THE SUMMER MEETING IN MADISON

The fifty-fourth Summer Meeting and thirtieth Colloquium of the American Mathematical Society was held at the University of Wisconsin, Madison, Wisconsin, Tuesday to Friday, September 7–10, in conjunction with the summer meetings of the Mathematical Association of America, the Institute of Mathematical Statistics and the Econometric Society. The Mathematical Association of America met on September 6–7, the Institute of Mathematical Statistics and the Econometric Society on September 7–10. About 700 people attended these meetings, among whom were the following 431 members of the Society.

C. R. Adams, R. B. Adams, V. W. Adkisson, J. E. Adney, J. G. Adshead, R. P. Agnew, A. A. Albert, F. E. Allen, W. R. Allen, C. B. Allendoerfer, R. V. Andree, Richard Arens, K. J. Arnold, M. C. Ayer, W. L. Ayres, H. M. Bacon, R. H. Bardell, A. S. Barr, J. J. Barron, R. C. F. Bartels, Walter Bartky, M. A. Basoco, Leon Battig, H. P. Beard, H. M. Beatty, E. F. Beckenbach, E. G. Begle, J. H. Bell, A. A. Bennett, A. F. Bentley, Stefan Bergman, D. L. Bernstein, Lipman Bers, H. R. Beveridge, F. C. Biesele, R. H. Bing, M. C. Blackstock, A. L. Blakers, J. W. Bradshaw, Richard Brauer, R. W. Brink, Foster Brooks, M. C. Brown, R. H. Bruck, H. E. Buchanan, R. C. Buck, P. B. Burcham, F. J. H. Burkett, L. J. Burton, L. E. Bush, Hobart Bushey, J. H. Bushey, W. H. Bussey, A. S. Cahn, S. S. Cairns, W. D. Cairns, R. H. Cameron, C. C. Camp, H. E. Campbell, R. E. Carr, W. B. Carver, E. D. Cashwell, T. E. Caywood, Subrahmanyan Chandresekhar, K. Chandrasekharan, J. O. Chellevold, Herman Chernoff, Sarvadaman Chowla, R. V. Churchill, C. E. Clark, F. M. Clarke, H. E. Clarkson, Nathaniel Coburn, C. J. Coe, H. J. Cohen, I. S. Cohen, L. A. Colquitt, B. H. Colvin, E. G. H. Comfort, H. H. Conwell, G. R. Costello, V. F. Cowling, C. C. Craig, J. S. Cronin, A. B. Cunningham, J. H. Curtiss, E. H. Cutler, G. B. Dantzig, J. A. Daum, P. H. Daus, Robert Davies, D. R. Davis, James Elmer Davis, L. A. V. DeCleene, D. B. DeLury, R. F. Deniston, A. H. Diamond, L. L. Dines, G. P. Dinneen, J. M. Dobbie, C. L. Dolph, J. L. Doob, H. L. Dorwart, D. M. Dribin, R. J. Duffin, Nelson Dunford, W. L. Duren, W. H. Durfee, Ben Dushnik, P. S. Dwyer, W. F. Eberlein, M. C. Eide, Samuel Eilenberg, Churchill Eisenhart, M. P. Emerson, J. H. Engel, Paul Erdös, W. S. Ericksen, H. P. Evans, H. S. Everett, G. M. Ewing, A. B. Farnell, J. V. Finch, C. H. Fischer, S. G. Fleming, L. R. Ford, W. C. Forman, J. S. Frame, Evelyn Frank, J. E. Freund, W. H. J. Fuchs, R. E. Fullerton, David Gale, H. L. Garabedian, R. E. Gaskell, H. M. Gehman, H. H. Germond, B. E. Gillam, M. A. Girschick, Wallace Givens, A. M. Gleason, Casper Goffman, V. D. Gokhale, R. A. Good, R. E. Goodman, Cornelius Gouwens, A. A. Grau, L. M. Graves, W. L. Graves, Edison Greer, L. W. Griffiths, V. G. Grove, S. G. Hacker, Elizabeth Hahnemann, D. T. Haimo, Franklin Haimo, Edwin Halfar, N. A. Hall, P. R. Halmos, E. S. Hammond, A. G. Hansen, W. L. Hart, G. E. Hay, Camilla Hayden, N. A. Haynes, C. T. Hazard, Erik Hemmingsen, D. E. Henriques, J. G. Herriot, I. N. Herstein, Fritz Herzog, A. D. Hestenes, Edwin Hewitt, E. H. C. Hildebrandt, T. H. Hildebrandt, Einar Hille, J. J. L. Hinrichsen, G. P. Hochschild, D. L. Holl, M. W. Hopkins, Harold Hotelling, K. O. Househam, A. S. Householder, C. C. Hsiung, R. C. Huffer, E. M. Hull, Ralph Hull, M. G. Humphreys, Witold

Hurewicz, J. W. Hurst, M. A. Hyman, M. H. Ingraham, A. W. Jacobson, W. C. Janes, H. F. S. Jonah, G. K. Kalisch, L. H. Kanter, Wilfred Kaplan, Irving Kaplansky, Samuel Karlin, Leo Katz, M. W. Keller, Claribel Kendall, J. F. Kenney, D. E. Kibbey, W. M. Kincaid, Fred Kiokemeister, J. R. Kline, E. R. Kolchin, M. Z. Krzywoblocki, R. E. Langer, Leo Lapidus, E. H. Larguier, H. D. Larsen, C. G. Latimer, W. G. Leavitt, H. L. Lee, J. R. Lee, Solomon Lefschetz, Walter Leighton, W. J. LeVeque, Harry Levy, F. A. Lewis, B. W. Lindgren, C. B. Lindquist, Charles Loewner, W. S. Loud, R. B. McClenon, Dorothy McCoy, E. A. McDougle, W. H. McEwen, L. H. McFarlan, J. V. McKelvey, M. M. McKelvey, J. C. C. McKinsey, J. K. L. MacDonald, C. C. MacDuffee, G. W. Mackey, Saunders MacLane, H. B. Mann, H. W. March, Morris Marden, A. M. Mark, Anna Marm, M. H. Martin, W. T. Martin, C. W. Mathews, A. E. May, Kenneth May, J. R. Mayor, A. E. Meder, A S. Merrill, B. E. Meserve, A. N. Milgram, H. J. Miser, W. L. Mitchell, E. E. Moise, J. T. Moore, R. L. Moore, T. W. Moore, Max Morris, D. R. Morrison, D. J. Morrow, A. P. Morse, D. S. Morse, Leo Moser, E. J. Moulton, H. T. Muhly, M. E. Munroe, A. L. Nelson, C. J. Nesbitt, John von Neumann, C. V. Newsom, Jerzy Neyman, K. L. Nielsen, Rufus Oldenburger, E. J. Olson, T. G. Ostrom, G. K. Overholtzer, F. W. Owens, H. B. Owens, J. C. Oxtoby, J. S. Oxtoby, Gordon Pall, S. T. Parker, W. V. Parker, G. A. Parkinson, E. W. Paxon, A. J. Penico, P. M. Pepper, Sam Perlis, B. J. Pettis, H. P. Pettit, C. R. Phelps, George Piranian, Everett Pitcher, G. B. Price, L. D. Pruett, F. M. Pulliam, A. L. Putnam, Gustave Rabson, Tibor Rado, E. D. Rainville, J. F. Randolph, R. B. Rasmussen, G. E. Raynor, M. O. Reade, C. J. Rees, M. S. Rees, P. V. Reichelderfer, I. M. Reiner, Irving Reiner, Eric Reissner, J. G. Renno, C. N. Reynolds, R. R. Reynolds, C. E. Rhodes, J. S. Rhodes, C. E. Rickart, P. R. Rider, F. D. Rigby, R. F. Rinehart, E. K. Ritter, H. A. Robinson, V. N. Robinson, G. F. Rose, Arthur Rosenthal, J. B. Rosser, W. E. Roth, E. H. Rothe, Herman Rubin, L. L. Runge, Arthur Sard, A. C. Schaeffer, H. M. Schaerf, A. T. Schafer, R. D. Schafer, Robert Schatten, Henry Scheffé, M. M. Schiffer, E. R. Schneckenburger, I. J. Schoenberg, Lowell Schoenfeld, K. C. Schraut, E. G. Schuld, A. L. Schurrer, G. E. Schweigert, W. T. Scott, I. E. Segal, M. E. Shanks, A. S. Shapiro, I. M. Sheffer, L. W. Sheridan, Seymour Sherman, M. F. Smiley, F. C. Smith, G. W. Smith, H. L. Smith, W. N. Smith, Ernst Snapper, L. J. Snell, W. S. Snyder, Andrew Sobczyk, E. S. Sokolnikoff, T. H. Southard, E. H. Spanier, George Springer, R. H. Stark, M. P. Steele, N. E. Steenrod, H. E. Stelson, C. F. Stephens, Rothwell Stephens, B. M. Stewart, R. R. Stoll, M. H. Stone, E. B. Stouffer, W. J. Strange, A. F. Strehler, R. L. Swain, Otto Szasz, A. H. Taub, Olga Taussky-Todd, J. S. Taylor, H. P. Thielman, J. M. Thomas, J. R. Thompson, R. M. Thrall, W. J. Thron, H. S. Thurston, E. W. Titt, C. J. Titus, John Todd, Leonard Tornheim, J. I. Tracey, G. R. Trott, P. L. Trump, A. W. Tucker, Bryant Tuckerman, J. W. Tukey, H. L. Turrittin, J. L. Ullman, J. I. Vass, Bernard Vinograde, L. I. Wade, R. W. Wagner, G. L. Walker, R. J. Walker, J. L. Walsh, J. A. Ward, S. E. Warschawski, Marjorie Watson, André Weil, M. J. Weiss, George Whaples, G. W. Whitehead, P. M. Whitman, L. R. Wilcox, R. L. Wilder, J. E. Wilkins, S. S. Wilks, R. L. Wilson, R. M. Winger, L. A. Wolf, M. A. Woodbury, Theodore Wysocki, G. S. Young, J. W. T. Youngs, Daniel Zelinsky, J. L. Zemmer, M. A. Zorn.

On Tuesday afternoon, Wednesday, Thursday and Friday mornings, Professor Richard Brauer of the University of Michigan delivered the Colloquium Lectures on *Representations of groups and rings*. The presiding officers at these lectures were Professor C. C.

MacDuffee, President Einar Hille, Professor Saunders MacLane, and Professor A. A. Albert respectively.

The Society held a joint session with Section A of the American Association for the Advancement of Science on Wednesday morning, at which time Professor R. L. Moore of the University of Texas delivered a retiring address (as Vice President of the American Association for the Advancement of Science and Chairman of Section A) entitled *Spirals*. Professor R. L. Wilder presided.

On Thursday afternoon Professor J. W. T. Youngs of the University of Indiana lectured on *Topological methods in the theory of Lebesque area*, with Professor Tibor Rado in the chair.

Presiding officers for the sessions of contributed papers were as follows. Tuesday afternoon: Algebra, Professor George Whaples; Analysis, Professor J. L. Walsh; Probability and Statistics, Professor P. S. Dwyer. Thursday morning: Analysis, Professor L. M. Graves; Topology, Professor R. L. Wilder. Thursday afternoon: Applied Mathematics, Professor R. V. Churchill; Algebra, Professor Ralph Hull. Friday morning: Analysis, Professor G. B. Price; Topology, Geometry and Analysis, Professor J. W. T. Youngs; Unclassified Late Papers, Professor Gordon Pall.

Except for one sectional session in Room 102, Biology Building, all sessions were held in Bascom Hall, the general sessions in Room 272 and sectional sessions in Rooms 272, 165 and 312.

Registration headquarters were in Tripp Hall. Rooms were available for those attending the meetings in Tripp and Adams Halls, on the edge of Lake Mendota, and the dormitory pier was much in use for swimming. Meals were served in the Cafeteria of Van Hise Hall. Arrangements were made for tennis and golf. On Wednesday afternoon there were trips to the Forest Products Laboratory of the United States Government and to the offices of the United States Armed Forces Institute and a boat ride on Lake Mendota. On Tuesday and Wednesday evenings there were visits to the University observatory. All these activities were well attended. In addition there were during the week several spontaneously organized excursions to cheese factories within easy driving distance of Madison.

On Tuesday afternoon the ladies of the Mathematics Department served tea on the lawn behind Tripp Hall.

On Tuesday evening a capacity audience were the guests of the University of Wisconsin at a concert given in Music Hall by the Pro Arte String Quartet of the University of Wisconsin.

On Wednesday evening a dinner was held for the four organizations in the Crystal Ball Room of the Loraine Hotel, with an assembled group of over four hundred people. Dean Walter Bartky acted as toastmaster. Dean M. H. Ingraham welcomed the mathematical groups on behalf of the University of Wisconsin. Professor J. L. Walsh responded for the Society, Dr. H. M. Gehman for the Association, Professor P. S. Dwyer for the Institute and the Econometric Society. Before concluding the dinner, Dean Bartky warmly commended the efforts of the Local Committee on Arrangements and brought to their feet in turn for a hearty clap from the audience the three most valiant members of that Committee, namely Professors H. P. Evans (Chairman), B. H. Colvin and Elizabeth S. Sokolnikoff.

A group photograph of those attending the meeting was taken on Wednesday at 1:30 P.M.

A second informal luncheon meeting of the applied mathematicians of the midwestern area was held on Thursday in the small dining room of Van Hise Hall, with an attendance of about forty. Those present approved the principle of continuing such discussion groups, decided against formation of a formal organization and for continued cooperation with the Applied Mathematics Committee of the Society. They also added two new members to their informal committee of three, one addition being Associate Secretary J. W. T. Youngs.

On Thursday afternoon Professor G. B. Price proposed to those assembled for Professor Youngs' invited address a resolution, unanimously approved, expressing the appreciation of the members of the four organizations to the administration of the University of Wisconsin, the local committee and all who contributed to the success of the meetings.

A well attended picnic supper was held Thursday at Picnic Point on Lake Mendota.

On Thursday evening about two hundred people gathered in the Pine Room of Van Hise Hall for a beer party officially sponsored by the Institute of Mathematical Statistics but well attended by Society members. The atmosphere was one of song and friendly talk, and many expressed the hope that such an event would become a regular feature of the Summer Meetings.

The Council met in the lounge of Slichter Hall at 9:00 P.M. on Tuesday and at 10:00 P.M. on Wednesday.

The Secretary announced the election of the following sixty-three persons to ordinary membership in the Society:

Mr. Anderson B. Alexander, Purdue University;
Dr. Nachman (Natan) Aronszajn, Centre National de la Recherche Scientifique, Paris;
Mr. Augustus Francis Bausch, Princeton University;
Mr. Jack Bazer, New York, N. Y.;

Mr. John William Benedick, Manhattan College, New York, N. Y.;
Mrs. Maude Bryan Blondeau, University of Arkansas;
Miss Olga Bocanegra-Saldaña, University of Puerto Rico;
Mr. Clifford Breuner, Temple University;
Mr. Robert Lloyd Broussard, Louisiana State University;
Mr. James William Butler, Oak Ridge National Laboratories, Oak Ridge, Tenn.;
Professor Lucia Virginia Carlton, Centenary College, Shreveport, La.;
Mr. Salvatore Rosario Chiefa, Columbia University;
Mr. Alfred Halstead Cockshott, Manhattan College, New York, N. Y.;
Miss Natalie Coplan, National Bureau of Standards, New York, N. Y.;
Mr. John Willis Crispin, Jr., Wayne University;
Mr. Howard J. Dietsche, South Ozone Park, N. Y.;
Mr. Avron Douglis, Institute of Mathematics and Mechanics, New York University;
Dr. Helen Margaret Elliott, Harvard University;
Mr. Chester Feldman, University of Chicago;
Mr. Ivey Clenton Gentry, Duke University;
Mr. Lincoln J. Gerende, Naval Medical Research Institute, Bethesda, Md.;
Dr. Cástor Segundo Goa, Ministerio de Obras Publicas de Venezuela, Caracas, Venezuela;
Professor Ulysses S. Grant, University of California at Los Angeles;
Mr. Gerald B. Haggerty, Rhode Island State College;
Mr. Cyrus B. Hailperin, Duquesne University;
Miss Mary-Elizabeth Hamstrom, University of Texas;
Mr. Hugh G. Harp, Ohio State University;
Mr. Herbert H. Hinman, City College, New York, N. Y.;
Mr. Robert Ellsworth Holdman, New York, N. Y.;
Mr. Donald Bruce Houghton, Franklin Institute, Philadelphia, Pa.;
Professor Thomas Roland Humphreys, Bergen Junior College, Teaneck, N. J.;
Mr. Mario Leon Juncosa, Cornell University;
Mr. Richard Ellis Kloss, U. S. Weather Bureau, Raton, N. M.;
Mr. Leo Lapidus, University of Maine;
Miss Margaret Mary LaSalle, Louisiana State University;
Mr. William Vincent Leamon, Western Union Telegraph Company, New York, N. Y.;
Mr. James Edward Martin, University of Pittsburgh, Johnstown, Pa.
Mr. Emanuel Mehr, Johns Hopkins University;
Mr. Saad Luka Mikhail, University of Illinois;
Mr. Bertram William Miller, New York, N. Y.;
Mr. George Constantinovich Miloslavsky, Princeton Textile Printing Co., New York, N. Y.;
Mr. James Henry Mulligan, Jr., Allen B. DuMont Laboratories, Inc., Passaic, N. J.;
Mr. Donald Joseph Myatt, Clarkson College of Technology, Potsdam, N. Y.;
Professor Zeev Nehari, Washington University;
Professor Otton Martin Nikodým, Kenyon College;
Dr. Ilse L. Novak, Wellesley College;
Mr. Albert Raymond O'Connor, Manhattan College, New York, N. Y.;
Mr. Charles Oister, Jr., Keystone Junior College, La Plume, Pa.;
Professor Edgar Phibbs, University of Alberta, Edmonton, Alberta, Canada;
Mr. Frank Proschan, Arlington, Va.;
Dr. Clarence Ross, Dahlgren, Va.;
Mr. Walter Rudin, Duke University;

Professor Socrates Watter Saunders, Morgan State College, Baltimore, Md.;
Major Raymond Ira Schnittke, University of Chicago;
Mr. Keeve Milton Siegel, Rensselaer Polytechnic Institute;
Miss Roselyn Adeline Siegel, Institute for Numerical Analysis, University of California at Los Angeles;
Professor Donovan Bradshaw Sumner, Louisiana State University;
Mr. Paul McCullough Sutton, New York, N. Y.;
Professor Garnet Louis Tiller, Utica College, Syracuse University;
Professor Lambuth Reilly Towson, North Georgia College, Dahlonega, Ga.;
Miss Marion Treon, Murray State Teachers College, Murray, Ky.;
Mr. John Edmund Warren, Teleregister Laboratories, New York, N. Y.;
Mr. Vincent Andrew Zora, Blaw-Knox Construction Company, Pittsburgh, Pa.

It was reported that the following had been elected to membership on nomination of institutional members as indicated:

Institute for Advanced Study: Dr. Walter Rudolf Habicht, Dr. Luis Antonio Santoló.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: French Mathematical Society: Professor Marcel Emile Brelot, Institut Fourier, Grenoble; Professor Michel Loève, University of California; Dr. Oleg I. Yadoff, Columbia University; Wiskundig Genootschap te Amsterdam: Dr. Jan Arnoldus Schouten, Epe, Netherlands.

The following actions of the Council taken by mail vote since the April meeting were reported: that the Proceedings of the Second Symposium on Applied Mathematics be published under the auspices of the Society; that the Society contribute to the Canadian Journal of Mathematics an annual subvention of $1,000 for a period of three years, with presumption of renewal for two additional years; that Professor Oswald Veblen be named the nominee of the American Mathematical Society for President of the International Congress of Mathematicians to be held in Cambridge, Massachusetts, in September, 1950.

The following appointments by President Einar Hille of persons to represent the Society were reported: Professor G. C. Evans at the Pacific Regional Conference on UNESCO in San Francisco on May 13–15, 1948; Professor C. V. Newsom at the inauguration of T. Keith Glennan as President of Case Institute of Technology on May 29–31, 1948; Professor E. F. Beckenbach at the inauguration of Fred D. Fagg, Jr., as President of University of Southern California on June 11, 1948; Professor Marston Morse at Third National Congress of Unione Matematica Italiana in Pisa, Italy, on September 23–26, 1948; Professor Richard Courant on Advisory Board of Applied

Mechanics Reviews for three years beginning July 1, 1948.

The following additional appointments by President Hille were reported: Professor J. W. T. Youngs as an additional member of the Committee on Arrangements for the 1948 Annual Meeting; Professors S. S. Cairns (Chairman), G. B. Price and P. A. Smith as a Committee on Revision of By-Laws; Professor J. R. Kline (Chairman), Dean W. L. Ayres, Professor W. T. Martin as a Committee to nominate an Executive Director of the Society; Professor T. H. Hildebrandt as Chairman (to replace Professor M. H. Stone) and Professor L. V. Ahlfors as an additional member of the Committee on the Award of the Bôcher Prize; Dean Walter Bartky as Chairman of the Committee on Applied Mathematics; Professor R. V. Churchill as a member of the Committee on Applied Mathematics, for unexpired term of Professor J. L. Synge, August 1, 1948–December 31, 1949; Professor A. H. Taub as Chairman for one year and as a member for three years, beginning July 1, 1948, of the Editorial Committee for Applied Mathematics Symposium Proceedings.

The Secretary reported that the Society had been listed as a sponsor of the Symposium on Modern Calculating Machinery and Numerical Methods, sponsored by the University of California at Los Angeles and the National Bureau of Standards in Los Angeles on July 29–31, 1948.

The Council voted that the Society should agree to be listed as a sponsor of a meeting of the American Society for Engineering Education, to be held at Rensselaer Polytechnic Institute in June, 1949.

A bequest of $1,000 to the Society under the will of the late Professor J. K. Whittemore of Yale University was reported to the Council and the Secretary was authorized and requested to express its appreciation to Mrs. Whittemore.

A contract has been signed by the Society and the Office of Naval Research whereby a sum of $25,000 will be available to the Society for the project of providing translations of important mathematical articles in Russian or other unfamiliar languages and for the project of the preparation of a pamphlet to aid mathematicians in reading Russian mathematical articles.

The following dates of meetings in 1949 were approved: February 26 in New York City; February 26 in Chicago; April 1–2 in Durham, North Carolina; April 29–30 in New York City; April 29–30 in Lawrence, Kansas; April 30 at Stanford University.

On recommendation of the Committee on Places of Meetings, the Council voted to hold the 1950 Annual Meeting at the University of Florida and the 1951 Summer Meeting at the University of Minnesota.

It was voted that the practice of having a group photograph at Summer Meetings should no longer be mandatory.

Certain invitations to deliver hour addresses were announced: Professor Raphael Salem for the October, 1948, meeting in New York City; Professor Herbert Busemann for the November, 1948, meeting in Los Angeles; Professors A. S. Besicovitch and Lamberto Cesari for the 1948 Annual Meeting in Columbus, Ohio; Professor S. B. Myers for the February, 1949, meeting in Chicago; Professors E. J. Mickle and R. M. Thrall for the April, 1949, meeting in Lawrence, Kansas.

The Policy Committee for Mathematics reported regarding its activities in connection with the possible formation of an International Mathematical Union. The Council voted to reaffirm its assignment to the Policy Committee of the problem of the International Mathematical Union and its designation of that Committee as the sole agent to carry on negotiations and bring recommendations to the governing boards of the organizations represented in the Policy Committee, leading to the formation of the Union. It was reported that the Mathematical Association of America would be represented in the Policy Committee in the future and the Council voted to approve a change in the original plan for the Policy Committee, thus giving the Association a representation of three members.

For the Organizing Committee of the International Congress, Vice Chairman W. T. Martin reported that the Committee had decided to hold Conferences in the following fields: Algebra, Analysis, Topology, and Applied Mathematics. Secretary Kline, for the Financial Committee, reported that the Rockefeller Foundation had increased its pledge of $7,500 (made for the Congress scheduled for 1940) to $12,000, that the Carnegie Corporation had increased its pledge from $12,000 to $18,000 and that the Mathematical Association of America had increased its pledge from $600 to $1,000. It was reported that Professor J. L. Doob had been added to the membership of the Organizing Committee for the International Congress.

The Council adopted, with minor changes, the report of the Committee on Revision of By-Laws. This committee was appointed to revise the By-Laws in order (1) to incorporate changes made necessary by the adoption of the report of the Committee on Reorganization and (2) to make other changes in the By-Laws as seemed necessary to make the By-Laws conform to existing practices of the

Society. The President was authorized and requested to appoint a new committee to study suggestions for additional changes made by the Committee on Revision of By-Laws.

The Council voted to accept the invitation to hold the Third Annual Symposium on Applied Mathematics at the University of Michigan in the summer of 1949.

A proposal that the Society provide competent survey articles every four or five years in each of the various fields of mathematics was referred to the Committee on the Role of the Society in Mathematical Publication.

Professors R. P. Agnew and W. M. Whyburn were elected representatives of the Society on the Council of the American Association for the Advancement of Science for the year 1949.

Professors Marston Morse and Oscar Zariski were nominated as representatives of the Society in the Division of Mathematical and Physical Sciences of the National Research Council for a three-year period beginning July 1, 1949.

The Council voted to invite Professor Deane Montgomery of the Institute for Advanced Study to deliver a series of Colloquium Lectures at the first Summer Meeting after that of 1949.

Abstracts of the papers read follow below. Papers whose abstract numbers are followed by the letter "*t*" were read by title. Paper number 422 was read by Professor Schafer, number 432 by Professor Durfee, number 452 by Professor Beckenbach, number 453 by Dr. Chandrasekharan, number 455 by Professor Cameron, number 460 by Professor Piranian, number 491 by Professor Scott, number 497 by Professor Todd, and number 528 by Professor Scheffé. Mr. Strehler was introduced by Professor C. C. MacDuffee and Mr. Pa by Professor G. T. Whyburn.

Algebra and Theory of Numbers

416*t*. A. A. Albert: *Absolute valued algebraic rings.*

An algebraic ring is a vector space A over a field F such that every element of A generates a finite dimensional algebra over F. A quadratic ring is a ring with a unity quantity such that the algebra generated by each element (not a scalar) is a quadratic field. We show that all alternative quadratic rings are finite dimensional, and use this result to prove that every absolute-valued real algebraic ring with a unity element is finite dimensional and thus is the real field, the complex field, the algebra of real quaternions, or the Cayley-Dickson algebra. The results may be extended immediately to division rings, that is, to algebraic rings whose nonzero elements form a quasigroup. (Received June 19, 1948.)

417. J. H. Bell: *Families of solutions of the unilateral matrix equation.*

A recent investigation of a problem described in the author's Bull. Amer. Math.

Soc. Abstract 47-3-115 leads to more general results regarding the existence of infinite families of solutions of the unilateral matrix equation $\sum A_i X^i = 0$, where the A_i and X have elements in a field of characteristic zero. It is found that the unilateral matrix equation will have an infinite family of solutions if and only if there are at least two distinct solutions which are similar. Also, all matrices, similar to a given solution X_1, will be solutions if and only if the minimum polynomial $m(\lambda)$ of X_1 is a common divisor of the elements of the matrix $A(\lambda) = \sum A_i \lambda^i$. In the proof of the latter result the following theorem arises: The equation $\sum A_i \cdot x X^i = 0$ has a solution if and only if the elements $A(\lambda)$ have a greatest common divisor which is not a unit. It is also proved that if the determinant of $A(\lambda)$ is identically zero and the unilateral equation has a solution, there is an infinite number of solutions. (Received May 4, 1948.)

418. J. W. Bradshaw: *Note on Kummer's second illustration.*

The author calculates the series $\sum (u_n/v_n)^n$, where $u_n = 1 \cdot 3 \cdots (2n-1)$, $v_n = 2 \cdot 4 \cdots (2n)$, by means of a continued fraction formula for the remainder after summing a fixed number of terms. He compares the method with the calculation in terms of gamma functions. (Received July 21, 1948.)

419*t*. A. T. Brauer: *Limits for the characteristic roots of a matrix.* III.

It was proved in the first two parts (Duke Math. J. vol. 13 (1946) pp. 387–395 and vol. 14 (1947) pp. 21–26) that the characteristic roots ω_v of an arbitrary matrix must lie in certain n circles, and more exactly in certain $n(n-1)/2$ ovals of Cassini. In this paper much smaller regions are obtained which must contain all the ω_v. These results are sometimes very sharp for numerically given matrices. This is shown by an example of a matrix of order 5 where for the absolute greatest root ω the interval $9.061 < \omega < 9.215$ is obtained while actually this root satisfies $9.187 < \omega < 9.188$. (Received July 26, 1948.)

420*t*. R. C. Buck: *Approximation in a class of polynomials.* Preliminary report.

The problem studied is that of the minimal value of $\|P\|$ for all polynomials P having integral coefficients and leading coefficient 1, under various choices of the norm $\| \;\|$. Results parallel those of Fekete (Math. Zeit. vol. 17 (1923) pp. 228–249). (Received July 29, 1948.)

421*t*. Leonard Carlitz: *Representations of arithmetic functions in* $GF[p^n, x]$. II.

In a previous paper (*Representations of arithmetic functions in* $GF[p^n, x]$, Duke Math. J. vol. 14 (1947) pp. 1121–1137), the writer proved that an arbitrary arithmetic function $f(A)$ can be written in the form $f(A) = \sum^* \alpha_{GH} \epsilon_{GH}(-A)$, the summation extending over a "fundamental" set of functions ϵ_{GH}, deg $A < r$. Now in certain applications considerable difficulty is caused by the presence of ϵ's with $h > r/2$ in this formula. It is now shown how in at least some problems this difficulty can be overcome by setting up a correspondence between ϵ's of degree $h > r/2$ and those with $h \leqq r$. This correspondence is effected by means of the equation $G'H - GH' = \theta$, where θ is in $GF(p^n)$. Applications are made to the sums $\sum_{B} \epsilon_{GH}(B^m)$, where $r = mk$, and the summation is either over all primary B of degree k or all B of degree less than k. (Received July 6, 1948.)

422. Claude Chevally and R. D. Schafer: *The exceptional simple Lie algebras F_4 and E_6.*

There is given a characterization of the exceptional simple Lie algebras of dimension 52 and 78 over an algebraically closed field K of characteristic 0. Let $\mathfrak{J}$ be the exceptional simple Jordan algebra of dimension 27 over K. It is shown that the derivation algebra $\mathfrak{D}$ of $\mathfrak{J}$ is the algebra F_4 (of rank 4 and dimension 52) in Cartan's list of simple Lie algebras over K. For X in $\mathfrak{J}$, denote by R_X the corresponding right multiplication. Since $[R_X, R_Y]$ is a derivation, the set $\mathfrak{D}+\{R_T\}$ for T of trace zero is a Lie algebra, which is shown to be E_6 (the simple Lie algebra of rank 6 and dimension 78 over K). (Received July 23, 1948.)

423*t*. Sarvadaman Chowla: *A new proof of a theorem of Siegal.*

Let $\chi(n)$ denote a real *primitive* character (mod k) where $k>1$. Siegel's celebrated theorem asserts that if ϵ is any positive number, we have $\sum_1^\infty \chi(n)n^{-1}>k^{-\epsilon}$ $[k>k_0(\epsilon)]$. Siegel's proof (Acta Arithmetica vol. 1 (1936)) used class-field-theory. In this paper the author gives a proof which is based on simple properties of Dirichlet's L-functions defined by $L(s, \chi)=\sum_1^\infty \chi(n)n^{-s}$ which is valid for $R(s)>0$. (Received July 27, 1948.)

424*t*. Sarvadaman Chowla: *On the least prime quadratic non-residue of a prime.*

Let q denote the smallest positive prime for which $x^2\equiv q$ (mod p) has no solution. Vinogradoff proved that for large p, $q<p^{1/2e^{1/2}}\log^2 p$. The author proves the sharper result, $q<p^\epsilon$ $[q>q_0(\epsilon)]$ (where ϵ is an arbitrary positive number), *assuming*, however, the "*extended Riemann hypothesis.*" (Received July 27, 1948.)

425. I. S. Cohen: *Prime factorization of ideals.*

It is proved that a commutative ring in which every ideal is a product of prime ideals is a direct sum of a finite number of rings each of which is either an integral domain with Dedekind-Noether ideal theory or else a local ring whose maximal ideal is principal and nilpotent. As a consequence, such rings are Noetherian. Among Noetherian rings, such rings can be characterized by the distributivity of the lattice of ideals, or by the validity of the relation $A:(B\cap C)=A:B+A:C$ among any three ideals, or by the validity of several other similar relations. (Received July 29, 1948.)

426. Ben Dushnik: *Isoperimetric sets.*

A set of different pairs of natural numbers—$p_1, q_1; \cdots; p_n, q_n, n>1$—is called a set of "isoperimetric" pairs (briefly, an isoperimetric set) if, for every pair, (I) $p<q$; (II) p and q are relatively prime and $p+q$ is odd; (III) $pq+q^2$ is constant for all pairs. Concerning such sets, the following two questions are discussed. If $N>1$ is any natural number, does there exist an isoperimetric set with N pairs? If p is any natural number, does there exist a natural $q>p$ such that the pair p, q satisfies (II) above and belongs to an isoperimetric set? Both questions are answered affirmatively; as to the second one, there are indeed infinitely many q's with the desired property. The constant value of $pq+q^2$ is called the "index" of the set. The smallest index is 858, corresponding to the pairs 7, 26 and 17, 22; this answers a question by Dernham in Amer. Math. Monthly vol. 55 (1948) p. 248. (Received August 4, 1948.)

427*t*. Franklin Haimo: *Division topology in an Abelian group.* Preliminary report.

Let G be an abelian group in which the patterns of the nonzero elements form a directed set with no maximum. (For information on patterns, see Duke Math. J. vol. 15 (1948) pp. 347–356.) For an element g in G, let V_g be the set of all h in G with the same pattern as g or with a "better" pattern. Then the V_g serve as nuclei with respect to which G is a topological group, the topology being called the division topology of G. If G has torsion elements and a division topology, then the torsion part of G is primary. A division topology is also a division-closure topology but not a connected topology. The possibility of a group having both a division topology and a compact topology is discussed. (Received July 26, 1948.)

428. I. N. Herstein: *Divisor algebras.*

In this paper the divisor algebras of algebraic function fields of degree of transcendence one over algebraically closed fields of constants are characterized axiomatically. Prime divisors, equivalence and dependence are taken as primitive. The free Abelian group generated by the prime divisors is considered. The elements of this group are called divisors. The axioms are of such a nature that: (1) equivalence classes of divisors form projective geometries under the dependence relation; (2) group multiplication induces a collineation; (3) a general Pappus theorem is true; (4) in every integral line of divisors there is at least one multiple of every prime divisor. It is shown that with the appropriate definition, functions on the set of prime divisors to lines of these geometries can be introduced and their sums and products defined so that a field is obtained which is an algebraic function field of degree of transcendence one over the algebraically closed projective field constructed on any line of these geometries. (Received May 14, 1948.)

429*t*. A. P. Hillman: *Reduction to standard form of an aerodynamical matrix characteristic value problem.*

The system $\sum_{j=1}^{n} d_{ij}y_j=\lambda(y_1+\sum_{j=1}^{n} c_jy_j)$ $(i=1, \cdots, n)$ arises in the National Bureau of Standards Report *Flexural transients in model wing following vertical "landing impact" at point below center of gravity*, NBS Lab 65181 PR1, BuAer TED NBS 2410, June 1946, p. 8. The present paper reduces it, under the assumption that $\sigma=\sum_{r=1}^{n} c_r$ is different from -1, to the standard matrix characteristic value system $\sum_{j=1}^{n} a_{ij}z_j=\lambda z_i$ by the substitutions $a_{ij}=d_{ij}-c_j(1+\sigma)^{-1}\sum_{s=1}^{n}d_{is}$, $y_j=z_j-(1+\sigma)^{-1}\sum_{i=1}^{n}c_iz_i$. (Received July 6, 1948.)

430*t*. R. E. Johnson: *The product of groups and rings.* Preliminary report.

For any two sets M and N, let (M, N) be the free abelian semi-group with generators (x, y), x in M, y in N. A multiplication is naturally introduced into (M, N) in case M and N are multiplicative sets. Any group (or ring in case M and N are multiplicative) that is a homomorphic image of (M, N) is called a product of M and N. The study of these products is reduced to the study of "admissible" subgroups of the free abelian group generated by (M, N). Results similar to those holding for tensor products and classical products of algebras are obtained. (Received July 27, 1948.)

431*t*. B. W. Jones: *A theorem on integral symmetric matrices.*

The following theorem is proved: Let A and B be symmetric integral nonsingular matrices with respective dimensions n and m $(n>m)$ and S an n by m matrix of rank m with rational elements such that s is the l.c.m. of the denominators and $S^TAS=B$.

Then there is an n by n matrix T with rational elements the prime factors of whose denominators all divide s, whose determinant is 1 and which takes A into an integral matrix A_0 which represents B integrally, that is, $U^TA_0U=B$ for some integral matrix U. Though the proof is independent of the theory of quadratic forms it immediately implies the following known result: If f and g are two quadratic forms with integral coefficients, having nonzero determinants and n and m variables respectively, $n>m$, and if there is an n by m matrix with rational elements whose denominators are prime to $2|f|$ and taking f into g, there is a form f_0 in the same genus as f whose matrix has integral elements and which represents g integrally. (Received June 7, 1948.)

432. B. W. Jones and William H. Durfee: *A theorem on quadratic forms over the ring of 2-adic integers.*

Let f, g and h be quadratic forms over the ring $R(2)$ of 2-adic integers, g and h having no variables in common with f and such that the symmetric matrices of f, g and h have their elements in $R(2)$. The authors show that if f is unimodular, then $f+2g$ equivalent to $f+2h$ implies g equivalent to h. The proof depends on the following lemma, of some interest in itself: If $T'FT\equiv I \pmod 2$, where F is the matrix of f and T is a square matrix over $R(2)$ of the same order, then there is an automorph D of F over $R(2)$ such that $2(T+D)^{-1}$ has its elements in $R(2)$. (Received June 1, 1948.)

433t. Joseph Lehner: *Further congruence properties of the Fourier coefficients of the modular invariant $j(\tau)$.*

Let $j(\tau)=x^{-1}+\sum_0^\infty c_nx^n$, $x=\exp 2\pi i\tau$, where $j(\tau)$, with $\operatorname{Im}\tau>0$, is the absolute modular invariant on the full modular group: $j(\tau')=j(\tau)$ for every $\tau'=(a\tau+b)/(c\tau+d)$ with integer a, b, c, d, and $ad-bc=1$. Continuing the results of a previous paper (Bull. Amer. Math. Soc. Abstract 54-7-231), the author proves: $c_n\equiv 0 \pmod{2^{2a+9}}$ if $n\equiv 0 \pmod{2^a}$, $c_n\equiv 0 \pmod{3^{2a+3}}$ if $n\equiv 0 \pmod{3^a}$, for $a, n=1, 2, 3, \cdots$. The modular equations for the univalent functions on the groups $\Gamma_0(p)$ (defined by $c\equiv 0 \pmod p$) are derived for the primes p for which $\Gamma_0(p)$ is of genus zero. Sufficient conditions are given for a modular function to possess the congruences of this and the previous paper. (Received August 9, 1948.)

434t. W. J. LeVeque: *A metric theorem on uniform distribution* (mod 1).

A new proof is given of a theorem of Koksma (Compositio Math. vol. 2 (1935) pp. 250–258) that the sequence x^n, $n=1, 2, \cdots$, is uniformly distributed (mod 1) for almost all $x>1$. This is contained in the following theorem, which is slightly weaker than Koksma's Theorem 3: Let $g(x, 1)$, $g(x, 2)$, $\cdots$, be any sequence of twice-differentiable functions of x, such that $h(x, j, k)=g'(x, j)-g'(x, k)$ is either nonincreasing or nondecreasing and is different from zero in (a, b) for $j\neq k$, and such that for $x=a$ and for $x=b$, $|h(x,j,k)|\geqq C|j-k|^\epsilon$ for some $C>0$, $0<\epsilon\leqq 1$. Then the sequence is uniformly distributed (mod 1) for almost all $x\in(a, b)$. This theorem is deduced from a lemma used by Kac, Salem and Zygmund (Trans. Amer. Math. Soc. vol. 63 (1948) pp. 235–244) in their investigations of quasi-orthogonal functions. (Received August 2 1948.)

435. C. C. MacDuffee: *Orthogonal matrices in four-space.*

Every real proper 4 by 4 orthogonal matrix can be written $A=R(\alpha)\cdot S^T(\beta) =S^T(\beta)\cdot R(\alpha)$ where α and β are unit quaternions, and $R(\alpha)$ and $S(\beta)$ are their first and second regular representatives. Every such product is orthogonal and proper. The coefficients of α and β can be obtained from the elements of A merely by the solution of linear and quadratic equations. (Received July 26, 1948.)

436. H. B. Mann: *On the field of origin of an ideal.*

Equality of ideals is defined as in Hecke: *Theorie der algebraischen Zahlen*, § 37. An ideal $\mathfrak{a}$ is said to originate in $\mathfrak{F}'$ over $\mathfrak{F}$, if it is not contained in any proper subfield of $\mathfrak{F}'$ which contains $\mathfrak{F}$. The numbers in $\mathfrak{a}$ which are also in $\mathfrak{F}$ form an ideal $\mathfrak{A}$ in $\mathfrak{F}$ called the ideal corresponding to $\mathfrak{a}$ in $\mathfrak{F}$. If $\mathfrak{A}=\mathfrak{a}^e\mathfrak{c}$, $(\mathfrak{a}, \mathfrak{c})=1$, then $\mathfrak{a}$ is called of order e with respect to $\mathfrak{F}$. If $e=1$ then $\mathfrak{a}$ originates over $\mathfrak{F}$ in one and only one field. If $\mathfrak{P}=\mathfrak{p}^g$ is the highest power of the ideal $\mathfrak{p}$ which is prime ideal in any field over $\mathfrak{F}$ then $\mathfrak{P}$ originates in a unique field $\mathfrak{F}'$ over $\mathfrak{F}$, whilst $\mathfrak{p}$ itself originates in infinitely many fields over $\mathfrak{F}$. Every field $\mathfrak{F}''$ in which $\mathfrak{p}$ originates over $\mathfrak{F}$ must be of the form $\mathfrak{F}'(\alpha)$ where α satisfies an equation of degree rg (r integral) with integral coefficients a_i $(i=0, 1, \cdots, rg)$, $a_0=1$, in $\mathfrak{F}'$ for which $a_{lg+k}\equiv 0$ $(\mathfrak{P}^{l+1})$, for $k>0$ and $a_{rg}\not\equiv 0$ $(\mathfrak{P}^{r+1})$. (Received July 1, 1948.)

437. Leo Moser: *Number of distances determined by n points.*

Let $f(n)$ be the minimum number of distances determined by n points in a plane. P. Erdös has shown that $f(n)>(n-1)^{1/2}-1$ (Amer. Math. Monthly vol. 53 (1946) pp. 248–250). By considering the possible distribution of points in concentric rings, center at one of the points determining the least distance, it is shown that $f(n)>cn^{2/3}$, where c is a fixed constant. For the n points vertices of a convex polygon Erdös conjectured $f(n)\geqq[n/2]$. It is shown that $f(n)\geqq[(n+2)/3]$. (Received July 16, 1948.)

438*t*. H. T. Muhly: *The irregularity of an algebraic surface and a theorem on regular surfaces.*

It is shown by elementary methods that if $|A_m|$ is the completion of the m-fold of the system of hyperplane sections of a normal model U of a field Σ/k of algebraic functions of two variables then the deficiency $\alpha(m)$ of the characteristic series of $|A_m|$ has a constant value $\delta(U)$ when m is large (a result due originally to Castelnuovo). The character $\delta(U)$ of U is shown to be a relative invariant of U, in the sense that it is invariant under regular birational transformations of U. Moreover, the value of $\delta(U)$ is the same for all nonsingular models of Σ, and it is this value which is defined to be the irregularity of Σ. If $\delta(U)=0$, then derived normal models U_h of U belonging to sufficiently high characters of homogeneity h have the property that their generic hyperplane sections are arithmetically normal. If O is the ring of homogeneous coordinates along U_h and if (y_0, y_1, y_2) is any triple of homogeneous elements of degree one in O such that O depends integrally on $k[y_0, y_1, y_2]$, then O has an independent integral base over $k[y_0, y_1, y_2]$. If x_1, x_2 is an arbitrary transcendence base for a regular field Σ/k, and if I is the integral closure in Σ of the ring $k[x_1, x_2]$, then there exist integers h such that I has an independent integral base over $k[x_1^h, x_2^h]$. (Received July 21, 1948.)

439. W. V. Parker: *Sets of complex numbers associated with a matrix.*

If A is a square matrix with complex elements and x and y are vectors such that $x\bar{x}'=y\bar{y}'=1$, the set of complex numbers $xA\bar{y}'$ is the set of all numbers in or on the circle of radius ρ about zero in the complex plane, where ρ^2 is the greatest of the characteristic roots of $A\bar{A}'$. The set $xA\bar{x}'$ is a convex set known as the field of values of A and is the set of all diagonal elements of matrices UAU' where U is unitary. The set $xA\bar{y}'$, where $x\bar{y}'=0$, is the set of all non-diagonal elements of matrices $UA\bar{U}'$ where U is unitary. If A is Hermitian with characteristic roots $\lambda_1 \leqq \lambda_2 \leqq \cdots \leqq \lambda_n$ this latter set consists of all numbers in or on the circle of radius $(\lambda_n-\lambda_1)/2$ about zero in the complex plane. (Received July 26, 1948.)

440*t*. C. E. Rickart: *Isomorphism of groups of linear transformations.*

Consider a system $(\mathfrak{X}, D, \mathfrak{X}^*; \mathcal{G})$ in which $\mathfrak{X}$, $\mathfrak{X}^*$ are respectively right and left linear vector spaces over the division ring D, which are dual in the sense of Jacobson (Ann. of Math. vol. 48 (1947) pp. 8–21) and have dimension greater than 3, and $\mathcal{G}$ is a group of linear transformations on $\mathfrak{X}$ under the operation $A \circ B = A+B-AB$. Assume further that every $A \in \mathcal{G}$ possesses an adjoint (loc. cit. p. 16) and that $\mathcal{G}$ contains every finite-dimensional involution (T is an involution if $T \circ T=0$) which possesses an adjoint. Let $(\mathfrak{Y}, E, \mathfrak{Y}^*; \mathcal{H})$ be a second such system and assume $\mathcal{G}$ and $\mathcal{H}$ isomorphic as groups. Then either $\mathfrak{X}$ and $\mathfrak{Y}$ (also $\mathfrak{X}^*$ and $\mathfrak{Y}^*$) are isomorphic as linear spaces or $\mathfrak{X}$ and $\mathfrak{Y}^*$ (also $\mathfrak{X}^*$ and $\mathfrak{Y}$) are anti-isomorphic (here D and E are anti-isomorphic instead of isomorphic). The proof uses methods developed by Mackey (Ann. of Math. vol. 43 (1942) pp. 244–260) in studying normed linear spaces. For the finite-dimensional case in which D and E are fields, the above follows from results of Schrier and van der Waerden (Abh. Math. Sem. Hamburgischen Univ. vol. 6 (1928) pp. 303–322.) (Received July 19, 1948.)

441. W. E. Roth: *On the eliminant of $f(x)$ and $x^nf(1/x)$.*

Elliot, in Proc. London Math. Soc. vol. 25, showed that Sylvester's dialytic eliminant of the two polynomials $f(x)$ and $x^nf(1/x)$ is the product $f(1)f(-1)F$ where F is the square of a rational function of the coefficients of $f(x)$. This theorem was again taken up by Taylor and by Muir. In the present paper we write $f(\lambda x)=a_0+a_1\lambda x +a_2\lambda^2x^2+ \cdots +a_n\lambda^nx^n$ and express the Sylvester eliminant of $f(\lambda x)$ and $x^nf(\lambda/x)$ as the product $f(\lambda)f(-\lambda)c^2D^2$ where c is a constant and D is a determinant of order $n-1$, whose elements are given in terms of $f(\omega^i\lambda)$, where ω is a primitive $2n$th root of unity. (Received June 30, 1948.)

442. B. M. Stewart: *A note on least common left multiples.*

Consider n-by-n matrices with elements in a principal ideal ring. C. C. MacDuffee (Bull. Amer. Math. Soc. vol. 39 (1933) p. 574) presents a method, due in essence to E. Cahen and A. Châtelet, for finding a least common left multiple M of two given matrices A and B, provided that both A and B are nonsingular. In this note a different proof shows that the stated method for finding an M is correct provided merely that the greatest common right divisor D of A and B is nonsingular. In the case that D is singular the stated method for finding an M may fail; in this case a modified method is suggested for finding an M by operating in a "simplest" way on the Hermite triangular forms of A and B. In every case M is unique up to a unimodular left factor. (Received April 27, 1948.)

443. R. R. Stoll: *Homomorphisms of semigroups.*

Homomorphisms of a semigroup S onto groups with a zero element adjoined are investigated. With no loss of generality it is assumed that S has a zero element. Then the above homomorphisms exist if and only if S contains a prime ideal. The main result obtained is a set of sufficient conditions for the existence of maximal elements among these homomorphic images. These conditions are then applied to the various classes of semigroups. Methods due to Dubreil (Memoires de l'Academie des Sciences de l'Institut de France (2) vol. 63 (1941) pp. 1–52) are used. (Received September 10, 1948.)

444. A. F. Strehler: *A result in "factorisatio numerorum."*

An explicit expression is found for the number-theoretical function $f(n)$, which is the number of different factorizations of the natural number n into factors each >1, the ordering of factors being considered. This formula is a summation over the μ-functions of Souriau (Revue Scientifique vol. 82 (1944) pp. 204–211), which are in turn functions of the exponents in the prime-power factors of n. The coefficients in this summation are closely related to the binomial coefficients: they are, in fact, sums of the diagonal arrays in the first k-rows of a Pascal triangle, and they themselves form a new Pascal-type triangle. The previously used expressions for the function $f(n)$ have been recursion formulas. An expression similar to that for $f(n)$ is derived for the closely related Moebius μ-function. (Received June 28, 1948.)

445. Olga Taussky: *On a theorem of Latimer and MacDuffee.*

The theorem (Ann. of Math. vol. 34 (1933) 313–316) concerns the 1-1 correspondence between the classes of matrices $S^{-1}AS$ and the classes of ideals in the ring of polynomials in A where A is a matrix solution of an algebraic equation $f(x)=0$. All the polynomials are assumed to have rational integers as coefficients, $f(x)$ is of degree n and A is an $n\times n$ matrix with integral elements which has $f(x)=0$ as minimum equation, S is a unimodular matrix with integral elements. An alternative proof in the case that $f(x)=0$ is irreducible is given. (Received July 24, 1948.)

446. Leonard Tornheim: *Construction of an integral basis of any algebraic number field by means of valuations.*

A construction for the valuations of an algebraic field has been given by S. MacLane (Trans. Amer. Math. Soc. vol. 40 (1936) pp. 363–395; Duke Math. J. vol. 2 (1936) pp. 492–510). By means of it some of the ideas of Berwick (*Integral bases*, 1927) are restated and results extended to give a construction of an integral basis of any algebraic number field without the exceptions he had and a basis for any ideal of the field. (Received July 27, 1948.)

447*t*. Bernard Vinograde: *The maximum of certain determinants.*

The maximum determinant of a positive definite matrix P, whose diagonal elements are ones, under the transformations TPT', where T is diagonal $(A, B, C, \cdots)$, $A, B, C, \cdots$ nonsingular of preassigned degrees, is attained when the corresponding blocks in TPT' are unit matrices. For the case of two blocks, this coincides with the condition under which linear functions of two sets of variates have maximum correlation, P being the variance-covariance matrix. (Received July 27, 1948.)

448*t*. N. A. Wiegmann: *Some theorems on infinite normal matrices.*

Some theorems and analogs of theorems which are known to be true for finite normal matrices are shown to hold for fully continuous infinite normal matrices as defined by Hellinger-Toeplitz (that is, for the matrix A formed by the coefficients of a fully continuous normal bilinear form $A(x, y)$). Among these are: If A, B and AB are fully continuous infinite normal matrices, then BA is a fully continuous infinite normal matrix; if A and B are normal, then AB is normal if and only if each commutes with the hermitian polar matrix (as defined for the infinite normal matrix) of the other; a set $\{A_i\}$ of fully continuous infinite normal matrices can be brought into diagonal forms by the same unitary transformation if and only if they commute in pairs. In addition, a canonical form under a unitary transformation is obtained for a set of fully continuous infinite normal matrices $\{A_i\}$ which is closed under multiplication; and an analog of the representation of the conjugate transpose of a finite normal matrix as a polynomial in the matrix is obtained for this type of infinite normal matrix. (Received July 21, 1948.)

449t. L. R. Wilcox: *A class of partially ordered sets.*

Systems are considered which exhibit properties of biological and other hierarchies with the order relation that of ancestry. These systems are defined by postulating a set L together with an unrestrictedly additive "parental" function Π on $\mathcal{L}$ to $\mathcal{L}$, where $\mathcal{L}$ is the set of all subsets of L. An "ancestral" function Π^∞ is defined by $\Pi^\infty(S)=\sum(\Pi^n(S); n=1, 2, \cdots), S\subset L$, where Π^n is the nth iterate of Π. Axioms are as follows: (1) for no x in L is x in $\Pi^\infty([x])$; (2) if $\Pi([x])$ is not empty, it consists of exactly two elements; (3) the partial ordering introduced in L by Π^∞ satisfies an appropriate chain condition. A symmetric "parental" relation P is defined so that xPy means $[x, y]=\Pi([z])$ for some z in L. Necessary and sufficient conditions are derived so that the domain of P may be divided into two disjoint sets ("sexes") with properties suggested by the biological instances. These systems are not lattices, except in trivial cases. Structure of the partial ordering is studied, and it is found that non-isomorphic systems exist whose partial orderings are nevertheless isomorphic. (Received July 27, 1948.)

450. L. R. Wilcox: *A problem concerning binary relations.*

Let L be a set and P a symmetric binary relation on $L\times L$. The equation $R+R^*=P$ ($+$being set addition) is considered for R a relation on $L\times L$ such that R and its transpose R^* have disjoint domains. It is found that a solution exists if and only if P is *alternating* in the following sense: Define, for any $S\subset L$, $\phi(S)$ as the set of all y in L for which x in S exists with xPy; define ϕ^n as the nth iterate of the set function ϕ. Then for every one element set $[x]\subset L$, and for $n=1, 2, \cdots, \phi^{n-1}([x])$ and $\phi^n([x])$ are disjoint. If P is alternating, a constructive method for finding all solutions R is found. Define P_{rt} as the smallest reflexive and transitive relation containing P. Let E be the set of equivalence classes in L defined by P_{rt}, and $\mathcal{E}$ the set of all subsets of E. Then the set of all solutions R is in one-to-one correspondence with $\mathcal{E}$. (Received July 27, 1948.)

ANALYSIS

451t. Warren Ambrose: *The L_2-system of a unimodular group.* I.

Some theorems are proved about the algebraic system made up by L_2 with respect to the Haar measure, with the operation of convolution (which is only defined for certain pairs of elements) for multiplication. (Received June 1, 1948.)

452. E. F. Beckenbach and E. W. Graham: *On subordination in complex variable theory.*

A general result is established, including as special cases the theorem of Study concerning conformal maps on convex domains, the analogous theorem concerning star-shaped maps, the general theorem of L. R. Ford (Duke Math. J. vol. 1 (1935) pp. 103–104), and the fundamental subordination theorem, and including also new results of which the following is typical: Let the analytic function $w=f(z)$ map $|z|<1$ on a plane domain D, and let the map of $|z|<r$, $0<r<1$, be denoted by $D(r)$. If for some r_0, $0<r_0<1$, the convex hull of $D(r_0)$ is contained in D, then for all ρ, $0<\rho<1$, the convex hull of $D(r_0\rho)$ is contained in $D(\rho)$. (Received July 26, 1948.)

453. Salomon Bochner and K. Chandrasekharan: *Summations over lattice-points in k-space.*

The convergence properties of expansions in Bessel functions, like $\sum r_k(n, h)J_{\alpha+k/2} \cdot (2\pi(nx)^{1/2})n^{-2\alpha+k)/4}$, where $r_k(n, h) = \sum \exp\{2\pi i(n_1h_1 + \cdots + n_kh_k)\}$ $(\sum n_i^2 = n)$, occurring in lattice-point problems are studied. The results obtained generalize or sharpen those of G. H. Hardy, Proc. London Math. Soc. (2) vol. 15 (1916) pp. 192–213; J. R. Wilton, Proc. London Math. Soc. (2) vol. 29 (1929) pp. 168–188; A. L. Dixon and W. L. Ferrar, Quart. J. Math. Oxford Ser. vol. 5 (1934) pp. 48–63, 172–185. The method employed is based on the theory of Fourier series in several variables. (Received July 8, 1948.)

454*t*. R. H. Cameron: *A "Simpson's rule" for the numerical evaluation of Wiener's integrals in function space.*

In this paper the author gives an approximate formula expressing the Wiener integral $\int_C^w F(x)d_wx$ of a smooth functional $F(x)$ as an $(n+1)$-fold ordinary integral whose first n variables go from $-\infty$ to ∞ and whose last variable goes from -1 to 1. This last variable enters in a very peculiar way, and the integration with respect to it makes the formula exact for third degree polynomial functionals. For general functionals satisfying certain conditions of smoothness and order of growth at ∞, a specific estimate of the error is given, and it turns out to be $O(n^{-2})$. Thus, for such a functional, one would expect that tripling the multiplicity of the integral would improve the accuracy by about one decimal place. (Received July 27, 1948.)

455. R. H. Cameron and W. T. Martin: *The transformation of Wiener integrals by nonlinear transformations.*

Let C be the space of functions $x(t)$ continuous on I: $0\leqq t\leqq 1$, and vanishing at $t=0$. Let $y=Tx=x+\Lambda(x|\cdot)$ be a transformation which takes a Wiener measurable set $\Gamma\subset C$ into $T\Gamma\subset C$ in a 1-to-1 manner. Here $\Lambda(xt)$ is a functional defined for all functions $x\in\Gamma$ and all numbers t on I, and having its first variation $\delta\Lambda$ expressible in the form $\delta\Lambda(x|t|y)=\int_0^1 K(x|t, s)y(s)ds$. Then if $D(x)$ is the Fredholm determinant of $K(x|t, s)$ (with parameter $\lambda=-1$) and Λ satisfies certain smoothness conditions, the authors call $D(x)$ the "volume element ratio" and define the "measure element ratio" $J(x)=D(x)\int_0^1 (d/dt)[x(t)-y(t)]d[x(t)+y(t)]$. In terms of this measure element ratio the following formula for transforming Wiener integrals is established under suitable hypotheses: $\int_{T\Gamma}^C F(y)d_wy=\int_\Gamma^C F(Tx)|J(x)|d_wx$. (Received July 19, 1948.)

456*t*. E. F. Collingwood: *Exceptional values of meromorphic functions.*

Suppose $f(z)$ is nonrational and meromorphic in $|z|<R\leqq\infty$ and denote by $G(a, \sigma)$ a typical domain defined by the inequality $|f(z)-a|<\sigma$ (or $1/|f(z)|<\sigma$ if $a=\infty$). A domain $G(a, \sigma)$ is "bounded" if its closure is contained in $|z|<R$. A recent theorem

(H. L. Selberg, Comment. Math. Helv. vol. 18, p. 313) shows that if the valency of $f(z)$ in any domain $G(a, \sigma)$ does not exceed $p<\infty$ and if all these domains are "bounded" then $m(r, a)<p \log r+\log^+ 1/\sigma+O(1)$. Hence, if $T(r, f)$ is unbounded, $\delta(a)=\Delta(a)=0$, where $\delta(a)$ is the deficiency of a and $\Delta(a)=\lim \sup_{r\to R} m(r, a)/T(r,f)$. In this paper the theorem is generalized in the following way. The number $\lambda(r)$ of the domains $G(a, \sigma)$ intercepted by $|z|=r$, if any, is finite; these domains are denoted by $G_\nu(r, a, \sigma)$ $(\nu \leqq \lambda(r))$ while $p_\nu(r, a, \sigma)$ is the valency of $f(z)$ in $G_\nu(r, a, \sigma)$ and, by definition, $P(r, a, \sigma)=\max_{\nu\leqq\lambda(r)} p_\nu(r, a, \sigma)$. $E(a, \sigma, p)$ is the set of values of r in $0\leqq r\leqq R$ in which all the $G_\nu(r, a, \sigma)$ $(\nu\leqq\lambda(r))$ are "bounded" and $P(r, a, \sigma)\leqq p$. These definitions remain applicable when σ and p are made single-valued functions, $\sigma(r)$ and $p(r)$, of r. It is proved here that *for a nonincreasing* $\sigma(r)>0$ *and* $0\leqq p(r)<\infty$ *such that* R *is a limit point of* $E=E(a, \sigma(r), p(r))$ then $m(r, a)<p(r)(\log r+O(1)) +\log^+ 1/\sigma(r) +O(1)$ *for* $r\in E$); *while if* $f(z)$ *is regular in* $|z|<R\leqq\infty$ *then* $m(r, a) <(\pi/2)p(r) +\log^+ 1/\sigma(r)+O(1)$ *for* $r\in E$). A new class of inequalities relating the deficiencies $\delta(a)$ and $\Delta(a)$ with the limits of the ratios $\log^+ (1/\sigma(r))/T(r, f)$ and $P(r, a, \sigma(r))\log r/T(r,f)$ or $P(r,a,\sigma(r))/T(r,f)$ follow from this theorem. (Received May 17, 1948.)

457. Jane S. Cronin: *Branch points of solutions of equations in Banach space.*

A generalization of the theory of ramifications of solutions of non linear integral equations, developed by E. Schmidt, is obtained by studying the local solutions x of the equation in Banach space (1) $(I-C)x+S(y)+T(x, y)=\theta$. I is the identity, C is linear, completely continuous, S satisfies a Lipschitz condition. T is kth-order $(k\geqq 2)$, and y is fixed. By applying the Riesz theory of completely continuous transformations, a continuous mapping M of complex Euclidean n-space into itself, where n is the dimension of the null space of $I-C$, is derived. The number of solutions of $M(z)=0$ equals the number of solutions of (1). If $n=1$, and a certain nonzero condition is satisfied, the topological degree of M at 0 is k; if $n=k=2$, and certain nonzero conditions are satisfied, the degree of M is four. If $S(y)=-y$, $T(x, y)=T(x)$, and the Leray-Schauder degree is defined for $I-C+T$, the degree of M equals the Leray-Schauder degree times ± 1. (Received July 30, 1948.)

458. W. F. Eberlein: *Weak almost periodic functions.*

Let G be a locally compact Abelian group and $C(G)$ be the Banach space of complex-valued bounded continuous functions on G. $x(t)$ in $C(G)$ is said to be weakly almost periodic (w.a.p.) if the set of translates of x is conditionally compact in the weak topology. The set of w.a.p. functions is a closed invariant *subring* of $C(G)$ and contains, in addition to the ordinary a.p. functions, all continuous positive definite functions and all continuous functions vanishing at infinity. Every w.a.p. function is uniformly continuous and possesses a mean value in the sense of von Neumann. The convolution of two w.a.p. functions is then defined and turns out to be an ordinary a.p. function. Every w.a.p. function has an expansion in terms of countably many characters for which the Parseval equation holds. When G is the real line the w.a.p. functions form a subclass of the Weyl W^2 class of generalized a.p. functions. (Received May 21, 1948.)

459t. Arthur Erdélyi: *Inversion formulae for the Laplace transformation.* Preliminary report.

Let $L_{k,t}$ be a numerically valued linear operator on a sufficiently large class of functions of the complex variable s. $L_{k,t}$ depends on the positive variable t and on the (discrete or continuous) parameter k. If $L_{k,t}$ can be applied under the integral sign in $f(s)=\int_0^\infty \phi(t)e^{-st}dt$ and if $N(u,t,k)=L_{k,t}[e^{-su}]$ is a singular kernel, that is, $\lim \int_0^\infty N(u,t,k)\phi(u)du=\phi(t)$ in some sense as $k\rightarrow\infty$, then we have the inversion formula $\phi(t)=\lim L_{k,t}[f(s)]$ for the Laplace transformation. Application of known singular kernels (and of others which can be obtained by Laplace's asymptotic evaluation of integrals, by the method of stationary phase, or otherwise) gives all the known inversions of the Laplace transformation and many new ones. The extension to the Laplace-Stieltjes transformation and the establishing of representation theorems runs on comparatively well known lines. The same principle can be applied for the inversion of other functional transformations. (Received July 10, 1948.)

460. Paul Erdös and George Piranian: *Topologizations of a sequence space by convergence fields of Toeplitz transformations.*

Two bounded sequences x and y of complex numbers are defined to be equivalent provided there exists a nonzero constant λ and a convergent sequence α such that $y_n=\lambda x_n+\alpha_n$ $(n=1, 2, \cdots)$. The space m/L of equivalence classes of bounded sequences is topologized on the basis of the convergence fields of certain selected Toeplitz transformations, and some properties of the topological space are investigated. The investigation yields theorems concerning Toeplitz transformations whose fields of convergence are small. The basic device used in the paper is the principle of aping sequences: If A is a regular Toeplitz transformation and ξ a bounded sequence which oscillates sufficiently slowly; and if x is a bounded sequence and y the transform of x by A; then $A(x\xi)=y\xi+\alpha$, where α denotes a null sequence, $x\xi$ the sequence whose general element is $x_n\xi_n$, and $A(x\xi)$ the transform of $x\xi$ by A. (Received June 1, 1948.)

461. Evelyn Frank: *On the continued fractions* $k_0\gamma_0+K_0^\infty(k_n(1-\gamma_n\bar{\gamma}_n)\cdot z/\bar{\gamma}_n z-1/k_{n+1}\gamma_{n+1})$.

In this paper it is shown that every power series (1) $\sum_{n=0}^\infty c_n z_n$ admits an expansion into a continued fraction of the form (2) $k_0\gamma_0+k_0(1-\gamma_0\bar{\gamma}_0)z/\bar{\gamma}_0 z-1/k_1\gamma_1+k_1\cdot(1-\gamma_1\bar{\gamma}_1)z/\bar{\gamma}_1 z-1/k_2\gamma_2+\cdots$, where the γ_p are certain rational functions of the c_p, and the k_p are constants chosen so that $|\gamma_p|\neq 1$. Conversely, to every continued fraction (2) there corresponds a power series (1). The power series expansions for the $2p$th and $(2p+1)$th approximants of (2) agree with (1) up to and including the term involving z^p and z^{p-1} respectively. A simple algorithm is shown for the expansion of the power series (1) into the continued fraction (2). Conditions under which the continued fraction (2) converges uniformly to the function represented by the power series (1) are found. Also conditions on the γ_p and k_p are given for which (2) converges uniformly in certain specific regions. (Received July 23, 1948.)

462. R. E. Fullerton: *A characteristization of L spaces.*

By using methods similar to those of Clarkson in characterizing spaces of continuous functions it is shown that necessary and sufficient conditions that a Banach space X be a space of integrable functions are: (1) The surface of the unit sphere contains a maximal closed convex set F which contains at least one point not on its boundary such that if C is the set $E\{\lambda x|x\in F, \lambda\geqq 0\}$ then C has the property that for any points $u, v\in X$ there exists a $z\in X$ such that $(u+C)\cap(v+C)=z+C$. (2) If $\|x\|=\|y\|=1$ and $-[(x-C)\cap(y-C)]=C$ then all point of the segments joining x and y and

x and $-y$ lie entirely on the surface of the unit sphere, (3) The unit sphere is the closed convex set determined by F and $-F$. In such a space X the family of maximal convex sets on the surface of the unit sphere may be made into a Boolean algebra in such a way that X is equivalent to the space of all completely additive functions defined over the algebra which vanish at $-F$. If F is a convex set determined by a denumerable set of independent points then X is equivalent to the space l. (Received July 24, 1948.)

463*t*. P. R. Garabedian: *A problem of Robinson.*

Let D be a domain of the z-plane bounded by analytic curves $C_1, \cdots, C_n$, $n \geqq 2$. Let z_0 be a point of D, and let Ω be the class of analytic functions $F(z)$ in D with a possible simple pole at $z=z_0$ and with boundary values which are in modulus not greater than 1. Denote by $\sigma(\zeta)$ the maximum of $|F(\zeta)|$ for $F \in \Omega$, and denote by A the set of points $\zeta \in D$ such that $\sigma(\zeta)=1$. Then the region $D-A$ is simply-connected. This result has been proved by Robinson (Duke Math. J. vol. 10 (1943) pp. 341–354) for the case $n=2$. The method of the present paper is based on a continuity argument and the uniqueness of the extremal function $F_0 \in \Omega$ such that $F_0(\zeta)=\sigma(\zeta)$. It is further shown that for $n \geqq 3$ the set A has, except in special cases, interior points. If the class Ω is extended to include functions with $m \geqq 1$ possible simple poles, further results can be obtained by a method of contour integration. If the poles of $F(z)$ are located at the critical points of the Green's function of D, $G(z; \zeta)$, then $|F(\zeta)| \leqq 1$. (Received May 25, 1948.)

464*t*. J. J. Gergen and F. G. Dressel: *Mapping by p-regular functions.*

Let $p(x, y)$ be positive in the domain R. If $u(x, y) \in C''$, and if $(pu_x)_x+(pu_y)_y=0$, in R, then u is p-harmonic in R. If $u, v \in C''$, and if $pu_x=v_y$, $pu_y=-v_x$, in R, then $F(z)=u+iv$, $z=x+iy$, is p-regular in R. This paper is concerned with some of the properties of functions, p-harmonic or p-regular in the interior S of a circle, under continuity and analytic conditions on p. The fundamental problem considered is that of extending the classical theorem on the conformal mapping of S onto a domain bounded by a simple, closed, rectifiable curve. An extension of this theorem is obtained for p positive and analytic in a domain containing the closure of S, and sup. bound $(|\nabla p|/p)$ on S sufficiently small. The analysis is based on the methods of Douglas and Courant. (Received August 2, 1948.)

465. L. M. Graves: *A mapping theorem.*

Let $G(x)$ be a function of class C' near the origin $x=0$ in a Banach space X, with values in a Banach space Y, and suppose for convenience that $G(0)=0$. It is well known that the equation $y=G(x)$ gives a one-to-one correspondence between neighborhoods of $x=0$ and $y=0$ whenever the differential relation $dy=dG(0; dx)$ does so. The present paper is concerned with the case when the condition of one-to-one-ness is omitted in hypothesis and conclusion. The method of successive approximations is still applicable in the proof. The result makes it possible to derive a multiplier rule for minima of abstract functions without the use of projections. (Received July 26, 1948.)

466*t*. P. R. Halmos: *A nonhomogeneous ergodic theorem.*

If X is a non-atomic measure space with measure μ, such that $\mu(X)=1$, and T is a metrically transitive measure preserving transformation of X onto itself, then there exists a function f in $L_2(X)$ such that $\int f d\mu = 0$ and such that $\sum_{n=1}^{\infty} (1/n)f(T^n x)$ is *not* convergent in the mean of order two. This result shows that not only is Izumi's nonhomogeneous ergodic theorem vacuous, but even that there are no general hypotheses on T which imply its conclusion; to ensure the convergence of such series, special hypotheses on f are necessary. (Received June 8, 1948.)

467*t*. Israel Halperin: *Hamel's basis and nonmeasurable sets.*

Let a set of real numbers E be called s-nonmeasurable if $m_*(E)=0$ and $m^*(EI) = m(I)$ for every measurable set I. Suppose that $f(x+y)=f(x)+f(y)$ for all x, y, that $f(x)$ is not identically 0 and that $f(x)$ has only a countable number of distinct values $c_1, c_2, \cdots, c_n, \cdots$. Let E_n be the set of x for which $f(x)=c_n$. Then the E_n are all s-nonmeasurable. Let $a_1, a_2, \cdots, a_\alpha, \cdots$ be a Hamel's basis so that every real x has a unique representation $\sum_\alpha r_\alpha a_\alpha$, with all r_α rational and only a finite number of them different from zero. Then the x for which $r_1=0$ form an s-nonmeasurable set. More generally, any restrictions, which are not self-contradictory and are not vacuous, and which involve a fixed finite or countable set of the r_α, will define an s-nonmeasurable set. This gives a construction to subdivide the set of real numbers (or any interval) into a countable family of mutually exclusive, congruent (under translation) s-nonmeasurable sets. (Received August 3, 1948.)

468*t*. Israel Halperin: *Non-finite solutions of the equation $f(x+y) = f(x)+f(y)$.*

The complete family of solutions as given by G. Hamel, in Math. Ann. vol. 60 (1905) p. 461, seems to include many functions which have infinite values for some x. However the indicated non-finite solutions are not unambiguous. It is now shown that, even with the widest possible allowance for non-finite solutions, the only ones are the trivial ones (1) $f(x)=(+\infty)$ for all x with the convention $(+\infty)+(+\infty) = (+\infty)$, and (2) $f(x)=(-\infty)$ for all x with the convention $(-\infty)+(-\infty)=(-\infty)$. (Received August 3, 1948.)

469*t*. W. L. Hart: *Line integrals independent of the path in real Hilbert space.*

Let H be the set of all $x=(x_1, x_2, \cdots)$ in real Hilbert space for which $\|x\| \leqq r$. The paper considers $\mathfrak{J} = \int_C \sum_{i=1}^{\infty} h_i(x)dx_i$, where C is a continuous arc $x=x(t)$ in H for $0 \leqq t \leqq 1$, and consists of a finite number of regular pieces (on which there exists a continuous derivative $x'(t)$ with $\|x'(t)\| \neq 0$). It is proved that $\mathfrak{J}$ is independent of the path C in case (1) $h_i(x)$ is continuous in H and $\sum_{i=1}^{\infty} h_i^2(x)$ converges uniformly on every compact subset of H; (2) there exists $\partial h_i/\partial x_j$, continuous in H; (3) $\partial h_i/\partial x_j = \partial h_j/\partial x_i$. There is a corresponding theorem on the solution of the system $\partial f(x)/\partial x_i = h_i(x)$ $(i=1, 2, \cdots)$. In form, (3) is not new. The novelty of the paper consists of the elimination of an assumption that $\sum_{i,j} (\partial h_i/\partial x_j)^2$ converges. This condition is involved if the independence of the path is proved as a special case of results phrased for much more general situations in Banach spaces by Kerner (Ann. of Math. (2) vol. 34 (1933) p. 546) and Michal (Acta Math. vol. 68 (1937) p. 71). (Received July 27, 1948.)

470. J. G. Herriot: *The polarization of a lens.* Preliminary report.

Consider a conductor placed in an electrostatic field uniform at infinity and having a given direction there. The intensity of the disturbance produced by the conductor may be measured by a quantity called the polarization in the given direction. (A more precise definition of this concept will be given in a forthcoming paper by M. M. Schiffer and G. Szegö. See Bull. Amer. Math. Soc. Abstract 54-7-354.) The mean polarization P_m is the average of the polarizations in any three mutually orthogonal directions. If V is the volume and C the electrostatic capacity, then for a sphere it is known that $P_m = 2V = (8\pi/3)C^3$. It is the purpose of this paper to compare P_m with V and C for the lens which is a solid determined by the intersection of two spheres. For the bowl (limiting case of lens) it is shown that $P_m \geqq (8\pi/3)C^3$. For a lens with dielectric angle $\pi/2$ and for two tangent spheres (another limiting case of lens) it is shown that $P_m + V \geqq 4\pi C^3$. This implies that $P_m \geqq (8\pi/3)C^3 \geqq 2V$. In the first two cases the quantities were expressed in terms of elementary functions whereas in the third case the logarithmic derivative of the gamma function was involved. P_m was obtained from the above-mentioned paper. (Received July 23, 1948.)

471*t*. Einar Hille: *Remarks on a paper by Zeev Nehari.*

Zeev Nehari has proved in a paper to appear in Bull. Amer. Math. Soc. that $f(z)$ is univalent in the unit circle if its Schwarzian derivative satisfies the inequality $|\{f(z), z\}| \leqq 2[1-|z|^2]^{-2}$. In the present note it is shown that 2 is the best possible constant in the sense that for every $C>2$ there exists a function, holomorphic in the unit circle, which satisfies $|\{f(z), z\}| \leqq C[1-|z|^2]^{-2}$ and takes on certain values infinitely often in the unit circle. (Received August 28, 1948.)

472*t*. I. I. Hirschman: *The behaviour at infinity of certain convolution transforms.* Preliminary report.

Let $a_1, a_2, \cdots$, be any sequence of real constants for which $\sum_1^\infty a_k^{-2} < \infty$. We define $E(s) = \prod_1^\infty (1-s/a_k)e^{s/a_k}$, $G(t) = (2\pi i)^{-1}\int_{-i\infty}^{i\infty} [E(s)]^{-1}e^{st}ds$, $\overline{E}(s) = \pi(1-s/|a_k|)e^{s/|a_k|}$, $\overline{G}(t) = (2\pi i)^{-1}\int_{-i\infty}^{i\infty} [\overline{E}(s)]^{-1}e^{st}ds$. $G(t)$ is said to belong to class Ia if there are both positive and negative a_k's and if $\sum_1^\infty |a_k|^{-1} = \infty$. $G(t)$ is said to belong to class II if there are only positive a_k's and if $\sum_1^\infty a_k^{-1} = \infty$. Let $\alpha(t)$ be a function of bounded variation in every finite interval and set (*) $f(x) = \int_{-\infty}^{\infty} G(x-t)d\alpha(t)$ where we suppose only that this integral converges conditionally for some one value of x. If $G(t) \in$ class II it is known that the transform (*) converges for $(\gamma_c < x < \infty)$ and diverges for $(-\infty < x < \gamma_c)$ where γ_c is a constant depending on $\alpha(t)$. The author proves that if, in this case, $f(x) = O[G(x-r)]$ as $(x \to \infty)$, then $\alpha(t)$ is constant for $(r < t < \infty)$. If $G(t) \in$ class Ia then the transform (*) necessarily converges for all x $(-\infty < x < \infty)$. It is shown, in this case, that if $f(x) = O[\overline{G}(x-r)]$ as $(x \to +\infty)$ for arbitrarily large negative values of r, then $f(x) \equiv 0$. (Received July 9, 1948.)

473*t*. I. I. Hirschman and D. V. Widder: *A representation theory for a general class of convolution transforms.*

Let $E(s) = e^{cs}s^p \prod_1^\infty [1-(s/a_k)]e^{s/a_k}$, where the constants a_k are real and such that $\sum_1^\infty 1/a_k^2 < \infty$. The authors showed earlier (see Bull. Amer. Math. Soc. Abstract 54-5-193) that $1/E(s)$ is the bilateral Laplace transform of some function $G(t)$. The present paper obtains necessary and sufficient conditions for the representation of a function $f(x)$ as a convolution transform, $f(x) = \int_{-\infty}^{\infty} G(x-t)\, d\beta(t)$, where the function $\beta(t)$ is in a variety of special classes. In particular Bernstein's familiar representation of a completely monotonic function as a Laplace transform is generalized. (Received June 22, 1948.)

474. Witold Hurewicz: *Linearization problems.*

Let U be a neighborhood of the origin 0 in Euclidean E_n, and F a continuous mapping of U with $F(U)\in E_n$ and $F(0)=0$. When is F topologically equivalent to a linear transformation in the neighborhood of the origin, that is, when does there exist a topological mapping T defined over a certain n-hood of the origin such that TFT^{-1} is an affine mapping? In case F is represented by analytic functions of coordinates a sufficient condition can be given. In general the problem is unsolved. (Received July 27, 1948.)

475. M. A. Hyman: *On the identification principle for partial differential equations with singular coefficients.* Preliminary report.

A. Weinstein (Trans. Amer. Math. Soc. vol. 63 (1948) pp. 342–354) has considered the equation (I) $y\phi_{yy}+y\phi_{xx}+p\phi_y=0$ and proved the following identification principle for $p>0$: *Let $\phi_1(x, y)$ and $\phi_2(x, y)$ be two solutions of* (I) *even in y and analytic throughout a region R containing a segment L of the x-axis: then if ϕ_1 and ϕ_2 coincide on L, they coincide everywhere in R.* The author has extended Weinstein's result to cases where $p<0$ (but not one of the eigenvalues $p=-1, -2, -3, \cdots$) and obtained a necessary and sufficient condition that $\phi(x, y)$ be a solution of (I) analytic in R; from this condition it follows that ϕ must be even in y and uniquely determined by its values on L. The chief tools are expansions in powers of y. Using the same methods, the author has investigated the general equation (II) $y^n\phi_{yy}+A\phi_{xx}+B\phi_x+C\phi_y+D\phi=0$ where A, B, C, D are analytic functions of x, y in R and n is an integer not less than 1. He has found conditions on A, B, C, D such that an analytic solution $\phi(x, y)$ of (II) shall be even or shall be uniquely determined by its values on L. Extensions and applications of these results are now being studied. (Received July 28, 1948.)

476. W. G. Leavitt: *On matrices with elements holomorphic in an unbounded region and a generalization to Prüfer rings.*

Let $\mathfrak{E}$ be the set of all functions holomorphic in and on the boundary of every finite portion of a region R of the complex z-plane. Let A be any matrix with elements in $\mathfrak{E}$ and with characteristic equation each of whose roots is in $\mathfrak{E}$. It is shown that there exists a matrix $T(z)$ with elements in $\mathfrak{E}$ whose determinant $|T|$ is nonvanishing in R, such that $T^{-1}AT$ is in the normal form with zeros below the main diagonal. The theorem is proved by generalizing the proof (Duke Math. J. vol. 14 (1948)) for R a bounded region, requiring the additional theorem: there exists an integral function whose expansion is specified to a finite number of terms at each of a set of points having no finite accumulation point. An alternative proof is provided by showing that $\mathfrak{E}$ is a Prüfer ring, that is, a ring satisfying all postulates of a principal ideal ring with the exception of the infinite chain condition. It is found that the proof of Leavitt and Whaples (to appear in Bull. Amer. Math. Soc.) for principal ideal rings is also applicable to Prüfer rings, and thus to the ring $\mathfrak{E}$. (Received June 28, 1948.)

477t. Walter Leighton: *A substitute for the Picone formula.*

An idea first employed by Marston Morse is developed to provide a substitute and generalization for the Picone formula. It is shown that this generalization enables one to establish comparison theorems for solutions of self-adjoint differential systems to which the classical Sturm-Picone methods do not apply. (Received July 28, 1948.)

478t. G. R. MacLane: *Polynomials with zeros on a Jordan curve.*

Let D_1, D be bounded simply-connected domains in the z-plane, $D_1 \subseteq D$, and let the boundary of D be a rectifiable Jordan curve Γ. Let $f(z)$ be holomorphic and never zero in D_1. There exists a sequence of polynomials $P_n(z)$ with *all* zeros on Γ, such that $\lim_{n\to\infty} P_n(z) = f(z)$ uniformly in any closed subset of D_1. The zeros cannot be essentially more restricted since in general all the zeros of the sequence must be everywhere dense on Γ. In some cases results on the rapidity of convergence are obtained, for example: if Γ is analytic, $f(z)$ holomorphic and never zero in the closed domain $D+\Gamma$, then one can find $P_n(z)$ of degree n such that $|P_n(z)-f(z)| < c/nd^2$ where c is a constant and $d = \operatorname{dist}(z, \Gamma)$. Similar results are obtained for a piecewise analytic boundary. The results are extended to multiply-connected domains D, the approximating functions now being rational with exactly one pole (multiple) in each simply-connected domain exterior to D and with all zeros on the boundary of D. The approximating functions may be written explicitly in terms of an appropriate parametrization of the boundary. (Received June 24, 1948.)

479. Morris Marden: *On the polynomial solutions of the generalized Lamé differential equation.*

The generalized Lamé differential equation considered is $P(z)w''+Q(z)w'+R(z)w = 0$ where $P(z)$, $Q(z)$ and $R(z)$ are real polynomials of degrees p, at most $p-1$ and at most $p-2$ respectively. $P(z)$ is assumed to have only simple zeros a_j $(j=1, 2, \cdots, p)$ of which the first q $(q \leqq p/2)$ lie in the upper half plane, the next q are the conjugate imaginaries of the first q and the remaining are real. In the expansion $Q(z)/P(z) = \sum_{j=1}^{p} [A_j e^{i\alpha_j}/(z-a_j)]$ it is assumed that all $A_j > 0$ and all $|\alpha_j| < \pi/2$. As is well known, choices of $R(z)$, to be denoted by $V(z)$, exist corresponding to which the differential equation has a polynomial solution $S(z)$. In a previous paper (M. Marden, Trans. Amer. Math. Soc. vol. 33 (1931) pp. 934–944) the zeros of the $V(z)$ and the $S(z)$ were shown to lie in the smallest convex region enclosing both the real a_j and the ellipses with foci at the conjugate imaginary pairs a_j and $\bar{a}_j$ and with eccentricities of $\cos \alpha_j$. In the present paper, k denotes the number of pairs of conjugate imaginary zeros of $S(z)$ and for each $a_j = b_j + ic_j$, $j = 1, 2, \cdots, q$, $E(a_j, k)$ denotes an ellipse with center at $e_j = b_j + c_j \tan \alpha_j$, with minor axis of $m_j = 2|c_j - e_j|$ parallel to the y-axis and with major axis of $k^{1/2} m_j$. It is shown that every non-real zero of $S(z)$ lies in at least one of the ellipses $E(a_j, k)$, that every non-real zero of the rth derivative of $S(z)$ lies in at least one of the ellipses $E(a_j, k+r)$ and that every non-real zero of the corresponding $V(z)$ lies in at least one of the ellipses $E(a_j, k+2)$. (Received July 19, 1948.)

480t. Mary K. Peabody: *Differential operators of infinite order.*

Let $G(\theta) = \sum_{k=0}^{\infty} g_k \theta^k$ where the coefficients are complex numbers generated by an entire function $G(w) = \sum_{k=0}^{\infty} g_k w^k$. In the most general case $\theta = p(z)d/dz + q(z)$; $p(z)$ and $q(z)$ being analytic functions and $\theta^k = \theta \cdot \theta^{k-1}$. Four modifications of θ are considered: $\theta_1 = d/dz + q(z)$, $\theta_2 = d/dz + P(z)$, $\theta_3 = zd/dz + C$, $\theta_4 = zd/dz + P(z)$, where $P(z)$ is a polynomial of degree n. Necessary and sufficient conditions (taking the form of restrictions upon the order and type of $G(w)$) are determined for the applicability of the operators $G(\theta_i)$ to various classes of functions such as holomorphic functions, entire functions, and entire functions of specified order and type. For instance: If ρ and σ are conjugate orders and α and β conjugate types, a necessary and sufficient condition that $G(\theta_2)$ apply to the class of entire functions $\mathcal{J}_{\rho,\alpha}$, ρ fixed $> n+1$, α fixed

$(0<\alpha<\infty)$, is that $G(w)\in\mathfrak{G}_{\sigma,\gamma}$, where $\gamma<\beta$. Estimates of the order and type of the transforms $G(\theta_i)f(z)$ are made when such exist as entire functions. Many of the results of this paper are analogous to those of H. Muggli (Comment. Math. Helv. vol. 11 (1938) pp. 151–179) for $G(d/dz)$ and E. Hille (Duke Math. J. vol. 7 (1940) pp. 458–495) for $G(z^2-d^2/dz^2)$. (Received July 15, 1948.)

481. B. J. Pettis: *On rings and half-fields in measure theory.*

This paper deals with the existence and uniqueness of extensions of a modular real function ϕ defined on a subset E of a generalized Boolean algebra L, where $E\ni 0$ and $\phi(0)=0$, and with the preservation of modularity, lattice continuity, and bounded variation. Extensions to the smallest field and smallest σ-field containing E when E is a ring or a half-field are particularly considered. One by-product is the following. Suppose E is a ring of subsets of a topological space, ϕ is monotone increasing, and given x in E and $\epsilon>0$ there exist x', x'' in E, an open y, and a compact z such that $x'\geqq y\geqq x\geqq z\geqq x''$ and $\phi(x')-\epsilon\leqq\phi(x)\leqq\phi(x'')+\epsilon$; then ϕ has a unique completely additive extension to the Borel sets. When E equals the open sets this corrects a theorem of Caccioppoli (Rado, *Length and area*) and provides a somewhat shorter proof of the Riesz-Markoff theorem on the representation of linear functionals over continuous functions. (Received August 12, 1948.)

482*t*. B. J. Pettis: *Remarks on abstract Carathéodory measurability.*

Let D be a generalized Boolean algebra, R any space having a single associative binary operation $+$, and ϕ an arbitrary but fixed function on D to R. Call an element a in D *measurable* if $\phi(x)=\phi(x\cap a)+\phi(x-xa)$ holds for every x in D, and *c-measurable* if $\phi(x)=\phi(x\cap a)+\phi(x-xa)=\phi(x-xa)+\phi(x\cap a)$ for every x in D. Let M and C be the sets of measurable and c-measurable elements; then M is a sub-lattice of D and C is a field in D. Further results deal with relationships between M, C, and properties of ϕ, and include sufficient conditions that C be a σ-field. These results include certain standard elementary theorems concerning Carathéodory outer measure (real-valued) functions and are connected with, but independent of, certain results of M. F. Smiley concerning measurable elements in lattices. (Received July 26, 1948.)

483*t*. V. C. Poor: *The extension of certain theorems in analytic function theory to polygenic functions.*

The polygenic functions considered in a domain of the complex plane are to be single-valued continuous and regular. They are regular in the sense that they possess a differential at every point of the domain. The application of the theory of residues over an area developed in a previous paper permits the proof of the analogue to the Lieuville theorem. The Rouché and the Hurwitz theorems are also proved for polygenic functions. This extension essentially justifies the definitions for residues of polygenic functions over an area. (Received May 14, 1948.)

484*t*. M. H. Protter: *A class of harmonic polynomials.*

Harmonic polynomials are considered which arise in the solution of the Cauchy problem for the Laplace equation in three independent variables. A generating function, recursion and differential expressions as well as explicit expressions for the polynomials are obtained. The relations between these polynomials and Bessel, Legendre, Lamé, and Mathieu functions are discussed. The polynomials are transformable to "wave polynomials" without the introduction of complex quantities; the general solution of the Cauchy problem for the wave equation may be represented as the limit of sequences of such polynomials. (Received July 24, 1948.)

485. Tibor Rado: *On convergence in area.*

Let $x^j=x^j(u, v)$, $j=1, 2, 3$, $(u, v)\in Q$: $0\leqq u\leqq 1$, $0\leqq v\leqq 1$, be a representation of a continuous path-surface S in Euclidean 3-space x^1, x^2, x^3, and let $A(S)$ be the Lebesgue area of S. Let S^1, S^2, S^3 be the flat surfaces S^1: $x^1=0$, $x^2=x^2(u, v)$, $x^3=x^3(u, v)$, S^2: $x^1=x^1(u, v)$, $x^2=0$, $x^3=x^3(u, v)$, S^3: $x^1=x^1(u, v)$, $x^2=x^2(u, v)$, $x^3=0$ (the projections of S upon the coordinate planes). Let S_n: $x^1=x^1_n(u, v)$, $x^2=x^2_n(u, v)$, $x^3=x^3_n(u, v)$ be a sequence of surfaces such that $x^j_n(u, v)\rightarrow x^j(u, v)$ uniformly in Q, and let S^j_n, $j=1, 2, 3$, be the projections of S_n upon the coordinate planes. Suppose that $A(S)<\infty$ and $A(S_n)\rightarrow A(S)$. Theorem: Under the conditions just stated, we have the relations $A(S^j_n)\rightarrow A(S^j)$, $j=1, 2, 3$. This theorem is an extension, to continuous path-surfaces in general parametric representation, of similar results established previously in the literature for arc-length, and for surfaces given in non-parametric form. (Received March 11, 1948.)

486. P. V. Reichelderfer: *Law of transformation for essential generalized jacobians.*

Let $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$ be a right-hand rectangular coordinate system obtained in euclidean three-space by applying a rigid motion to the right-hand system x_1, x_2, x_3—thus $\bar{x}_j=c_j+c_{jk}x_k$, where the constants c_j, c_{jk} are real and the matrix (c_{jk}) is normal and orthogonal. Given a continuous surface of the type of a circular disc; choose any representation for it, and denote the jacobians (assuming these exist) of its projections on the $\bar{x}_2\bar{x}_3$, $\bar{x}_3\bar{x}_1$, $\bar{x}_1\bar{x}_2$ planes respectively by $\bar{J}_1$, $\bar{J}_2$, $\bar{J}_3$, the jacobians of its projections on the x_2x_3, x_3x_1, x_1x_2 planes respectively by J_1, J_2, J_3. When these are ordinary jacobians it is well known that $\bar{J}_j=c_{jk}J_k$. The purpose of this note is to prove that the same formula holds almost everywhere for the essential generalized jacobians introduced by T. Rado and P. Reichelderfer (*A theory of absolutely continuous transformations in the plane*, Trans. Amer. Math. Soc. vol. 49 (1941) pp. 258–307). In making this proof the writer uses an unpublished result by T. Rado on convergence in area for surfaces (see abstract 54-11-485). (Received March 15, 1948.)

487t. P. V. Reichelderfer: *On the semi-continuity of double integrals.*

The purpose of this note is to extend the results of T. Rado, *On the semi-continuity of double integrals in parametric form*, Trans. Amer. Math. Soc. vol. 51 (1942) pp. 336–361, by using essential generalized jacobians instead of ordinary jacobians. In particular, it is shown that the value of the integral I of an admissible integrand is independent of the choice of an essentially absolutely continuous representation for an oriented continuous surface oS; thus the integral may be denoted by $I(oS)$. If oSo is an oriented continuous surface having an essentially absolutely continuous representation in a neighborhood of whose elements the integrand is non-negative and at which the Weierstrass E-function is also non-negative, then $I(oS)$ is lower semi-continuous at oSo in the class of all oriented continuous surfaces having essentially absolutely continuous representations. (Received April 29, 1948.)

488t. T. D. Reynolds: *Analytic solutions of integral equations having nonanalytic kernels.*

By real variable methods the author proves the following theorem. Let R denote the rectangle $a<x<b$, $c<y<d$. Suppose that $K(x, y, u, v)$ has the following properties in R: (1) $K(x, y, u, v)$ is continuous with its partial derivatives $\partial^{m+n+p+q}/\partial x^m \partial y^n \partial u^p \partial v^q K(x, y, u, v)$, $m, n, p, q=0, 1, 2, \cdots$, for (x, y), (u, v) in R, $(x, y)\neq(u, v)$; (2) there exist constants λ, M such that $|\partial^{m+n}/\partial x^m \partial y^n K(x, y, u+\sigma(x-u), v+\sigma(y-v))| \leqq Mm!n!\lambda^{m+n}/(1-\sigma)\ [(x-u)^2+(y-v)^2]^{(m+n+1)/2}$ for $0\leqq\sigma<1$, (x, y) (u, v) in R, $(x, y)\neq(u, v)$, $m, n=0, 1, 2, \cdots$. Let $V(x, y)$ be analytic in R. Let $Z(x, y)$ be continuous and bounded in R and satisfy in R the integral equation $Z(x, y) = V(x, y)+\iint_R K(x, y, u, v) Z(u, v)dudv$. Then $Z(x, y)$ is analytic in R. The above theorem, in conjunction with some of Seward's work (Amer. J. of Math. vol. 66 (1944) pp. 255–267), yields a real variable proof of Seward's theorem (ibid. p. 255) on harmonic continuation in space of a harmonic function which assumes analytic values on an analytic surface. (Received August 9, 1948.)

489. E. H. Rothe: *Critical points and gradient fields in Hilbert space.*

The paper is concerned with the theory of nondegenerated critical points of a scalar $i(x)$ in a Hilbert space H. Especially a relation is derived between the "Morse numbers" of such critical points and the mapping degree of the mapping of H into itself induced by the gradient of $i(x)$. The theory is applied to a nonlinear integral equation, and an existence and uniqueness theorem generalizing an earlier one by M. Golomb is proved. (Received July 26, 1948.)

490. I. J. Schoenberg: *Variation-diminishing Stieltjes integral operators of the convolution type.*

Let $L(t)$ be a real function of bounded variation in $-\infty<x<\infty$. We say that the transformation (1) $g(x)=\int_{-\infty}^{\infty} f(x-t)\, dL(t)$ is *variation-diminishing* if and only if the inequality $V(g)\leqq V(f)$ always holds for every continuous and bounded $f(x)$; here $V(f)$ and $V(g)$ represent the numbers of changes of sign of $f(x)$ and $g(x)$, respectively. Following an oral suggestion of A. Dvoretzky and I. I. Hirschman, the author's results, as announced in Proc. Nat. Acad. Sci. U. S. A. vol. 34 (1948) pp. 164–169, may be generalized as follows: The transformation (1) is variation-diminishing if and only if $L(t)$ has a Laplace-Stieltjes transform of the form (2) $\int_{-\infty}^{\infty} \exp(-st)dL(t)=C\cdot\exp(\gamma s^2+\delta s)\prod_{\nu=1}^{\infty}\exp(\delta_\nu s)/(1+\delta_\nu s)$, where all constants are real, $\gamma\geqq 0$, $\sum\delta_\nu^2$ converges, and where (2) is valid in a certain strip containing the imaginary s-axis. (Received July 29, 1948.)

491. W. T. Scott and R. H. Stark: *Infinite linear systems and continued fractions.*

The authors consider two systems of linear forms $L_p(X)\equiv -a_{p-1}X_{p-1}+b_pX_p-a_pX_{p+1}$, $a_{-p}=1$, $a_p\neq 0$, and $M_p(Y)\equiv -c_{p-1}Y_{p-1}+d_pY_p-c_pY_{p+1}$, $p=0, 1, 2, \cdots$, where a_p, b_p, c_p, d_p, X_p, Y_p are complex numbers. By means of a Green's formula similar to that used by Hellinger and Wall (Ann. of Math. vol. 44 (1943) pp. 103–127) for one system of linear forms, comparison theorems relating solutions of the systems $L_p(X)=0$ and $M_p(Y)=0$ are obtained. For example, if there exist $\epsilon_p>0$ for which $\sum[\epsilon_p^2+|b_p-d_p|^2\epsilon_p^{-2}+|a_p-c_p|^2\epsilon_{p+1}^{-2}+|a_{p-1}-c_{p-1}|^2\epsilon_{p-1}^{-2}]|X_p|^2$ converges for every solution X of $L_p(X)=0$, then $\sum\epsilon_p^2|Y_p|^2$ converges. This yields as special cases invariability theorems and other known results for J-fractions. A theory of continued fractions $-K_0^{\infty}(-a_{p-1}^2/b_p)$ is developed under conditions which insure that a related quadratic form is positive definite. When a_p and b_p are suitably chosen analytic functions of z for $I(z)>0$, Stieltjes integral representations for the equivalent functions of the continued fraction are obtained. (Received June 16, 1948.)

492. I. E. Segal: *Two-sided ideals in operator algebras.*

It is shown that a closed two-sided ideal in a C^*-algebra (= a uniformly closed self-adjoint algebra of operators on a Hilbert space) is the intersection of kernels of irreducible representations of the algebra. The quotient of the algebra modulo the ideal is isomorphic to a C^*-algebra whose state space is linearly and topologically equivalent to the collection of states of the original algebra which vanish on the ideal. The correspondence between ideals and closed convex sets of states which arises in this way has the property that there is an identity modulo the ideal (that is, the ideal is regular) if and only if the set is compact, and that the set corresponding to the intersection of two ideals is the closed convex sum of the corresponding sets. The intersection of a finite number of regular ideals in a C^*-algebra is regular. (Received July 20, 1948.)

493. I. M. Sheffer: *On the theory of sum-equations.* II.

This paper complements the work of an earlier one of the same title. The sum-equation system (1) $\sum_{j=0}^{\infty} a_{nj}x_{n+j}=c_n$ $(n=0, 1, 2, \cdots)$ is related to the problem of expanding an analytic function $f(t)$ in a series of form (2) $f(t)=\sum_0^{\infty} c_nt^nA_n(t)$ where (3) $A_n(t)=\sum_{j=0}^{\infty} a_{nj}t^j$. When system (1) is k-periodic (that is, $A_{nk+j}(t)=A_j(t)$), solutions for (1) and (2) can be expressed in "closed" form; and, under certain conditions, the nonperiodic case can be solved by a limiting process based on the periodic case. It is also shown to be possible to treat the nonperiodic case (under certain conditions) independently of the periodic case. (Received July 26, 1948.)

494. Andrew Sobczyk: *Generalized analytic functions and conformal manifolds.* Preliminary report.

Let E_n and E_m be differentiable manifolds, having real local coordinate systems, and satisfying an additional requirement as suggested below. Consider functions $y=f(x)$ on E_n to E_m, defined by $y_1=f^{(1)}(x_1, \cdots, x_n), \cdots, y_m=f^{(m)}(x_1, \cdots, x_n)$, in terms of the local coordinates, the $f^{(i)}$'s being assumed to be of class $C^{(1)}$. Generalizing a familiar property of regular functions of a complex variable, such a function will be called *orthomorphic* in case at each point of the x-domain in E_n the m gradient vectors are of equal length and mutually orthogonal. It is shown that the graph in $E_n \oplus E_m$ of an orthomorphic function is a special *conformal* manifold; that is, if local Euclidean coordinate systems are chosen in the natural way on the graph, the transformation between any two overlapping systems is of the form of a translation plus a constant times a linear orthogonal transformation. Examples of orthomorphic functions are studied. Numerous new properties of the functions, and analogues of classical theorems on analytic functions on Riemann surfaces, are obtained. (In case $n=m=3$, it is a known theorem of Liouville that the $C^{(2)}$ conformal group is generated by the motions and inversions. This drastically limits the extent of the class of orthomorphic functions.) (Received August 9, 1948.)

495*t*. D. B. Sumner: *The inversion of convolution transforms by means of on integral.*

The generalized Stieltjes transform $f(x)=\int_0^{\infty}\phi(t)dt/(x+t)^{\rho}$, $\rho>0$, has an inversion formula $-(2\pi i)^{-1}\int_C (z+t)^{\rho-1}f'(z)dz$, where C is the circle $|z|=t$ cut at the point $-t$, a generalization of the original result of Stieltjes for $\rho=1$. A similar type of inversion formula is obtained for the convolution transform $f(x)=\int_{-\infty}^{\infty}g(x-t)\phi(t)dt$, where it is assumed that the bilateral Laplace transform $\int_{-\infty}^{\infty}\exp(-su)g(u)du$ has simple poles at λ_n, $-\mu_n$, $0<\lambda_1<\lambda_2<\cdots$, $0<\mu_1<\mu_2<\cdots$, and $\lambda_n, \mu_n\to\infty$. The inversion formula utilizes the closure properties of the set of functions $\{\exp(i\omega\lambda_n), \exp(-i\omega\mu_n)\}$ over $(-\pi, \pi)$. It supplements recent discoveries by Widder and Hirschmann, who have treated the problem of inverting the convolution transform as the solution of a differential equation of infinite order with operator $e^{cD}D^p\prod_1^{\infty}(1-D/a_n)e^{D/a_n}$. (Received August 12, 1948.)

496. Otto Szasz: *Inequalities concerning ultraspherical polynomials and Bessel functions.*

Recently Turán and Szegö proved the following inequality for Legendre polynomials: $(P_n(x))^2-P_{n-1}(x)P_{n+1}(x)\geqq 0$, $-1\leqq x\leqq +1$; analogous inequalities were given by Szegö for ultraspherical and other orthogonal polynomials (cf. Szegö, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 401–405). In this paper sharper inequalities are given for Legendre polynomials and for a class of ultraspherical polynomials. The basic tool is the well known recurrence formula. The method is applicable to other sequences of functions, satisfying a recurrence formula. We prove in particular for Bessel functions the inequality: $(\mu+1)\{J_\mu^2(x)-J_{\mu-1}(x)J_{\mu+1}(x)\}\geqq J_\mu^2(x)$. (Received July 29, 1948.)

497. Otto Szasz and John Todd: *The convergence of Cauchy-Riemann sums to Cauchy-Riemann integrals.*

A set of conditions sufficient to ensure that $\lim h\sum_{\nu=1}^{\infty}f(\nu h)=\int_0^{\infty}f(x)dx$, as $h\to 0$, where the integral is taken in the Cauchy-Riemann sense, is derived. These are sufficient to cover such cases as these when $f(x)=\sin x/x$, $f(x)=J_0(x)$, where the integrals are not absolutely convergent. (Received July 24, 1948.)

498*t*. W. J. Thron: *Singularities of functions represented by continued fractions.* Preliminary report.

Let $0\leqq m\leqq d_n\leqq M\leqq\infty$. The function $f(z)$ defined by $1+d_0z+K(z/(1+d_nz))$ is easily seen to be holomorphic for all z not on the negative real axis. It is shown that $f(z)$ has at least two singular points (not poles) on the negative real axis. Denote these points by z_1 and z_2 $(z_1>z_2)$. Concerning the location of these points the following inequalities have been established: $-1/(2+M+2(1+M)^{1/2})\geqq z_1\geqq -1/(1+M)$; $-1/m\geqq z_2\geqq -1/(2+m-2(1+m)^{1/2})$. (Received July 24, 1948.)

499. C. J. Titus: *A topological characterization of a class of affine transformations.*

Let the symbols $\{x_\sigma\}_m$ and $\{y_\sigma\}_m$ denote real-valued sequences of period m, $m\geqq 2$; that is, $x_{\sigma+m}=x_\sigma$, $y_{\sigma+m}=y_\sigma$ $(\sigma=0, \pm 1, \pm 2, \cdots)$. Any pair of such sequences defines in the xy-plane the sequence of points (x_σ, y_σ) $(\sigma=1, 2, \cdots, m)$. If one connects these points by going from the σth to the $(\sigma+1)$st point $(\sigma=1, 2, \cdots, m)$, first by a horizontal line segment and then by a vertical line segment, a closed oriented polygon is formed. A closed oriented polygon is said to be of *non-negative circulation* (Loewner, Charles, *A topological characterization of a class of integral operators*. Ann. of Math. (1948)) if the order, with respect to the polygon, of every point not on the polygon is non-negative. Consider the transformation $y_\sigma=-\sum_{\rho=1}^{m}a_\rho x_{\sigma-\rho+1}$, where the a_p are arbitrary real numbers. The transformation maps a periodic sequence $\{x_\sigma\}_m$ into the periodic sequence $\{y_\sigma\}_m$. These two sequences, as previously described, define a sequence of points which in turn determine a closed oriented polygon. The principal theorem can now be stated. *A necessary and sufficient condition that the transformation "generate" only polygons of non-negative circulation for all* $\{x_\sigma\}_m$ *is that there exist non-negative* C_p, β_p *and* γ_p *such that* $a_{k+1}-a_k=\sum_{p=1}^{q} C_p(\beta_p)^{q-k+1}(\gamma_p)^{k-1}$ $(k=1, 2, \cdots, m-1)$, $q=[m/2]$. (Received July 21, 1948.)

500t. H. S. Wall: *Convergence of continued fractions in parabolic domains.*

It is shown that the determinate case holds for a positive definite continued fraction $K[-a_{p-1}/(b_p+z_p)]$, in which the $|b_p|$ are bounded, if, and only, if, the series $\sum |d_p|$ diverges, where $d_1=1$, $d_{p+1}=1/(d_p a_p^2)$, $p=1, 2, 3, \cdots$. This result is used to establish the following extension of the "parabola theorems." For each $p=1, 2, 3, \cdots$, let $|c_p|-R(c_p \operatorname{Exp} \theta_p) \leqq 2r \cos \phi_p \cos \phi_{p+1}(1-g_{p-1})g_p$, where $\theta_p=i(\phi_p+\phi_{p+1})$, $-(\pi/2)+c \leqq \phi_p \leqq (\pi/2)-c$, $0<c<\pi/2$, $0<r<1$, $0 \leqq g_{p-1} \leqq 1$, r and c being independent of p. The continued fraction $K(c_{p-1}/1)$, $c_0=1$, converges if, and only if, the series $\sum |k_p|$ diverges, where $k_1=1$, $k_{p+1}=1/(c_p k_p)$. (Received May 24, 1948.)

501. S. E. Warchawski: *On conformal mapping of variable regions.*

The paper is a contribution to the study of the degree of variation of the mapping function of a simply connected region under a deformation of the boundary curve. A typical result is the following: Suppose that C_i $(i=1, 2)$ denote two simple closed curves with continuously turning tangent, which satisfy the hypotheses: (i) For some $\epsilon>0$, C_1 is in the "ϵ-neighborhood" of C_2 (that is, the region obtained by describing a circle of radius ϵ about every point of C_1), and C_2 in that of C_1; (ii) If $\alpha_i(P)$ denotes the tangent angle to C_i at P, then $|\alpha_1(P_1)-\alpha_2(P_2)|<m\epsilon$, provided $\overline{P_1P_2}<\epsilon$ (m is a constant); (iii) The ratio of any arc s of C_i to the corresponding chord c satisfies the inequality $s/c \leqq a$ (=constant); (iv) C_i contain the circle $|z|<r$ and are contained in $|z|<R$. (v) The curvature of one of the curves, say $K_1(s)$ of C_1, exists for almost all values of the arc length s, and $\int_{C_1}|K_1(s)|^p ds$ exists for some $p>1$. Then, if $f_i(z)$ maps $|z|<1$ onto C_i, $f_i(0)=0$, $f_i'(0)>0$, one has $\int_0^{2\pi}|f_1'(re^{i\theta})-f_2'(re^{i\theta})|^p d\theta \leqq M^p \epsilon^p$ for $0 \leqq r<1$, where M depends on m, a, r, R and p *only*. In particular, this implies: $|f_1(z)-f_2(z)| \leqq (M/2)\epsilon$ for $|z| \leqq 1$. (Received July 27, 1948.)

502t. Antoni Zygmund: *On the existence of boundary values for regular functions of several complex variables.* I.

Let $f(z_1, z_2, \cdots, z_k)$ be a function regular for $|z_1|<1$, $|z_2|<1, \cdots, |z_k|<1$. Suppose that the integral $J=\int_0^{2\pi} \cdots \int_0^{2\pi} \log^+|f|(\log^+\log^+|f|)^{k-1}dx_1 \cdots dx_k$ is bounded for $r_1, \cdots, r_k \rightarrow 1$, where f stands for $f(r_1e^{ix_1}, \cdots, r_ke^{ix_k})$. Then, for almost every point $(\eta_1, \eta_2, \cdots, \eta_k)$ in the cell $0 \leqq \eta_1 \leqq 2\pi, \cdots, 0 \leqq \eta_k \leqq 2\pi$, the function $f(z_1, \cdots, z_k)$ tends to a finite limit $f(e^{i\eta_1}, \cdots, e^{i\eta_k})$ as the points $z_1, \cdots, z_k$ approach $e^{i\eta_1}, \cdots, i^{i\eta_k}$, each along a nontangential path. If in the integral J we drop the iterated logarithm, the boundedness of the resulting integral suffices for the existence, for almost every $(\eta_1, \cdots, \eta_k)$, of the *iterated* limit of $f(z_1, \cdots, z_k)$ as each of the variables $z_1, \cdots, z_k$ tends successively and nontangentially to the corresponding point $e^{i\eta_j}$. Let $\alpha>0$, and let $F(x_1, \cdots, x_k)=\sup_{r_1, \ldots, r_k}|f(r_1e^{ix_1}, \cdots, r_ke^{ix_k})|$. If the integral $J_\alpha=\int_0^{2\pi} \cdots \int_0^{2\pi}|f|^\alpha dx_1 \cdots dx_k$ is bounded for all $r_j<1$ by a finite number M^α, then the function F is of the class L^α, and the integral of F^α over $0 \leqq x_1 \leqq 2\pi, \cdots,$

$0 \leqq x_k \leqq 2\pi$ does not exceed $C_\alpha M^\alpha$, where C_α depends on α only. In particular, if J_α is bounded, then $\lim \int_0^{2\pi} \cdots \int_0^{2\pi} |f(r_1 e^{ix_1}, \cdots, r_k e^{ix_k}) - f(e^{ix_1}, \cdots, e^{ix_k})|^\alpha dx_1 \cdots dx_k = 0$, as $r_1, \cdots, r_k \rightarrow 1$. (Received June 25, 1948.)

Applied Mathematics

503t. R. C. F. Bartels and O. Laporte: *Supersonic flow past a delta wing at angles of attack and yaw.*

The paper deals with certain solutions of the linearized potential equation for a supersonic flow which define conical flow fields. The method outlined in an earlier treatment is employed in the determination of the flow around a delta wing at angles of attack and yaw. The velocity field is expressed in terms of a function $F(\zeta)$ of the complex variable ζ which is analytic in a doubly connected region of the ζ-plane corresponding to the disturbed region of flow between the wing and its Mach cone. In this representation the components of velocity form the real parts respectively of the triplet of analytic functions defined by the integrals $\beta\int(1+\zeta^2)F(\zeta)d\zeta$, $i\beta\int(1-\zeta^2)F(\zeta)d\zeta$, $-2\int\zeta F(\zeta)d\zeta$, which resemble the formulas of Weierstrass for the solution of the minimal surface problem; here β is the cotangent of the Mach angle. The specification of $F(\zeta)$ is formulated as a boundary value problem for the doubly connected region of the ζ-plane in which the boundary conditions require that the velocity vanish along the Mach cone and be tangential to the surface of the wing. It is shown that the solution of this problem is not unique. There is, however, one and only one solution for which the normal force coefficient of the wing is finite. This solution is given in terms of elliptic functions. (Received July 27, 1948.)

504. Lipman Bers: *The Dirichlet problem for a partial differential equation of mixed type.* Preliminary report.

Let D be a plane domain situated in the upper half-plane whose boundary B lies partly on the line $y=0$. Let $K(y)$ be a sufficiently well-behaved function such that $K(0)=0$, $K(y)>0$. It is shown (by a method imitating Poincaré's "balayage") that in D the first boundary value problem for the equation $K(y)\psi_{xx}+\psi_{yy}=0$ always has a (unique) solution. Until now only the case $K(y)=y$ has been considered, by Tricomi, Hellerstedt and Frankl (see Frankl, Bull. Acad. Sci. USSR. vol. 10 (1946) pp. 166–182 and the literature quoted therein). These authors used rather complicated methods and imposed severe restrictions on the domain D. The author requires merely that for every sufficiently small $\epsilon>0$ the intersection of D with the half-plane $y>\epsilon$ be a connected Dirichlet domain. (Received July 27, 1948.)

505. A. S. Cahn: *The warehouse problem.*

John von Neumann, George B. Dantzig, T. C. Koopmans, and others are currently developing methods for solving the programming or management type problem in a rapidly changing "economy." This involves particularly the maximization of a linear form in variables subject to linear inequalities. It is shown that their methods are applicable to the problem of the optimum use of a storage warehouse for commodities subject to periodic price fluctuation. This example is important because the assumptions made in linearizing the economy are not seriously different from the realities of the situation. Also, the computation involved lies within the scope of present day computing machinery, so that the method could be immediately useful. The effects of various side conditions upon the size of the computation are discussed. (Received July 30, 1948.)

506t. G. B. Dantzig: *Programming in a linear structure.*

W. Leontief, Schlienger, Wald, von Neumann, and T. C. Koopmans have studied economic models of the type considered here. This paper differs essentially from those of the above authors in that it is concerned with the basic problem of programming in a rapidly changing "economy." The basic assumptions of the model lead to a fundamental set of linear equations expressing the conditions which must be satisfied by the various levels of activity, X_i, in the dynamic system. These variables are subject to the further restriction $X_i \geqq 0$. The determination of the "best" choice of X_i is made to depend on the maximization (or minimization) of linear form in X_i (a typical example would be the minimization of the total budget over several time periods). This problem is equivalent to the maximization of a linear form whose variables are subject to linear inequalities. It is also closely related to the problem of determining a Min-Max of a bilinear form. In is proposed that computational techniques such as those developed by J. von Neumann and by the author be used in connection with large scale digital computers to implement the solution of programming problems. (Received July 28, 1948.)

507. A. H. Diamond: *Reproducing kernels for certain classes of biharmonic functions.*

The author considers the reproducing kernels for real biharmonic functions such that $\int\int_D(\Delta u)^2dxdy$ and $u=0$ on the boundary of D. It is shown how to construct the kernel function for certain simply connected domains by orthonormalizing the system $\mathrm{Re}\{\bar{z}z^n\}$ and $\mathrm{Im}\{\bar{z}z^n\}$. The relationship between the kernel function and the Green's and Neumann's function is established and this is used to show how the kernel function can be used to solve the boundary value problem $\Delta\Delta u=0$ in D, $u=0$ and $\partial u/\partial\eta=f(s)$ on the boundary of D. (Received July 27, 1948.)

508. Wilfred Kaplan: *Dynamical systems with indeterminancy.*

In a previous paper (Bull. Amer. Math. Soc. Abstract 52-5-176) it was proposed that the differential equations $dx_i/dt=f_i(x_1 \cdots x_n)$ of a physical system be modified to allow for errors in the f_i. One is thus led to consider as ϵ-solutions curves $x_i=x_i(t)$ whose tangent vectors are close to f_i in a certain sense. The order symbol $x'<x''$ is used to signify that x'' can be reached from x' along such an ϵ-solution with increasing time; the notations $\alpha(x)=E\{x' \mid x'<x\}$ and $\alpha(X)=\bigcup\alpha(x)$, $x\in X$ (X any subset of the phase space M) are used. It is shown that there exist certain disjoint open sets $X_1 \cdots X_N$ (called "stable stationary states") in M with the properties: (1) $\alpha(X_j)$ is open; (2) $X_j \subset \alpha(X_j)$; (3) $\bigcup\alpha(X_j)=M$; (4) $X_j\cdot\alpha(X_k)=0$ for $j\neq k$. The nerve of the covering of M (assumed compact) by the $\alpha(X_j)$ is a complex K, each simplex of which corresponds to a class of points of M with equally uncertain future. This complex is thus a model for the qualitative structure of the system. In general, a sufficiently small change in the allowed error ϵ has no effect on K. It is shown that K can be found by an analysis on a finite subset of M. (Received July 21, 1948.)

509. M. Z. Krzywoblocki: *On the hodograph equation in adiabatic inviscid rotational flow.*

By a suitable choice of a function analogous to the velocity potential in a potential flow, the author succeeded in transferring the stream function equation into the hodograph plane. The final equation differs considerably from the known "Chaplygin equation" in the potential flow. (Received June 3, 1948.)

510*t*. J. P. LaSalle: *Relaxation oscillations.*

A case of relaxation oscillations which is simpler though not included in the general case of Levinson and Smith is studied. Sufficient conditions for the existence of stable periodic solutions (limit cycles) are found by the construction of regions each of which contains a single periodic solution. The regions approximate the location of the periodic solutions in the phase plane and provide bounds on the periods and amplitudes. The asymptotic case of relaxation oscillations is investigated by considering the behavior of the regions as a parameter approaches zero. (Received July 10, 1948.)

511*t*. C. C. Lin: *On the flow of compressible fluid through a group of regularly arranged airfoils.*

The complex potential function of the incompressible flow due to two complex sources in the ζ-plane at $\zeta=a_1$ and $\zeta=a_2$ outside the circular boundary $|\zeta|=1$ is $F(\zeta)=A_1 \log(\zeta-a_1)+\bar{A}_1 \log(\zeta-1/\bar{a}_1)+B_1 \log(\zeta-a_2)+\bar{B}_1 \log(\zeta-1/\bar{a}_2)$, where A_1 and B_1 are complex constants, and $\mathrm{Re}(A_1)=-\mathrm{Re}(B_1)$. It is shown that the mapping $dz=g(\zeta)(\zeta-a_1)^{-1+1/n}(\zeta-a_2)^{-1-1/n}d\zeta-\bar{f}, =-4^{-1}F'^2(\zeta)\{g(\zeta)\}^{-1}(\zeta-a_1)^{1-1/n}(\zeta-a_2)^{1+1/n}d\zeta$ gives a compressible flow (with von Kármán-Tsien approximation) past a circular series of n identical blades in the z-plane, with velocity potential ϕ and stream function ψ given by $\phi+i\psi=F(\zeta)$, provided that $g(\zeta)$ satisfies the following general requirements: (a) It is regular in the region R, defined by $|\zeta|\geqq 1$. (b) It does not vanish in R except possibly at one point on the circle where $F'(\zeta)=0$. (c) Along the circle $|\zeta|=1$, $\oint dz=0$. (d) Except in the neighborhood of $\zeta=a_1$, $|F'(\zeta)\{g(\zeta)\}^{-1}(\zeta-a_1)^{1-1/n}\cdot(\zeta-a_2)^{1+1/n}|<2$ in R. In the limiting case $n\to\infty$, condition (d) is to be satisfied throughout R, and the above scheme defines the compressible flow past a straight series of infinite number of blades. (Received July 15, 1948.)

512. M. H. Martin: *A new approach to the theory of two-dimensional flows.*

When the pressure p and stream function ψ are chosen for independent variables in the two-dimensional steady flow of an inviscid fluid subject to no external forces, the stream lines and velocity vector are presented parametrically by $x=-\eta_p$, $y=\xi_p$, $u=\xi_\psi$, $v=\eta_\psi$, where $\xi=\xi(\psi, p)$, $\eta=\eta(\psi, p)$ satisfy the pair of partial differential equations $\xi_\psi^2+\eta_\psi^2=2T$, $\xi_\psi\xi_{pp}+\eta_\psi\eta_{pp}=0$, and $T=T(\psi, p)$, the specific kinetic energy, is an assigned function of ψ, p. Eliminating one of ξ, η from the pair, a Monge-Ampère partial differential equation results for the other and the Cauchy-Kowalewski existence theorem leads to a local existence and uniqueness theorem for flows possessing an assigned stream line and pressure distribution along it, together with a method for calculation of such flows. For irrotational flows $T_\psi=0$ and the Monge-Ampère equation has an intermediate integral which yields a parametric representation for the well known Prandtl-Meyer flows. The theory is not restricted to irrotational flows. Applied to rotational flows, the Munk-Prim substitution principle is obtained and flows with one-dimensional hodographs are investigated. (Received July 26, 1948.)

513*t*. Leo Moser: *Linked rods and continued fractions.*

A set of n uniform rods AB, BC, $\cdots$, MN, each of mass m and length $2a$, are connected by smooth joints at B, C, $\cdots$, M, and lie in one straight line on a smooth horizontal table. A horizontal blow P is struck at N in a direction perpendicular to

BC. Let v_k and ω_k be the relative initial linear and angular velocities of the kth rod. Let p_k/q_k be the kth convergent to the continued fraction expansion of $3^{1/2}$. Recurrence formulae are found for the v's and ω's and by comparing these with the recurrence formulae for the p's and q's it is shown that $|v_k| = p_{2k-1}$, $|\omega_{k/3}| = q_{2k-1}$. (Received July 16, 1948.)

514. R. R. Reynolds: *Solution of boundary value problems for Laplace's equation in a multiply-connected domain.*

Methods developed in papers of Bergman and Schiffer (Duke Math. J. vol. 14 (1947) pp. 609–638) and Bergman (Quarterly of Applied Mathematics vol. 5 (1947) pp. 69–81) for obtaining the solution $u(x, y)$ of the Dirichlet problem for an m-ply connected domain B_m are applied to the case where B_m is the unit circle Γ_0 with $m-1$ circles Γ_r punched out. General formulas are given for determining a system of analytic functions $f_\nu(z) = \sum_{\mu=1}^{\nu} c_{\mu\nu} v_\mu(z)$ satisfying $\iint_B f_\mu(z)\overline{f_\nu(z)}dxdy = \delta_{\mu\nu}$ from a complete set $v_\mu(z)$. In particular, the $c_{\mu\nu}$ are calculated in a numerical example for B_3 and used to solve the boundary value problem in which $u(x, y)$ has given constant values u_r on $\Gamma_r (r=0, 1, 2)$. Finally, in connection with conformal mapping theorems, the coefficients $a_{pq}^{(N)}$ $(N=1, 2, \cdots, 15)$ of the approximating kernel $K_N(z, w) = \sum_{\nu=1}^{N} \sum_{q=1}^{N} a_{pq}^{(N)} v_p(z)\overline{v_q(w)}$ are computed for B_3. (Received September 3, 1948.)

515t. H. E. Salzer: *Coefficients for polar complex interpolation. I. Lagrangian coefficients.*

In a previous article by the author, namely *Formulas for direct and inverse interpolation of a complex function tabulated along equidistant circular arcs*, Journal of Mathematics and Physics vol. 25 (1945) pp.1 41–143, formulas for Lagrangian interpolation coefficients for complex analytic functions tabulated around a circular arc at points differing in phase by the constant amount θ are given as rather lengthy trigonometric functions of the angle θ. The present work supplements the preceding one by giving the numerical values, to eight decimals, of those functions of θ, for six cases quite likely to arise in practice, namely $\theta = 1°, 5°, 10°, 15°, 20°$ and $30°$, for the 3-, 4-, and 5-point Lagrangian interpolation formulas. (Received June 30, 1948.)

516t. H. E. Salzer: *Coefficients for polar complex interpolation. II. Auxiliary coefficients for Lagrangian interpolation.*

In the author's previous article, namely, *Alternative formulas for direct interpolation of a complex function tabulated along equidistant circular arcs*, Journal of Mathematics and Physics vol. 27 (1947) pp. 56–61, formulas for certain auxiliary coefficients are given for direct Lagrangian interpolation for complex analytic functions tabulated around a circular arc at points differing in phase by the constant amount θ. Those formulas are (1) long trigonometric expressions in terms of θ, and (2) when evaluated for smaller values of θ, very many significant figures are lost. For those two reasons, it is a great help to have the formulas for the auxiliary coefficients evaluated for the angular intervals θ which are most likely to arise in practice. The present work supplements the above mentioned article by giving those complex coefficients for $\theta = 1°, 5°, 10°, 15°, 20°$ and $30°$, with an accuracy in the modulus of nine significant figures (eight figures guaranteed), for the 3-point through the 9-point Lagrangian interpolation formulas. (Received July 30, 1948.)

517t. E. A. Trabant: *Vibrations of cantilever beam as geodesics.*

Using the methods of A. D. Michael (*Studies in geodesics in vibrations of elastic beams*, Proc. Nat. Acad. Sci. U.S.A. vol. 31 (1945) pp. 38–43) the transverse vibrations of a cantilever beam which satisfy its fourth order partial differential equation with the usual boundary conditions are investigated. By a suitably defined arc-parameter s, the generalized Euler-Lagrange equation is obtained. Starting with the Euler-Lagrange equation the original differential equation is obtained. It is shown that the vibrations of the cantilever beam with total energy C can be represented as geodesics in a "Riemannian" space with element of arc-length given in terms of two arbitrary continuous functions which satisfy the boundary conditions. (Received July 26, 1948.)

518t. C. A. Truesdell: *On Cauchy's vorticity formula.*

The following kinematic formula for the vorticity w of any continuous motion of any continuum is derived: $w/\rho = [w_0/\rho_0 + (1/\rho_0)\int_{t_0}^{t} \text{curl}\ (\text{grad}\ r \cdot a)dt] \cdot \text{grad}\ r$. Here ρ is the density, t_0 is the time at which $w = w_0$ and $\rho = \rho_0$, r is the Eulerian position vector, a is the acceleration, and all differential operations are taken with respect to the Lagrangian variables. In a circulation-preserving motion the integral, which represents the effect of diffusion of vorticity, vanishes, and there remains only the convective expression derived by Cauchy. The general formula is applied to a compressible viscous fluid. (Received August 3, 1948.)

Geometry

519. C. C. Hsiung: *Invariants of intersection of certain pairs of curves in n-dimensional space.*

The purpose of this paper is to derive and characterize metrically, as well as projectively, some projective invariants associated with an ordinary point O of intersection of two curves C, C' in n-dimensional space S_n $(n \geqq 3)$ for each of the following two cases: (a) C, C' have at O distinct osculating linear spaces S_k, S_k' $(k = 1, \cdots, n-1)$ of k dimensions. (b) C, C' have at O distinct tangents but $S_2 \equiv S_2'$, $\cdots$, $S_r \equiv S_r'$, where r is any fixed integer satisfying $2 \leqq r \leqq n-1$. (Received July 27, 1948.)

520t. H. T. Muhly and Oscar Zariski: *Hilbert's characteristic function and the arithmetic genus of an algebraic variety.*

The properties of the generalized characteristic function of Hilbert associated with a doubly homogeneous polynomial ideal are studied in the case where the homogeneous ideal is determined by a pair (U, V) of projective models of a field Σ of algebraic functions. This study yields the following results. (1) The virtual arithmetic genus $p_a(W)$ of a sectionally normal model W of Σ is a relative birational invariant, where by a sectionally normal model is meant a normal model which is such that its sections by almost all linear subspaces of its ambient space are also normal. (2) If Σ is of dimension two or three over the ground field k, then the virtual arithmetic genu- of a nonsingular model of Σ is an absolute birational invariant. (3) If Σ is of dimens sion two over k and if U and V are normal models of Σ such that $U < V$, then $p_a(U) \geqq p_a(V)$. The theorem of Riemann-Roch for a large class of linear systems on an algebraic surface follows as a corollary to these results. (Received July 6, 1948.)

521*t*. Chenkuo Pa: *Some theorems on rectilinear congruences and transformations of surfaces.*

Let S be a nondevelopable analytic surface in ordinary space; let L be a rectilinear congruence with each of its generators passing through a corresponding point of S but not tangent to S, and let S' be a surface generated by the harmonic conjugate point of a point of S with respect to the foci of L on the corresponding generator through the point of S. The author proves the following results: 1. If the asymptotic net of S corresponds both to that of the focal sheets of L, then S is an R-surface and L is an R-congruence with its developables cutting S in an R-net. Conversely, for a given R-surface, there exist ∞^4 R-congruences having the above properties. 2. If the developables of L conjugate to S, and if the asymptotic nets of S and S' are in correspondence, then both S and S' are Jonas surfaces and the developables of L cut S and S' both in Jonas nets. Conversely, for a given Jonas surface S, there exist ∞^8 rectilinear conjugate congruences to S which satisfy the above conditions. (Received July 8, 1948.)

Logic and Foundations

522*t*. A. R. Schweitzer: *An outline of the history and the philosophy of the concept of orientation.* II. Preliminary report.

This paper continues a paper reported in Bull. Amer. Math. Soc. Abstract 44-7-304. From a psychological point of view this paper is concerned with "orientative thinking" classified as follows: 1. Speculative thinking, as represented, for example, by questions in the Book of Job. 2. Philosophical thinking, as represented, for instance, by the "elemental thinking" outlined by Albert Schweitzer (*Out of my life and thought*, New York, 1933, p. 260). 3. Postulational thinking, as represented by the hypotheses of mathematical physics and the assumptions of geometry and mathematical analysis. On another basis of classification the subject matter of this paper refers to man's position in: A. the universe (religion, philosophy); B. the world (sociology, social psychology, ethics); C. nature (science, mathematics). The term "orientation" is conceived so as to include organization (arrangement, interrelationship) of parts in a whole. (Received July 27, 1948.)

523*t*. A. R. Schweitzer: *On a relation of mathematics to philosophy.*

The author aims to interpret Kant's *Kritik der reinen Vernunft* (1781) as a means of gradual transition from mathematics to metaphysics. The principal references to Kant's treatises (apart from the above) are: (1) *Von dem ersten Grunde des Unterschieds der Gegenden im Raum* (1768), (2) *Prolegomena zu einer jeden künftigen Metaphysik die als Wissenschaft wird auftreten können* (1783), (3) *Was heisst: Sich im Denken orientiren?* (1786). The transition is effected by classifying mathematics into: I. a science of extension: space (geometry) and time (arithmetic; in particular modular arithmetic), II. a science of discovery (heuristic mathematics) and abstraction from nature (applied mathematics; mathematical physics; analysis), III. a science of a priori principles; that is, axiomatic or postulational mathematics. These three types of mathematics are respectively associated with Kant's three universes of discourse, namely: I. sensibility, II. understanding, III. reason. The author discusses some of Kant's views concerning mathematical conceptions which seem unnecessarily restricted in the light of subsequent developments. In this connection reference is made to the monograph of Wilhelm Reinecke, *Die Grundlagen der Geometrie nach Kant*, Kantstudien vol. 8 (1903) pp. 345–395. (Received July 27, 1948.)

Statistics and Probability

524. Herman Chernoff: *Asymptotic studentization in testing of hypotheses.*

If H is a hypothesis for which $t \leqq c_1(\theta)$ would be a good test if the value of the nuisance parameter θ were known and $\hat{\theta}$ is an estimate of θ, then the following method of asymptotic studentization (obtaining critical regions of almost constant size) was suggested by Wald. Consider $t \leqq \phi(\hat{\theta})$ where $\phi(\hat{\theta}) = c_1(\hat{\theta}) + \cdots + c_s(\hat{\theta})$ and $\Pr\{t \leqq c_1(\theta)\} = \alpha$, $\Pr\{t - c_1(\hat{\theta}) \leqq c_2(\theta)\} = \alpha, \cdots$, $\Pr\{t - c_1(\hat{\theta}) - \cdots - c_r(\hat{\theta}) \leqq c_{r+1}(\theta)\} = \alpha$. It is shown that under reasonable conditions this test and various modifications, designed for those cases where the $c_r(\theta)$ are difficult to obtain exactly, have the asymptotic property that $\Pr\{t \leqq \phi(\hat{\theta}) = \alpha + O(N^{-s/2})$ where N is the size of the sample involved or an analogous variable. This property can be extended to the case where θ is a k-dimensional variable. (Received May 24, 1948.)

525. K. L. Chung: *An estimate concerning the Kolmogoroff limit distribution.*

This paper sharpens Kolmogoroff's result (1933) on the maximum deviation of the empirical distribution based on n samples from the theoretical distribution $F(x)$, assumed to be continuous. Let $nF_n(x)$ be the number of sample values not exceeding x and $d_n = \sup_x |n(F_n(x) - F(x)|$; Kolmogoroff proved that for a fixed λ, $\Pr(d_n \leqq \lambda n^{1/2})$ tends to the limit distribution $\Phi(\lambda) = \sum_{j=1}^{\infty} (-1)^j \exp(-2j^2\lambda^2)$ uniformly in λ as $n \to \infty$. It is shown in this paper that the result holds even if $\lambda = \lambda(n)$ varies with n but lies within the range $(A_0 \lg n)^{-1}$ to $(A_0 \lg n)$ where A_0 is any positive constant; moreover the difference $\Pr(d_n \leqq \lambda(n)n^{1/2}) = \Phi(\lambda(n))$ is of the order of magnitude of nearly $n^{-1/10}$. This estimate is obtained as consequence of a general theorem on "lattice distributions," after the problem is reduced (by Kolmogoroff) to one about the addition of independent random variables following a certain lattice distribution. From the asymptotic distribution follows a new law of the iterated logarithm, that is, $\Pr(\lim \sup d_n 2^{1/2}(n \lg n) = 1) = 1$, and also more refined statements. (Received August 30, 1948.)

526*t*. William Feller: *Fluctuation theory of recurrent events.*

Consider a sequence of independent or dependent trials but suppose that each has a discrete sample space. This paper studies recurrent patterns $\mathcal{E}$ which can be roughly characterized by the property that after every occurrence of $\mathcal{E}$ the process starts from scratch, the conditional probabilities coinciding with the original absolute probabilities. Typical examples are success runs, returns to equilibrium, zeros of sums of independent variables, passages through a state in a Markov chain. New methods are developed unifying and simplifying previous theories and applying to larger classes of recurrent events. It is shown in an elementary way that the probability that $\mathcal{E}$ occurs at the nth trial either has a limit or is asymptotically periodic. This theorem has many consequences. For example, the ergodic properties of discrete Markov chains follow in a few lines, and the difference between finite and infinite chains disappears. Several theorems of the renewal type are proved. Weak and strong limit theorems for the number N_n of occurrences of $\mathcal{E}$ in n trials are derived shedding new light on stable distributions. (Received July 29, 1948.)

527*t*. William Feller: *Infinitely divisible distributions.*

A simple derivation of P. Levy's formula is given starting from the following definition: a distribution function $F(x)$ is infinitely divisible if for every n it is possible to find finitely many distributions $F_{k,n}(x)$ such that $F(x)=F_{1,n}(x)*\cdots *F_{k_n,n}(x)$ and that $F_{k,n}(x)$ tends to the unitary distribution uniformly in n. This definition is more general than the one used by P. Levy and Khintchine. The equivalence of the two definitions was proved by Khintchine by deep methods. The new approach renders the equivalence obvious. Furthermore, a new characterization of infinitely divisible distributions is given; it is equivalent to Gnedenko's characterization but requires no special analytical tools. (Received July 29, 1948.)

528. E. L. Lehmann and Henry Scheffé: *Completeness, similar regions, and unbiased estimation.* Preliminary report.

A family $\mathfrak{M}$ of measures M on a space X of points x is defined to be *complete* if $\int_X f(x)dM=0$ for every M in $\mathfrak{M}$ implies $f(x)=0$ except on a set A for which $M(A)=0$ for every M in $\mathfrak{M}$. For a given family of measures the question of completeness may be regarded as the question of unicity of a related functional transform. Classical unicity results are applicable to many families of probability distributions that have been studied by statisticians. The notion of completeness throws light on the problem of similar regions and the problem of unbiased estimation. The concept of a *maximal* sufficient statistic—roughly, a sufficient statistic that is a function of all other sufficient statistics—is developed. A constructive method of finding such is given, which seems to apply to all examples ordinarily considered in statistical theory. A relation between completeness and maximality is found. (Received July 23, 1948.)

529. Herman Rubin (National Research Fellow): *Some results on the asymptotic distribution of maximum- and quasi-maximum-likelihood estimates.*

The author investigates the asymptotic normality of maximum- and quasi-maximum-likelihood estimates of parameters of systems of linear stochastic difference equations. The principal tool is the extension of the central limit theorem to dependent variables previously obtained by the author (Bull. Amer. Math. Soc. Abstract 54-7-280). The results obtained are analogous to those in the case in which no diferences are present. Some extensions are also made to systems of stochastic difference equations linear in the coefficients but not necessarily in the variables. If the complete system of stochastic difference equations is linear in the jointly dependent variables, asymptotic efficiency is demonstrated for maximum-likelihood estimates. (Received July 26, 1948.)

530. J. E. Walsh: *Some nonparametric tests of whether the largest observations of a set are too large.* Preliminary report.

Let $x(1),\cdots,x(n)$ represent the values of n observations arranged in increasing order of magnitude. By hypothesis these observations have the properties: (1) They are independent and form continuous symmetrical populations; (2) For large n the variances of the tail order statistics are either very large or very small compared with the variances of the central order statistics; (3) For large n the tail order statistics are approximately independent of the central order statistics; (4) Each observation is from a population whose median is either θ or ϕ, where $x(n-r+1),\cdots,x(n)$

are from populations with median θ while the central and smaller order statistics are from populations with median ϕ. The test is: *Accept* $\phi<\theta$ *if* min $[x(n-i_k)+x(j_k); 1\leqq k\leqq s\leqq r]>2x(t_\alpha)$, where $i_u<i_{u+1}$, $j_v<j_{v+1}$, $i_s=r-1$, and t_α is defined by $\Pr[x(t_\alpha)<\phi|\theta=\phi]=\alpha$. Here $\alpha=\Pr\{\min[x(n-i_k)+x(j_k); 1\leqq k\leqq s\leqq r]>2\phi|\theta=\phi\}$. For large n the significance level of the test is approximately α while the significance level does not exceed 2α for any value of n. Suitable values of α can be obtained for $r\geqq 4$. As $\theta-\phi\rightarrow-\infty$ the power function tends to zero, while the power function tends to unity as $\theta-\phi\rightarrow\infty$. For $\theta-\phi<0$ the power function is monotonically increasing. (Received July 29, 1948.)

Topology

531. Richard Arens: *Approximation in, and representation of, linear algebras.*

A BQ^*-algebra is a Banach algebra (real scalars) with a linear involutory anti-isomorphism (*) in which $\|f\|^2\leqq\|ff^*+gg^*\|$ for any f and g, and ff^* lies in the center for all f. The quaternions Q form the simplest noncommutative example. The main representation theorem for any BQ^*-algebra A is this: there is a compact Hausdorff space X on which the orthogonal group G acts leaving a point x_0 of X fixed such that A is equivalent to the ring of all continuous functions on X with values in Q for which $f(x_0)=0$ and $f(Tx)=T_Q(f(x))$, T_Q being the automorphism of Q corresponding to T of G. Indispensable for the proof is this generalization of the Stone-Weierstrass theorem: Let X be a compact Hausdorff space, and let C be the ring of continuous $n\times n$ complex-entried matrix valued functions on X. Let A be a closed linear subset of C such that if A contains f then it contains f^* and the trace of $f+f^*$. Then a necessary and sufficient condition that A include a subset B of C is that for each x and y of X and each b of B there is an a in A such that $a(x)=b(x)$, $a(y)=b(y)$. (Received July 7, 1948.)

532t. R. R. Bernard: *Probability in dynamical transformation groups.* Preliminary report.

Let T be a multiplicative abelian topological group which is separable, metric, locally compact and has a compact generating system. Let μ be the Haar measure on T. Let X be a metric space and let T act as a transformation group on X. Denote the image of $x\in X$ under $t\in T$ by xt. Let A be a subset of X, V a subset of T and let $x\in X$. Denote the set of all $t\in V$ such that $xt\in A$ by $T(x, A, V)$. Let s, t^* be arbitrary elements of T. Let W be any open generator of T which contains the identity and such that $K=Ws$ is compact. If $\mu T(x, A, K^nt^*)$ exists and if the limit as $n\rightarrow\infty$ of $\mu T(x, A, K^nt^*)/\mu(K^nt^*)$ exists and is independent of the choice of K and t^* this limit is termed the *probability that xt is in A*. Denote it by $P(x, A)$. It is shown that if T is connected the limit is necessarily independent of t^*. If T is the additive group of integers or reals this definition is equivalent to the classical definition. Every point of a minimal center of attraction, $A(x)$, of a point $x\in X$ is regionally recurrent with respect to $A(x)$. If X is compact, the minimal center of attraction, A, of X is a closed invariant set and if $x\in X$ and N is any open set containing A then $P(x, A)$ exists and equals one. The set of central orbits contains A. (For definitions cf. Gottschalk and Hedlund, Bull. Amer. Math. Soc. Abstract 54-3-169). (Received June 7, 1948.)

533t. Felix Bernstein: *The four color problem in a point lattice.* III.

In the second paper of this series it has been shown that configurations requiring five colors exist. The proof however necessitates the disjunction between a great number of cases. Here a general topological method is presented which permits complete classification into four color and five color configurations for a well defined class of configurations. The essential tool of this method is the concept of a "polygraph of order n" as an ordinary graph to whose elements (vertices and sides) numbers from a given finite set $S(0, 1, 2, \cdots, n)$ are affixed. The sequences of numbers which can be represented by a continuous way in the polygraph of the class correspond to the configurations of the class which can be colored with four colors. Thus necessary and sufficient conditions are derived. While the classical four color problem is confined to a finite number of two-dimensional regions, the problems of coloring solved here embrace as well the case of the finite as that of the infinite configurations of a certain class, and can also be generalized to the higher-dimensional space. In outlook as well as in methods they are more closely akin to the interests of the field of general topology. (Received July 1, 1948.)

534. R. H. Bing: *Partitioning a set.*

A set M can be partitioned if for each positive number ϵ there exists a finite collection of mutually exclusive open subsets of M such that the sum of the elements of this collection is dense in M and each element of this collection is of diameter less than ϵ and has property S. It is shown that any plane set with property S can be partitioned. Also, any compact locally connected continuum can be partitioned if it is not locally separated by any arc. As an application of the notion of partitioning, it is shown that a compact locally connected continuum can be assigned a convex metric if it can be partitioned. (Received July 27, 1948.)

535*t*. R. P. Dilworth: *The lattice of continuous functions.*

The set $C(S)$ of all bounded continuous real functions over a topological space S forms a lattice under the containing relation $f(x) \leqq g(x)$ all $x \in S$. $C(S)$ is, in general, not a complete lattice, that is, a bounded set of continuous functions need not have a least upper bound among the upper bounds in $C(S)$. In this paper we construct a set of real functions over S which forms a complete lattice under the above containing relation and which is isomorphic to the minimal completion of $C(S)$ by means of normal subsets. By this means it is proved that *the completion of $C(S)$ by normal subsets is distributive* (*indeed, completely distributive*!). (Received July 9, 1948.)

536*t*. E. E. Floyd: *A non-homogeneous minimal set.*

There is constructed a compact subset X of the plane and a homeomorphism T of X onto itself such that X is minimal with respect to T and such that X is of dimension 1 at some points and of dimension 0 at the others. This is a solution of the problem of constructing a compact minimal set which is not homogeneous. (Received June 14, 1948.)

537. A. M. Gleason: *A structure theorem for spaces with a compact Lie group of transformations.*

Let R be a Hausdorff space in which Tietze's extension theorem holds. Let G be a compact Lie group operating on R. If $x \in R$ denote by G_x the set of all elements of G leaving x fixed. Suppose that $p \in R$ is such a point that, for all q near p, G_q is a conjugate of G_p. Then there is a closed neighborhood of $G(p)$ (the orbit of p) which is the topological direct product of $G(p)$ and a closed set C in R. The principal tool is the construction of a matrix valued function K on the space R such that $K(p)$ is the identity matrix and $K(g(x)) = H(g)K(x)$ for all $g \in G$ and $x \in R$, where H is a faithful matrix representation of G. A corollary of particular interest arises if R is itself a topological group and G is a compact Lie subgroup. In this case there is a neighborhood of G in R homeomorphic to the direct product of G and a neighborhood of G in the factor space R/G. (Received July 24, 1948.)

538t. D. W. Hall: *A note on metrization.*

This note presents a new metrization theorem, the proof of which is very short and completely elementary. Known metrization theorems, the original proofs of which were much more difficult, are shown to follow from this theorem without difficulty. (Received July 26, 1948.)

539t. Marshall Hall: *A topology for free groups and related groups.*

Countable free groups are among the groups G possessing the following two properties: (1) G has a countable infinity of elements; (2) For each element $g \neq 1$ of G there is a subgroup U of finite index in G such that $g \notin U$. A topology may be defined for such groups in which a basis for open sets consists of cosets of subgroups U of finite index in G. This topology may be realized by a linear metric mapping the elements of G onto a linear set. The closure of this set is a Cantor set, C, and the points of C correspond to the completion $\widehat{G}$ of G which is a compact group. It is shown that C may be constructed in such a way that the Haar measure on $\widehat{G}$ is equal to the corresponding Lebesgue measure on C. A subgroup H of G is proved to be the intersection of the U's which contain it if and only if H is closed in the topology. Throughout this paper the central interest is attached to the particular properties of this topology and not to the previously known theorems on embedding in compact groups. (Received July 12, 1948.)

540. P. R. Halmos: *On the representation of σ-complete Boolean algebras.*

The purpose of this note is to give a simple proof of the following generalization of a theorem of Loomis. Every Boolean σ-algebra is σ-isomorphic to the class of all Baire sets, modulo Baire sets of the first category, in a totally disconnected, compact Hausdorff space. (Received July 14, 1948.)

541. Edwin Hewitt: *A general fixed point theorem.*

Let L be a locally convex linear topological space over the real numbers and let A be any closed convex subset of L. Then any continuous mapping ϕ of A into itself such that $\phi(A)$ has compact closure admits a fixed point. The proof employs methods of Tychonoff and a generalization of a theorem due to Mazur. (Received July 27, 1948.)

542. E. E. Moise: *Grille decomposition and convexification theorems for compact metric locally connected continua.*

The principal theorems of this paper are as follows: (I) Let S be a compact metric locally connected continuum. Then there is a sequence $G_1, G_2, \cdots$ of finite collections of mutually exclusive connected, uniformly locally connected open sets, such that for each i, (1) each point of S belongs to the closure of some element of G_i, (2) G_{i+1} is a refinement of G_i, and (3) each element of G_i has diameter less than $1/i$. (II) Let S be as in (I). Then S can be given a convex metric preserving the original topology. The convexification problem, of which (II) is the solution, was proposed by Menger in 1928 (Math. Ann. vol. 100, p. 75). (Received July 27, 1948.)

543*t*. G. D. Mostow: *A new proof of E. Cartan's theorem on the topology of semi-simple groups.*

A new proof is given for the theorem of Cartan: a semi-simple Lie group is topologically the direct product of a compact subgroup and a Euclidean space. The notion of a symmetric Riemannian space which plays an essential role in Cartan's proof is eliminated. (Received July 19, 1948.)

544*t*. B. J. Pettis: *On points of uniform convergence in topological spaces.*

Let S be a topological space (no separation axioms) and T a uniform space with ϕ' the cardinal number of the uniform neighborhoods $\{V_\alpha\}$ in T. Let $L=[\lambda]$ be a directed set having cardinal number ϕ''. For each λ in L let $f_\lambda(x)$ be a function on S to T and suppose $\lim_\lambda f_\lambda(x)=f(x)$ exists in the Moore-Smith sense for each x. Call p in S *a point of uniform convergence* if given any α there exists a neighborhood U of p and some λ in L such that x in U and $\mu>\lambda$ imply $f_\mu(x)$ is in $V_\alpha(f(x))$. The following then holds: if each f_λ is continuous the points in S not of uniform convergence form an I_ϕ set (Dunford, Ann. of Math. vol. 41 (1940)) for any $\phi\geqq\phi'$, ϕ''. Furthermore, every point of uniform convergence is a continuity point of $f(x)$. For countable L and metric S and T these results were given by Kuratowski (Fund. Math. vol. 5 (1924)) and earlier and more restrictedly by Banach, Osgood, and Baire. For complete metric S and metric T these results yield short direct proofs of Montgomery's theorem on the double continuity of xy in metrized groups, the Banach-Mazur generalized ergodic theorem, and the Nikodym-Saks theorems on the convergence of completely additive set functions. (Received July 26, 1948.)

545. E. H. Spanier: *The Mayer homology theory.*

W. Mayer has defined new homology groups based on a boundary operator whose pth power (p a prime) is zero, instead of the usual one whose square is zero. Mayer established the topological invariance of these groups but left unsettled the question of their relation with the classical homology groups. In this paper it is shown that the Mayer groups do not lead to new topological invariants but lead instead to interesting alternative definitions of the classical homology groups. This result is obtained by applying the axiomatic characterization of homology theory of Eilenberg and Steenrod to a suitably relabeled collection of Mayer groups. (Received July 20, 1948.)

546. G. W. Whitehead: *A generalization of the Hopf invariant.* II.

Let f be a mapping of the n-sphere S^n into S^r representing an element α of the nth homotopy group $\pi_n(S^r)$ of S^r, and suppose that the Freudenthal suspension Ef of f is inessential. To each nullhomotopy h of Ef there is associated (if $n<3r-2$) an element $\beta\in\pi_{n+2}(S^{2r+1})$. It is shown that $E^2H(\alpha)=\beta+(-1)^r\beta$, where $H(\alpha)$ is the generalized Hopf invariant of α (Bull. Amer. Math. Soc. Abstract 52-5-215). Conversely, if $\beta\in\pi_n(S^{2r-1})$ with r even and $n<3r-3$, then there exists $\alpha\in\pi_n(S_r)$ such that $E\alpha=0$ and $H(\alpha)=2\beta$. This construction is used to answer in the negative a question proposed by Hopf (Fund. Math. vol. 25 (1935) pp. 427–440): if $r=4k+2>2$ there is no element of $\pi_{2r-1}(S^r)$ with Hopf invariant 1. Moreover, S^{4k+1} does not admit a continuous multiplication with two-sided identity if $k>0$. (Received July 12, 1948.)

547*t*. G. T. Whyburn: *Continuous decompositions.*

Motivated by the fact that the decomposition of a region effected by an analytic function is l.s.c. but not necessarily u.s.c. in the open set sense, a study is made of decompositions G of a topological space A into disjoint closed sets which are (i) u.s.c. in the limit sense ($X \cdot \lim\inf X_i \neq 0$ implies $X \supset \lim\sup X_i$) and (ii) l.s.c. in the open set sense (sum of all elements intersecting an open set is open). The hyperspace A' of such a decomposition is a topological space and the natural mapping $\phi(A)=A'$ is continuous and interior (open). Accordingly, perfect separability as well as regularity plus local compactness go over from A to A'. For locally connected spaces A the condition (iii) for each region R in A' each component of $\phi^{-1}(R)$ maps onto R under ϕ, seems appropriate to add to (i) and (ii) to define a continuous decomposition. If A is locally compact, locally connected, separable and metric, G will be both u.s.c. and l.s.c. in the open set sense if and only if the elements of G are compact and G satisfies (i), (ii) and (iii). Further, the decompositions effected by important classes of analytic functions satisfy (i), (ii) and (iii). (Received May 11, 1948.)

548*t*. G. T. Whyburn: *Continuous decompositions and developability.*

Extending a concept of Stoïlow, we define a light interior mapping $f(A)=B$ on a locally connected generalized continuum A to be *developable* provided A is the sum of a strictly monotone increasing sequence of conditionally compact regions $[R_n]$, $\overline{R}_n \subset R_{n+1}$, such that $f(\overline{R}_n) \cdot f[F(R_{n+1})]=0$ and $f|\overline{R}_n$ is interior. It is shown that for f to be developable it is necessary and sufficient that non-compactness be invariant under f for closed generalized continua in A or, equivalently, that each component of the f inverse of a continuum in B be compact. This condition in turn implies that the decomposition generated by f is continuous in a natural and significant sense. Further, the condition is satisfied by well defined classes of analytic functions. (Received May 11, 1948.)

549*t*. G. T. Whyburn: *Interior mappings on locally compact spaces.*

An interior (open) mapping $f(A)=B$ on a locally connected generalized continuum is said to generate a continuous decomposition of A provided (*) for any region R in B, each component of $f^{-1}(R)$ maps onto R under f. This condition is implied by a similar local conditions at points of B and also by the condition (†) for any continuum K in B, each component of $f^{-1}(K)$ is compact. In fact, a localized (†) implies (*); but (*) does not imply (†) as is shown by the mapping $w=e^z$. If (*) holds, the decomposition generated in each component of the inverse of a region in B is itself continuous. If A is a 2-manifold, f is light and (*) holds, then either $f^{-1}(y)$ is infinite for every $y \in B$ or f has a finite degree k, uniform for all $y \in B$. If f is light, for any locally connected generalized continuum Y in B whose interior rel. B is dense in Y, $f^{-1}(Y)$ is locally connected; further, any $a \in A$ is interior to an arbitrarily small locally connected continuum in A on which f is interior. (Received May 11, 1948.)

550*t*. G. T. Whyburn: *Sequence approximations to interior mappings.*

Given a sequence of continuous transformations $f_n(x)$ of a locally compact separable metric space A onto open subsets of a similar space B which converges uniformly on each compact subset of A to a mapping $f(x)$. Necessary and sufficient conditions are first established in order that $f(x)$ be an interior (or open) mapping. The sequence is said to be uniformly approximately interior on a subset A_0 of A provided that for any $\epsilon>0$, a δ and an N exist such that the f_n image of the ϵ-neighborhood of any x in A_0 contains the δ-neighborhood of $f_n(x)$ for all $n>N$. It is shown that if A is locally connected and the mappings f_n are interior and f is light, the sequence is uniformly approximately interior and f is interior. (Received April 5, 1948.)

551. R. L. Wilder: *Monotone mappings of manifolds.*

Monotone mappings of 2-manifolds have been extensively studied. The present paper attempts to begin a general study of r-monotone mappings (G. T. Whyburn, Amer. J. Math. vol. 57 (1935) p. 904) of n-manifolds. Simple examples show that 2-monotone images of the 3-sphere may no longer be manifolds; they will, however, be generalized manifolds. Accordingly the n-gm is adopted as the basic configuration, and it is shown that an $(n-1)$-monotone image of an orientable n-gcm is again an orientable n-gcm with the same homology characters. It is also shown that (1) an $(n-1)$-monotone image of an lc^n compactum is lc^n; (2) nasc that a locally compact space S be lc^n is that if M is a compact subset of S such that $p^r(M)=0$ for some $r\leqq n$, then for any open set U containing M there exists an open set V such that $M\subset V\subset U$ and all r-cycles on V bound on U; (3) if S is an orientable n-gm and M a compact subset of S such that $p^r(M)=p^{n-r-1}(M)=0$ for some r, $1\leqq r\leqq n-2$, then for any open set U containing M there exists an open set V such that $M\subset V\subset U$ and all compact r-cycles in $V-M$ bound in $U-M$. (Received July 26, 1948.)

552. G. S. Young: *On regular convergence of sequences of 2-manifolds.* II.

Two new types of regular convergence are defined, W-r-regular, and S-r-regular. Let $\{M_n\}$ be a sequence of two-manifolds in a locally compact metric space converging to a set M. If $\{M_n\}$ converges $W-1$-regularly to M, and the sequence $\{B_n; B_n$ the boundary of $M_n\}$ converges $W-0$-regularly to a subset of M, then M is a 2-manifold. If W-regularity is replaced by S-regularity, then M is homeomorphic to almost all M_n. (Received July 26, 1948.)

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