see N. Levinson (Duke Math. J. vol. 11 (1944) pp. 729–733; vol. 12 (1944) p. 335). A further theorem of the same type is proved in which the unit circle is replaced by the domain consisting of the points distant less than h from the segment $-1\leqq x\leqq 1$, and the constant .7259 is replaced by $4^{-1}\pi \exp(-\pi^2 h/8)$. This is an extension of a theorem due to I. J. Schoenberg (Trans. Amer. Math. Soc. vol. 40 (1936) pp. 12–23). A shortened proof of Schoenberg's original result is also given. (Received September 7, 1948.)

22*t*. C. N. Moore: *On the functional behavior of a function defined by a certain Dirichlet's series.*

Form a sequence of the odd integers and place below them a sequence obtained by increasing the numbers above by two. Let p_c represent any prime in the second sequence which lies below a number in the first sequence which is a multiple of one of the first λ odd primes, $p_1, p_2, \cdots, p_\lambda$. Consider the function $L_c(s)=\sum p_c(\log p_c)/(p_c)^s$ on the line $s=1+it$. The series on the right converges absolutely for $R(s)>1$, and the analytic function thus defined can be extended to the left of the line $s=1+it$, thus furnishing a means of studying its functional behavior on that line. Such properties of the function have application to the problem of the infinitude of prime pairs. (Received October 11, 1948.)

23*t*. Leopoldo Nachbin: *On strictly minimal topological division rings.*

The author shows that several theorems usually established for real or complex topological vector spaces depend on the scalar domain being strictly minimal or complete. A topological division ring K is *strictly minimal* if, T_K denoting its topology, there exists no Hausdorff topology T on K such that $T<T_K$ and K endowed with T is a topological vector space over K endowed with T_K. *Completeness* is meant in Weil's sense for the additive group of K. The following results are typical and are generalizations of part of the recent work of J. Braconnier and I. Kaplansky: (1) If K is strictly minimal and f is a linear functional defined on a topological vector space over K, then f is continuous if and only if $f^{-1}(0)$ is closed; if K is not strictly minimal this need not be true. (2) In order that every finite-dimensional vector space over K should have only one admissible topology it is necessary and sufficient that K be strictly minimal and complete. (3) In order that every vector space automorphism of any finite-dimensional topological vector space over K should be homeomorphic it is necessary and sufficient that K be strictly minimal and complete. (4) Every topological division ring which admits a nontrivial valuation preserving the topology is strictly minimal. (Received September 9, 1948.)

24. Everett Pitcher: *The index and minimum properties of the characteristic roots of a quadratic function.*

Two approaches to the problem of existence and number of characteristic roots are formulated and the relation between them made clear in the following development. Let (A) $B(\eta, \phi, \lambda)$ be a real valued, symmetric, bilinear function of η, ϕ, differentiable in λ for $0\leqq\lambda<b$. Set $I(\eta, \lambda)=B(\eta, \eta, \lambda)$. Suppose (B) $I(\eta, 0)$ is positive definite; (C) if $I(\eta_0, \lambda_0)\leqq 0$ then $I_\lambda(\eta_0, \lambda)\leqq 0$ for $0\leqq\lambda<b$; (D) if $I(\eta_0, \lambda_0)=0$ and $\eta_0\neq 0$

then $I_\lambda(\eta_0, \lambda_0) < 0$. Of particular interest are the facts that λ need not enter linearly and the sense of variation of I with λ is not specified necessarily for all η. I may possess two properties: the *index property*, that the index of $I(\eta, \lambda_0)$ be finite and equal to the number of characteristic roots less than λ_0; the *minimum property*, that the characteristic roots in order are countable and that for any set $\eta_1, \lambda_1; \cdots; \eta_p, \lambda_p$ of characteristic pairs of I, the set of numbers λ which together with an $\eta \neq 0$ satisfies the conditions $I(\eta, \lambda) = 0$, $B(\eta, \eta_i, \lambda) = 0$ $(\lambda \neq \lambda_i)$, $B_\lambda(\eta, \eta_i, \lambda) = 0$ $(\lambda = \lambda_i)$ contains its greatest lower bound. The principal theorem states that if A, B, C, D hold, the index property and the minimum property are equivalent. Connections can be made with a variety of concrete characteristic value problems. (Received September 20, 1948.)

25. P. C. Rosenbloom: *The conformal mapping of nearly circular regions.*

If $f(z)$ maps the unit circle onto a domain D contained in $|w| < 1$ and containing D_1: $|w| < 1$, $|w - c| > \rho$, where $|c| = 1$, $0 < \rho < 1$, and if $f(0) = 0$, $f'(0) = a > 0$, then $f(z) = z - (1-a)z(c+z)/(c-z) + O((1-a)\rho^{1/2})$. This gives a very simple proof of Loewner's differential equation for functions mapping the unit circle on slit domains. (Received October 30, 1948.)

26t. I. E. Segal: *A kind of abstract integration pertinent to locally compact groups.* I. Preliminary report.

A *gage* is defined as a self-adjoint linear functional μ on a dense self-adjoint subalgebra A_0 of a C^*-algebra A, such that for $W \in A_0$, $\mu(W^*XW)$ is a continuous function of $X \in A_0$. The concept of gage includes that of state, and also that of a regular measure on a locally compact Hausdorff space. A further example is: A = operator group algebra of the locally compact group G, A_0 = subalgebra of all operators of the form T_f, where $T_f g$ = convolution of f with g, f being a continuous function which vanishes outside a compact set in G and g being an arbitrary element of $L_2(G)$, and $\mu(T_f) = f(e)$, e being the group identity. It is shown that all elements X of A of the form W^*VW, with $W \in A$ and $V \in SA(A_0)$, where $SA(A_0)$ is the collection of self-adjoint elements of A_0, are integrable in the sense that GLB $[Y | Y \geqq X, Y \in SA(A_0)]\mu(Y) = $ LUB $[Y | Y \leqq X, Y \in SA(A_0)]\mu(Y) < \infty$. In case the group G above is abelian, this is a lemma on which a proof of the generalized Plancherel theorem could be based. (Received August 19, 1948.)

27t. I. E. Segal: *Extension of representations of subgroups of locally compact groups.*

A continuous unitary irreducible representation (on a Hilbert space) of a compact subgroup of a locally compact group can be extended to the same kind of representation of the full group. The same is true for any subgroup of a discrete group. More generally, a sufficient condition for a continuous unitary irreducible representation of a closed subgroup of a locally compact group to be extendable to the same kind of representation of the full group is obtained, in terms of states of algebras related to the group and subgroup. The author's results naturally include the theorem of A. Weil covering the case in which the full group is compact, but his methods are unrelated to those of Weil, and the proof could be greatly simplified by restricting consideration to this case. (Received September 7, 1948.)

28*t*. Robert Sips: *Asymptotic representations of Mathieu functions and of spheroidal wave functions.*

Asymptotic representations are obtained for the periodic solutions, and for the corresponding separation constants, of the Mathieu equation $y''+(a-2^{-1}\lambda^2c^2\cos^2\eta)y=0$, for large values of the parameter λc. The asymptotic representation of the separation constant a, though obtained by an essentially different method, is identical to the one given earlier by S. Goldstein (Proc. Cambridge Philos. Soc. vol. 23 (1927) pp. 303–336). The Mathieu functions themselves are represented asymptotically, for all real values of the variable η, as a series of functions of the parabolic cylinder $D_n(\alpha)=\exp(-\alpha^2/4)\cdot H_n(\alpha)$, where $\alpha=(2\lambda c)^{1/2}\cdot\cos\eta$ and H_n is the nth Hermite polynomial. The spheroidal wave functions are the periodic solutions for large values of λc of the equation $(\sin\eta)^{-1}\cdot d(\sin\eta dH/d\eta)/d\eta+(A+\lambda^2c^2\sin^2\eta-m^2/\sin^2\eta)H=0$ for prolate wave functions, and the same equation with $\sin\eta$ replaced by $\cos\eta$ and a minus sign before the term $\lambda^2c^2\cos^2\eta$ for oblate wave functions. But, whereas the asymptotic expression of the prolate spheroidal wave functions is a series of parabolic functions, similar to the one representing the Mathieu functions, the corresponding development for the oblate wave functions is a series of functions of the form $\alpha^m\exp(-\alpha^2/4)\cdot L_p^{(m)}(\alpha^2/2)$ where $\alpha=(2\lambda c)^{1/2}\cdot\sin\eta$ and $L_p^{(m)}$ is the generalized Laguerre polynomial. Numerical results, obtained from the asymptotic expansions, are compared with exact values published by various authors. (Received May 10, 1948.)

29. Otto Szasz: *A generalization of Bernstein's polynomials to the infinite interval.*

A constructive proof of Weierstrass' theorem on the uniform approximation of a continuous function by polynomials over a finite interval is based on Bernstein's polynomials. There the values of the function at equidistant points are employed, with weights corresponding to the Bernoulli probability distribution. In the present paper the values of a function at equidistant points are employed over the infinite interval $(0, \infty)$, but the weights now correspond to the Poisson distribution. Uniform convergence to the function is proved at each point of continuity. A degree of approximation is given on introducing a weighted modulus of continuity. The process is connected with the problem of approximate integration by Cauchy-Riemann sums over the infinite interval. (Received September 6, 1948.)

30. J. L. Walsh, W. E. Sewell, and H. Margaret Elliott: *On the degree of convergence of harmonic polynomials to harmonic functions.*

Let $u(x, y)$ be harmonic inside C, continuous in the corresponding closed region $\overline{C}$; under suitable restrictions on C if the kth directional derivative of $u(x, y)$ satisfies on C a Lipschitz condition of order α $(0<\alpha\leqq 1)$, then a properly chosen sequence of harmonic polynomials $p_n(x, y)$ of respective degrees n converges to $u(x, y)$ in $\overline{C}$ with a degree of convergence $1/n^{k+\alpha}$. Conversely, if a sequence $p_n(x, y)$ converges to $u(x, y)$ in $\overline{C}$ with this degree of convergence, the kth directional derivative of $u(x, y)$ satisfies on C a Lipschitz condition of order α if $\alpha<1$. Furthermore, if $u(x, y)$ is harmonic inside C_R, continuous on $\overline{C}_R$, and if the kth directional derivative of $u(x, y)$ satisfies a Lipschitz condition of order α on C_R $(0<\alpha\leqq 1)$, then a suitable sequence $p_n(x, y)$ converges to $u(x, y)$ on $\overline{C}$ with a degree of convergence $1/n^{k+\alpha}R^n$; and conversely, if a sequence $p_n(x, y)$ converges to $u(x, y)$ on $\overline{C}$ with this degree of convergence, the $(k-1)$th directional derivative of $u(x, y)$ satisfies on C_R a Lipschitz condition of order α if $\alpha<1$. (Received September 7, 1948.)

APPLIED MATHEMATICS

31*t*. Milton Abramowitz: *Asymptotic expansions of Coulomb wave functions.*

The author deals with the problem of determining various expansions for the regular solution of the Coulomb wave equation when $L=0$ for values of the parameters ρ and η where the power series expansion is slowly converging. An asymptotic expansion is derived from the contour integral representations as derived by Sexl. Other approximating expansions are obtained from the differential equations using Sexl's previously determined saddle point approximations as starting point. Also included is a new short table of the zeros of the regular solution. (Received April 9, 1948.)

32*t*. Garrett Birkhoff: *Note on virtual mass.*

Let S be any solid, and Σ any solid of revolution about the x-axis containing S. Then the rotational virtual mass of S in an infinite fluid equals the moment of momentum about the x-axis of the fluid in $\Sigma-S$, when S is rotating about the x-axis with unit angular velocity. The cross-components of the virtual mass tensor satisfy similar identities. For example, that between rotation about the x-axis and translation along the y-axis is the y-momentum of the fluid in $\Sigma-S$, when S is rotated about the x-axis. The above is a generalization of a recent result of Theodorson. Again, if H and K are two infinitesimal rigid motions, and S^* is a sphere of radius r containing S, the K-component of pressure thrust for H-motion of S is $\mathrm{Lim}_{r\to\infty}\{\int\int_{S^*}(HUdS_K-KUdS_H)+\int\int\int_{S^*-S}[(HK-KH)U]dV$, where U is the velocity potential for H-motion, and dS_K the surface flux of the vector field defined by K. Using this identity, it is easy to prove that a rigid body in an infinite fluid behaves dynamically like a Lagrangian system with six degrees of freedom. (Received August 10, 1948.)

33*t*. Harvey Cohn: *Interaction of simple waves.*

The author shows how the interaction of simple waves can be solved in general in the hodograph space by means of constructions such as those of J. Giese (Aberdeen Ballistic Research Laboratories Report, no. 657). For instance, if the flow with velocity $q=(u, v, w)$ involves two waves represented by vectors $q_1(\sigma)$, $q_2(\tau)$ in hodograph space ($\sigma_0\leqq\sigma\leqq\sigma_1$, $\tau_0\leqq\tau\leqq\tau_1$, $q_1(\sigma_0)=q_2(\tau_0)=q_0$ and $q_1(\sigma_1)=q_1$, $q_2(\tau_1)=q_2$), then the resultant velocity q_{12} of the interaction is found by first spanning the two preceding curves by a surface $w(u, v)$ satisfying $Aw_{uu}-2Bw_{uv}+Cw_{vv}=0$, where $A=c^2(1+w_v^2)-(v+ww_v)^2$, $B=c^2w_uw_v-(u+ww_u)(v+ww_v)$, and $C=c^2(1+w_u^2)-(u+ww_u)^2$, and next drawing from the points q_1 and q_2 the characteristic curves which will determine q_{12}. The integration of the partial differential equation lends itself to the application of familiar (local) existence and uniqueness theorems. The author integrates the equation by power series to find the approximate interaction of Meyer waves. (Received August 23, 1948.)

34. Eugene Isaacson: *Water waves over a sloping bottom.*

Progressing gravity waves in water bounded by a plane bottom at an angle ω to the water surface can be described with the aid of a potential function $\phi(x, y)$ in the sector $-\omega\leqq\theta\leqq 0$ of the (x, y)-plane, satisfying the boundary conditions: $\partial\phi/\partial y-\phi=0$ for $\theta=0$; $\partial\phi/\partial\theta=0$ for $\theta=-\omega$. This potential function had been determined for rational multiplies of 90° and for two limiting cases by Miche, Stoker, Lewy, Friedrichs. For details and references—also to the work of A. E. Heins—see the papers in Communications on Applied Mathematics vol. 1 (1948). In the present paper a simple explicit representation of the solution for arbitrary angle ω, which comprises all previously treated cases, will be given. To obtain this representation, the solution for rational multiples p/q of 90°, first given by Lewy as a sum of many integrals, is expressed in terms of a single integral in such manner that the transition to the case of an arbitrary angle ω can be carried out. (Received June 3, 1948.)

35*t*. C. C. Lin: *On the theory of isotropic turbulence of high Reynolds numbers.* Preliminary report.

Kolmogoroff's theory as applied to isotropic turbulence is discussed in terms of the spectrum without reference to correlation functions. The inverse five-thirds power law for the intermediate range of the spectrum is rederived following the original ideas of Kolmogoroff without making specific assumptions regarding the dependence of the transfer term on the spectrum. A law of decay of turbulence is derived following this idea. It is shown that in general the square of Kolmogoroff's micro-scale η instead of Taylor's micro-scale λ increases linearly in time. The Reynolds number of turbulence R_λ generally decreases linearly in time. The product $u^{-2}R_\lambda$ increases linearly in time, where u is the intensity of turbulence. This law of decay includes the half-power law when R_λ is a constant. The decrease of R_λ seems to be indicated by the experiments of Batchelor and Townsend. Furthermore, other earlier experimental results not fitting the half-power law are very well described by the present theory. (Received June 28, 1948.)

36*t*. C. C. Lin, Eric Reissner, H. S. Tsien: *On two-dimensional non-steady motion of a slender body in a compressible fluid.*

It is well known that in problems of flow of a compressible fluid a perturbation approach often leads to linear differential equations. Recently von Kármán pointed out that for steady motion in the transonic range certain nonlinear terms have to be retained even in a perturbation theory (Journal of Mathematics and Physics, Massachusetts Institute of Technology, vol. 26 (1947) pp. 182–190). The present paper contains a systematic discussion of *non-steady* perturbation flows with a resultant classification of all possible cases including determination of their similarity parameters. Of particular interest is the result that for non-steady flow a linearized perturbation theory is obtained even as the Mach number of the undisturbed flow approaches unity, provided the reduced frequency parameter k is large compared with the 2/3 power of the amplitude ratio δ. (Received September 16, 1948.)

37*t*. E. J. McShane: *The differentials of certain functionals in exterior ballistics.*

If a projectile is launched with given initial conditions, its range, time of flight, and so on, are functionals of the range-wind, cross-wind, density, temperature (usually regarded as functions of the altitude y alone), and of other functions of position and velocity also. Ranges are ordinarily tabulated for certain simple "standard" choices of wind (identically 0), of density, and so on, and effects of departures from standard conditions are approximated by the differential of the functional. G. A. Bliss proved that the differential exists if the space of (for example) wind functions is normed by

$\|w(\cdot)\| = \max\ [\sup|w(y)|,\ |dw(y)/dy|\ ]$. The presence of dw/dy in this definition is undesirable for both esthetic and physical reasons. In the present paper it is proved that the differential exists even when the norm is the familiar one, $\|w(\cdot)\| = \sup\ |w(y)|$. (Received August 30, 1948.)

Geometry

38t. Edward Kasner and John DeCicco: *Osculating conics of the integral curves of equations of the type* (G).

The importance of differential equations of the type (G): $y''' = G(x, y, y')y'' + H(x, y, y')y''^2$, in geometry and physics, has been brought out by Kasner in his Princeton Colloquium Lectures, *Differential-geometric aspects of dynamics*, Amer. Math. Soc. Colloquium Publications, vol. 3, 1913, 1934, 1948, and in many later papers. The osculating conics (five-point contact) of the ∞^3 integral curves are studied. The ∞^1 osculating conics, constructed at a lineal element E to the ∞^1 curves of the system passing through E, not only pass through E, but also touch another conic in general position. Their centers describe still another conic passing through the point of E. The foci of these conics describe an algebraic curve of sixth degree with a singular point of fourth order at the point of E. The degenerate cases of these various loci are studied in detail. (Received September 18, 1948.)

Logic and Foundations

39. Samuel Linial and E. L. Post: *Recursive unsolvability of the deducibility, Tarski's completeness, and independence of axioms problems of propositional calculus.*

The primitive connectives are $\sim p$ and $(p \vee q)$, corresponding modus ponens and substitution the rules of deduction. Let S be a fixed normal system on a, b with non-null g's and g''s whose deducibility problem is recursively unsolvable (see Post, Bull. Amer. Math. Soc. vol. 50 (1944) pp. 286–287, 292; vol. 52 (1946) footnote 3). To a and b are made correspond $\sim\sim(p \vee \sim p)$ and $\sim\sim\sim\sim(p \vee \sim p)$ respectively. With $\sim(\sim p \vee \sim q)$ as connective, correspondents in propositional calculus result for each non-null string B on a, b, $B(p)$ designating a certain particular correspondent. A normal operation is simulated in propositional calculus by $(\sim, \vee)$ implications which have a corresponding effect for certain correspondents of the strings involved, and implications which transform any one correspondent of a string into any other. A finite set $\{A_i\}$ of tautologies result such that if $\{P_i\}$ is any particular complete finite set of axioms (tautologies) for propositional calculus, $c(p)$ the tautology $\sim\sim(\sim p \vee p)$, then B is asserted in S when and only when: $B(p)$, itself a tautology, is deducible from $\{A_i\}$; $\{A_i\}$, $\{(B(p) \supset T_i)\}$ is a complete set of axioms; $\{A_i\}$, $c(p)$, $(B(p) \supset c(p))$ is not an independent set of axioms. The title results follow for arbitrary finite sets of tautologies as axioms, hence also the first and third without the tautology restriction. (Received May 24, 1948.)

Statistics and Probability

40t. Aryeh Dvoretzky: *On the strong stability of a sequence of events.*

Let $\{A_n\}$ $(n = 1, 2, 3, \cdots)$ be any sequence of events (which may interdepend in an arbitrary manner) defined in the same set of trials. Put $\alpha_n = \Pr\{A_n\}$, $\alpha_{m,n} = \Pr\{A_m A_n\}$, $n\beta_n = \sum_{i=1}^{n} \alpha_i$, $C_{n,2}\gamma_n = \sum_{1 \leqq i < j \leqq n} \alpha_{i,j}$, $\delta_n = \beta_n - \alpha_n^2$ and $f_n = R_n/n$ where R_n is the number of those events among $A_1, A_2, \cdots, A_n$ which are realized. Let $\bar{f}_n = E\{f_n\}$ be the expected value of f_n and $\phi_n = f_n - \bar{f}_n$. The sequence $\{A_n\}$ is called *strongly stable* provided $\{\phi_n\}$ is strongly stable, that is (cf. Kolmogoroff, *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Berlin, 1933, p. 58), if for every $\epsilon > 0$, $\lim_{n\to\infty} \{\text{sup.}_{\nu > n} |\phi_\nu| > \epsilon\} = 0$. Extending classical results M. Loève (J. Math. Pures Appl. (9) vol. 24 (1945) pp. 249–318, p. 257) proved that if $n\delta_n$ is bounded then $\{A_n\}$ is strongly stable. Using the lemma that if $a_n \geqq 0$ and (*) $\sum_{n=1}^{\infty} a_n/n < \infty$ there exists a sequence of integers $\{n_i\}$ such that $0 < n_{i+1} - n_i = o(n_i)$ and $\sum_{i=1}^{\infty} a_{n_i} < \infty$, the author proves that even (**) $\sum_{n=1}^{\infty} \delta_n/n < \infty$ implies the strong stability of $\{A_n\}$. Since the sufficient condition (*) in the lemma becomes also necessary when $\{a_n/n\}$ is monotone it is possible to establish cases where (**) is necessary and sufficient for the strong stability of $\{A_n\}$. (Received September 20, 1948.)

41*t*. C. J. Everett and S. M. Ulam: *Multiplicative systems.* I.

Let $G(x)$: $g_i(x) = \sum p_1(i; j_1, \cdots, j_t)x^j$ (power-series in $x_1, \cdots, x_t$) be a transformation of the unit cube I_t, $p_1(i; j)$ being the probability that a particle of type i produce, upon transmutation, $j_1 + \cdots + j_t$ particles, j_s of type s. Then, in the i-component $g_i^k(x) = \sum p_k(i:j)x^j$ of the kth iterate $G^k(x)$, $p_k(i;j)$ gives the probability of the population $(j_1, \cdots, j_t)$ in the kth generation of progeny from one particle of type i. Weak restrictions on nonvanishing of $\partial g_i/\partial x_j$ and certain $\partial^2 g_i/\partial x_j \partial x_k$ imply: (1) if $x_i^0 \equiv \lim p_k(i; 0)$, x^0 and 1 are the only fix-points of G on I_t; (2) $\lim p_k(i; j) = 0$, all $j \neq 0$; (3) $\lim G^k(x) = x^0$, all $x \neq 1$ on I_t, (4) matrix $M \equiv [m_{ij}]$, $m_{ij} \equiv (\partial g_i/\partial x_j)_1$, has maximal characteristic root $r > 0$ and unique positive vectors v, v' of norm 1 such that $vM = rv$, $Mv' = rv'$, a result due to Frobenius; (5) $x^0 = 1$ if and only if $r \leqq 1$; (6)) $\lim jM^k/\|jM^k\| = v$ uniformly on the positive sector of t-space; (7) for every $e > 0$, $f > 0$, there exists a K such that for all $k \geqq K$, $\sum p_k(i; j) \geqq 1 - f$, j being summed over all vectors in the e-cone of v, including $j = 0$. This is the analogue of the weak law of large numbers in additive probability. (Received September 16, 1948.)

42*t*. C. J. Everett and S. M. Ulam: *Multiplicative systems.* II.

Under the condition $r < 1$ (hence $x^0 = 1$), the following results are obtained: (1) for $x \neq 1$ in I_t, vectors $1 - G^k(x)$ and $G^{k+1}(x) - G^k(x)$ approach v' in the sense of ray convergence; (2) if $q_k(i)$ is defined by $p_k(i; 0) = q_1(i) + \cdots + q_k(i)$, $q_k(i)$ is a probability density on $k = 1, 2, \cdots$ with moments of all orders; (3) if a source with generating function $S(x)$ acts alternatively with transmutation, the generating function for the kth population is $H_k(x) = S(G(x)) \cdots S(G^k(x))$, $H(x) \equiv \lim H_k(x)$ exists on I_t, $(\partial H/\partial x_i)_1$ exists, is computable from a linear system, and represents the k-limit of expectation of particles of type i in population k; (4) analogous theorems are valid for total progeny in systems with and without source; (5) H_k (and its analogues in (4)) may be realized as a component-function of the kth iterate of a single transformation. (Received September 16, 1948.)

43*t*. C. J. Everett and S. M. Ulam: *Multiplicative system.* III.

The set Z_i of all *possible* genealogies (graphs) z arising from one particle of type i possesses an intrinsic metric $d(z, z')$ under which it is a complete separable zero-dimensional metric space. Simple axioms (A) on intervals and (B) on measure of intervals are given from which completely additive measure theory is derived. Intervals in Z_i are defined intrinsically and shown to satisfy axioms (A). If a generating transformation $G(x)$ is given, the transition probabilities $p_1(i; j)$ serve to define a measure for the intervals of Z_i satisfying axioms (B). The latter is nontrivial due to non-local-compactness of Z_i. Thus $G(x)$ defines a G-measure for the additive class of G-measurable sets of Z_i. Among other results may be mentioned: the set of graphs which terminate has measure x_i^0. Almost all graphs either terminate or have kth generation population $(j_1, \cdots, j_t)$ approaching the ratios $v_1: \cdots :v_t$ of v. The latter is the analogue for multiplicative systems of the strong law of large numbers in additive probability. (Received September 16, 1948.)

TOPOLOGY

44. L. W. Cohen and Casper Goffman: *The topology of ordered abelian groups.*

The problem is to find complete group extensions of ordered abelian groups G. It is well known that the set of all lower segments will not serve for non-archimedean groups. A subset of the lower segments, called Dedekindean, is defined. These are the elements of the complete group extension G_D of the given group G. The groups G have an ordinal $\xi^*(G)$ in terms of which convergence and topological completeness are defined. G_D is topologically complete. It is shown that groups G complete in the archimedean sense are topologically complete, that the converse is not true but that if a topologically complete group is dense in every archimedean extension it is complete in the archimedean sense. The Hahn groups are examples of topologically complete ordered abelian groups and are of the second ξ^*-category. (Received August 19, 1948).

45. R. H. Fox: *A remarkable simple closed curve.*

In 3-dimensional Euclidean space there exists a simple closed curve whose complementary domain has a non-abelian fundamental group although, in the everyday sense of the word, this curve is not knotted at all. (Received September 18, 1948.)

46*t*. F. I. Mautner: *Completeness relations on locally compact groups.*

The completeness relations which have recently been obtained for the irreducible unitary representations of separable locally compact groups (Proc. Nat. Acad. Sci. U.S.A. vol. 34 (1948) pp. 52–54) are now studied in more detail. One of the results is that two methods which lead to the same Parseval relation for compact or locally compact commutative groups may lead to two essentially different completeness relations for certain classes of locally compact groups. As an illustration, it is shown that the "regular representation" of the group of linear transformations in one variable over any infinite discrete field can be decomposed in an explicit manner into uncountably many inequivalent irreducible unitary representations, despite the fact that a certain algebra of operators associated with the group (the weakly closed ring generated by the regular representation) is a factor of class II_1 in the sense of Murray and von Neumann (Ann. of Math. vol. 37 pp. 116–229) and thus a simple algebra. (Received March 13, 1948.)

47*t*. A. H. Stone: *On the analytic definition of degree of multicoherence.*

It is proved that the "analytic" and "geometric" definitions of the degree of multicoherence (Eilenberg, Fund. Math. vol. 27 (1936) pp. 159, 153) agree for all connected, locally connected, normal spaces. A number of geometrical deductions are made. The principal tool is a modification of the notion of the "nerve" of a system of sets, to take into account the numbers of components of their intersections. (Received August 13, 1948.)

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