THE APRIL MEETING IN ANN ARBOR

The four hundred thirty-fifth meeting of the American Mathematical Society was held at the University of Michigan, Ann Arbor, Michigan, on April 16–17, 1948. The attendance was approximately 200, including the following 175 members of the Society:

L. U. Albers, A. A. Albert, W. R. Allen, D. N. Arden, R. C. F. Bartels, J. H. Bell, Felix Bernstein, R. H. Bing, H. L. Black, C. J. Blackall, W. M. Boothby, D. G. Bourgin, J. W. Bradshaw, Richard Brauer, F. L. Brown, L. M. Browne, R. H. Bruck, W. K. Burroughs, R. E. Carr, E. D. Cashwell, Abraham Charnes, R. V. Churchill, Nathaniel Coburn, C. J. Coe, H. J. Cohen, A. H. Copeland, Max Coral, H. S. M. Coxeter, C. C. Craig, Jane Cronin, H. B. Curry, M. M. Day, D. J. Dickinson, Flora Dinkines, C. L. Dolph, Ben Dushnik, P. S. Dwyer, John Dyer-Bennett, W. F. Eberlein, B. J. Eisenstadt, Benjamin Epstein, Ky Fan, C. H. Fischer, K. W. Folley, J. S. Frame, R. E. Fullerton, Casper Goffman, Michael Golomb, S. H. Gould, L. M. Graves, L. J. Green, E. B. Grennan, P. E. Guenther, William Gustin, S. W. Hahn, Marshall Hall, G. E. Hay, Ruth Heinsheimer, R. G. Helsel, I. N. Herstein, Fritz Herzog, Edwin Hewitt, J. F. Heyda, T. H. Hildebrandt, J. D. Hill, D. L. Holl, L. A. Hopkins, C. C. Hsiung, H. K. Hughes, H. D. Huskey, R. E. Huston, A. W. Jacobson, Meyer Jerison, P. S. Jones, G. K. Kalisch, L. H. Kanter, Irving Kaplansky, Wilfred Kaplan, Leo Katz, D. E. Kibbey, W. M. Kincaid, C. F. Kossack, D. M. Krabill, Benjamin Lapidus, C. G. Latimer, H. D. Larsen, K. B. Leisenring, J. H. Levin, Madeline Levin, D. J. Lewis, B. J. Lockhart, E. D. McCarthy, S. W. McCuskey, H. B. Mann, Morris Marden, J. W. Marshall, L. E. Mehlenbacher, E. J. Mickle, D. D. Miller, J. M. Mitchell, E. E. Moise, D. C. Morrow, S. B. Myers, C. J. Nesbitt, K. L. Nielsen, E. A. Nordhaus, M. J. Norris, D. A. Norton, J. A. Nyswander, F. C. Ogg, J. M. H. Olmsted, E. J. Olson, E. H. Ostrow, G. K. Overholtzer, F. W. Owens, H. B. Owens, R. S. Pate, M. H. Payne, P. M. Pepper, Sam Perlis, C. L. Perry, George Piranian, E. C. Pixley, V. C. Poor, G. B. Price, Gustave Rabson, Tibor Rado, G. Y. Rainich, E. D. Rainville, M. O. Reade, O. W. Rechard, Arthur Rosenthal, A. E. Ross, E. H. Rothe, H. J. Ryser, Hans Samelson, W. C. Sangren, A. C. Schaeffer, H. M. Schaerf, K. C. Schraut, W. R. Scott, I. E. Segal, M. E. Shanks, A. S. Shapiro, L. W. Sheridan, Seymour Sherman, M. F. Smiley, F. C. Smith, G. W. Smith, W. F. Smith, W. N. Smith, T. H. Southard, R. D. Specht, R. H. Stark, H. E. Stelson, Rothwell Stephens, B. M. Stewart, Helen Sullivan, R. L. Swain, R. M. Thrall, C. W. Topp, Leonard Tornheim, G. L. Walker, M. S. Webster, M. T. Wechsler, D. V. Wend, N. A. Wiegmann, R. L. Wilder, J. E. Wilkins, C. S. Williams, M. A. Woodbury, J. B. Wright, G. S. Young, P. M. Young, J. W. T. Youngs.

At 2:00 P.M. Friday Professor H. S. M. Coxeter delivered an address entitled *Self-dual configurations and regular graphs*, with Professor T. H. Hildebrandt presiding. At 9:00 A.M. Saturday Professor Irving Kaplansky gave an address entitled *Topological rings*, with Professor Marshall Hall presiding.

Two sessions for short research papers were held at 10:00 A.M. with presiding officers Professors A. C. Schaeffer and Richard Brauer. There was also a session at 3:15 P.M. Friday and one at 10:15 A.M.

Saturday with presiding officers Professors Morris Marden and S. B. Myers, respectively.

The University of Michigan was host to those attending the meeting at an afternoon tea which took place at 4:30 P.M. Friday in the Rackham Building. At 6:30 P.M. Friday a dinner was held at the Smith Catering Service with an attendance of approximately 150. Professor T. H. Hildebrandt acted as toastmaster. The guests were welcomed in a warm and amusing manner by Dean Hayward Kenniston, of the University of Michigan. Professor Tibor Rado gave a serious and interesting talk on the language problem in Puerto Rico, recounting his impressions and conclusions during his stay on the island.

Local arrangements for the meeting were in charge of Professor S. B. Myers.

There follow the complete abstracts of the papers presented at this meeting. Papers whose abstract number is followed by the letter "*t*" were presented by title. Paper no 291 was presented by Dr. Ryser, no. 298 by Professor Perlis, no. 308 by Professor Mickel.

ALGEBRA AND THEORY OF NUMBERS

289. A. A. Albert: *On right alternative algebras.*

An algebra A over a field F is called right alternative if $y(xx)=(yx)x$ for every x and y of A. We show that if F has characteristic not 2 then every such algebra of degree two over F is alternative. If e is an idempotent of A the right multiplications R_e satisfy the usual relation $R_e^2=R_e$. However the left multiplications only need satisfy $(L_e^2-L_e)^2=0$. Examples are given of right alternative algebras which are not alternative. Let F be a nonmodular field and $t(x, y)$ be the trace of L_{xy}. Then we show that the maximal nilideal of A is the set of all x such that $t(x, y)=0$ for every y and coincides with the radical of the attached algebra $A^{(+)}$. We call A semisimple if this radical is zero and prove that every semisimple right alternative algebra over a nonmodular field is actually alternative. (Received February 16, 1948.)

290*t*. Reinhold Baer: *Direct decompositions into infinitely many summands.*

In this note certain results concerning themselves with direct decompositions of operator loops into a finite number of direct summands are extended to direct decompositions into infinitely many direct summands. (Received February 16, 1948.)

291. R. H. Bruck and H. J. Ryser: *On the number of points in a finite projective plane.*

The following result is obtained: Let the positive integer n be such that (i) $(-1, n; p)^k=-1$ for at least one odd prime p, where $(-1, n; p)=(-1, n)_p$ is the Hilbert symbol and $2k=n(n+1)$. Then there exists no projective plane geometry with $n+1$ points on each line. (Condition (i) is equivalent to the following pair of conditions: (ii) n is of the form $4m+1$ or $4m+2$; (iii) at least one prime factor of the form $4m+3$ occurs in n to an odd power.) The proof uses the Minkowski-Hasse theory of equivalence of rational quadratic forms. The result seems to be new except for the case $n=6$ (for critical comments see F. W. Levi, *Finite geometrical systems*, University of Calcutta, Calcutta, 1942). (Received February 21, 1948.)

292. J. S. Frame: *Congruence relations between the traces of matrix powers.*

A congruence $S(p^\beta)\equiv S(p^{\beta-1})$ mod p^β involving the sums of powers of roots of unity is shown to hold under the following conditions. We let $S(m)$ denote the trace of the mth power of a matrix A of finite degree $S(0)$ over a field of characteristic zero. The congruence is implied by the two conditions: (1) $A^n=1$ and (2) $S(k)=S(1)$ for every k such that $(k, n)=1$. These conditions imply that $S(1)$ is a rational integer, but not every matrix with rational integral trace satisfies (2). The congruence is a powerful aid in constructing the table of characters of a finite group, and generalizes the well known special case $\beta=1$. (Received March 18, 1948.)

293t. Franklin Haimo: *An inverse limit group.*

Let G_∞ be the inverse limit group of the groups G/nG, $n=1, 2, \cdots$, for an additive abelian group G; and let G' be the subgroup of completely divisible elements of G. (Cf. S. MacLane, Bull. Amer. Math. Soc. Abstract 54-1-6.) G_∞ has no nonzero elements which are completely divisible. Moreover, G/G' is isomorphic with a subgroup G'' of G_∞ in an obvious fashion. However, if G can admit a topology which makes it a compact topological group, then $G''=G_\infty$. If G is the group of integers, it is easy to show that $G''\neq G_\infty$. From this follows the well known fact that the group of integers is never a compact topological group. (Received March 12, 1948.)

294. Marshall Hall: *Studies in free groups.*

A standard representation for a subgroup U of a free group F was given in a recent paper written jointly by T. Rado and the author. This representation is not in general unique. In this paper a canonical representation is found. This will be unique and will have certain properties not possessed by other representations. A number of applications of this representation are made. By a generalization of a process due to J. Nielson it may be shown that if the decision problem is solvable for a subgroup U, then it will be solvable within a restricted set of operations. Special properties of normal subgroups U are investigated. (Received March 12, 1948.)

295. C. G. Latimer: *On zero ternary quadratic forms.*

Let $f=\sum a_{ij}x_ix_j$ (a_{ij} integers, $a_{ji}=a_{ij}$) be a properly primitive indefinite ternary form of determinant $D>0$ and with a properly primitive adjoint F. If (u_1, u_2, u_3) is a solution of $f=0$ and $U_i=\sum_j a_{ij}u_j$ $(i=1, 2, 3)$ then (U_1, U_2, U_3) is a solution of $F=0$. If the g.c.d. of the u's is unity and the same is true of the U's, (u_1, u_2, u_3) is called a doubly primitive solution of $f=0$. It is shown that such a solution exists if and only if every generic character of f is equal to unity. If (u_1, u_2, u_3) is a doubly primitive solution there are certain integers t_i, such that $\sum t_iU_i=1$. We determine two transformations, each of determinant unity, which transform f and F into $\phi=y_1^2-y_2^2-Dy_3^2$ and $\Phi=DY_1^2-DY_2^2-Y_3^2$ respectively. The coefficients of these transformations are integers which are determined explicity in terms of the u's and t's. (Received March 11, 1948.)

296t. Saunders MacLane: *Operator homomorphisms of kernels.*

Let (K, θ) be a Q-kernel with center H and obstruction $k^3 \in H^3(Q, H)$, in the sense of Eilenberg and the author (Ann. of Math. vol. 48 (1947) pp. 326–341). Its graph Γ consists of all pairs (x, α), with $x \in Q$ and the automorphism α in the automorphism class $\theta(x)$; Γ has a natural homomorphism $\phi(x, \alpha) = x$ onto Q. If G is an abelian group with operators in Q, consider the group A of all operator homomorphisms of the kernel (K, θ) into G, reduced modulo those operator homomorphisms which are obtained by "cutting down" crossed characters of Γ into G. THEOREM: If Γ is a free group, then A depends only on H, G, Q, and the obstruction k^3 is in fact isomorphic to the group $H^2(Q, H, k^3, G)$ introduced by Eilenberg and the author in the study of the second cohomology group of a space (Proc. Nat. Acad. Sci. U. S.A. vol. 32 (1946) pp. 277–280). (Received February 23, 1948.)

297t. Saunders MacLane: *The second homotopy kernel of a polyhedron.*

Let P be a polyhedron, S the one-dimensional skeleton of a sufficiently fine triangulation of P, π_1 and π_2 the first and second homotopy groups of P. The second relative homotopy group $\pi_2(P, S)$ has π_2 as center and has the free group $\pi_1(S)$ as a group of operators. Using the fact that $\pi_1(S)$ has a canonical homomorphism onto π_1, it follows that $\pi_2(P, S)$ is a π_1-kernel; its graph is the free group $\pi_1(S)$, and its obstruction is the 3-dimensional cocycle $k^3 \in Z^3(\pi_1, \pi_2)$ associated with P. The expression of the cohomology groups of P in terms of k^3, found by Eilenberg and the author, can then be formulated as follows: The second cohomology group of P, with coefficients in G, is the group of all operator homomorphisms of the kernel $\pi_2(P, S)$ into G, modulo those homomorphisms which can be obtained by cutting down crossed homomorphisms of $\pi_1(S)$. (Received February 23, 1948.)

298. Sam Perlis and G. L. Walker: *Finite Abelian group algebras.* I.

Let G be a finite group and F a field. The determination of all groups H such that the group algebras G_F and H_F are isomorphic is solved for the case in which F is non-modular and G is abelian of prime power order. The result is a corollary of the solution of another problem in which G and H are given and one seeks all fields F making G_F isomorphic to H_F. For each pair G and H (abelian of prime power order p^e) there is determined a unique field L, contained in the field of p^eth roots of unity, such that $G_F \cong H_F$ if and only if $F \geqq L$. The field L is defined in terms of the invariants of G and H. For cyclic G of arbitrary order n and for any F of characteristic not dividing n, the structure of G_F is expressed very simply in terms of the factorization of $x^n - 1$ over F. (Received March 11, 1948.)

299. A. E. Ross and Thomas Matthews: *On a constructive development of Galois theory.*

Pursuing further the ideas of S. Shatunovsky (*Algebra as the study of congruences with respect to functional moduli*, Odessa, 1913) the authors consider a separable polynomial $f(x)$ of degree n, without multiple factors in a field K, and they represent the splitting field K_n of $f(x)$ as the residue class ring $K[x_1, \cdots, x_n]/I$ where I is the ideal in $K[x_1, \cdots, x_n]$ whose basis consists of polynomials which are factors of $f(x)$ usually used to carry out successive extensions leading to the splitting field K_n. They denote these factors by $\mu_1(x_1), \cdots, \mu_n(x_1, \cdots, x_n)$ and show that a permutation π of the roots $\bar{x}_1, \cdots, \bar{x}_n$ ($\bar{x}_i$ is the residue class of x_i modulo I) of $f(x)$ in K_n determines an automorphism σ of K_n over K if and only if $\mu_i(\bar{x}_1, \cdots, \bar{x}_i)=0$ implies $\mu_i(\pi(\bar{x}_1), \cdots, \pi(\bar{x}_n))=0$ for $i=1, \cdots, n$. This observation leads to a direct determination of the number of admissible permutations of the roots, that is, of the order of the group G of automorphisms σ of K_n over K. It follows at once that K is the fixed field of G, and the fundamental theorem of Galois theory appears as a direct consequence of a simple lemma on substitutions. (Received March 18, 1948.)

300. R. M. Thrall: *Some generalizations of quasi-Frobenius algebras.*

Consider algebras $\mathfrak{A}$ over a field k and with 1 element. $\mathfrak{A}$ is called a quasi-Frobenius (QF) algebra if every primitive left ideal is dual to a primitive right ideal. Some of the most important properties of QF algebras do not characterize these algebras, but occur in more extensive classes, thus leading to the following definitions. $\mathfrak{A}$ is said to be a QF-1 algebra if every faithful representation is its own second commutator. $\mathfrak{A}$ is said to be a QF-2 algebra if every primitive left (right) ideal of A has a unique minimal subideal. $\mathfrak{A}$ is said to be a QF-3 algebra if it has a unique minimal faithful representation (a faithful representation is called minimal if deletion of any direct constituent leaves it unfaithful). Some of the elementary properties of these classes of algebras are investigated with special attention to relations between them; for example, the class of QF-3 algebras properly contains the class of QF-2 algebras. (Received February 13, 1948).

ANALYSIS

301. W. F. Eberlein: *Abstract ergodic theorems.*

Mean ergodic theorems for a semi-group G of linear transformations are studied. It is shown that relaxation of the uniform boundedness and countability restrictions customarily imposed on G leads to a more general theory, which not only embraces a significantly wider domain of phenomena but subsums previous results in a sharper and more transparent form. In addition to the standard mean ergodic theorems, the theory contains as special cases the most general mean ergodic theorem for Abelian semi-groups, the existence and uniqueness of the mean for almost periodic functions, Fejer's $(C, 1)$ summability theorem, and applications to the abstract theory of Markoff processes. (Received March 11, 1948.)

302. Caspar Goffman: *On Lebesgue's density theorem.*

The density theorem of Lebesgue may be stated in the following form: If S is a measurable linear point set, the metric density of S exists and is equal to 0 or 1 almost everywhere. We prove the converse that for every set Z of measure 0 there is a measurable set S whose metric density does not exist at any point of Z. We note, however, that in order for Z to be the set of points for which the metric density of some set S exists but is different from 0 or 1, Z must be both of measure 0 and of first category. As a converse, we show that for every F_σ type set of measure 0 (thus, of first category) there is a measurable set S whose metric density exists but is different from 0 or 1 at every point of Z. Several related results are given for functions. (Received March 9, 1948.)

303. L. M. Graves: *Minima of functionals in abstract spaces.*

Let $J(x)$ and $\phi(x)$ be defined near $x_0 \in X$, values of ϕ in Y, X and Y Hilbert spaces. If $J(x_0)$ is a minimum subject to $\phi(x)=0$, then there exist $\lambda_0 \geqq 0$ and a linear functional L on Y such that the first variation of $F(x)=\lambda_0 J(x)+L(\phi(x))$ vanishes at

x_0. A generalization of the Weierstrass condition is proposed, and it is shown that for each "strong variation," and for each "weak variation" satisfying $\delta\phi(x_0;\, \xi)=0$, there exist multipliers λ_0, L, for which the Weierstrass condition holds, and the second variation $\delta^2F(x_0;\, \xi)\geqq 0$. Normality is not required (cf. Goldstine, Bull. Amer. Math. Soc. vol. 46 (1940) pp. 142–149). A condition used both here and by Goldstine requires that the range C of the transformation $\delta\phi(x_0;\, \xi)$ is closed in the space Y. An example shows how easily this property may be destroyed by a transformation of the side conditions $\phi(x)=0$. However, at least in the case when the functions ϕ are linear, the norm in the subspace C of Y may always be so chosen that the required conditions are satisfied. From this it follows that when the equations $\phi=0$ are linear partial differential equations, and the functional J is a multiple integral with suitable properties, then multipliers λ_0, L exist for which the first variation δF vanishes at the point x_0. (Received March 17, 1948.)

304*t*. Deborah T. Haimo: *Singularities of mappings of E^n into E^n.*

Let f be a mapping of E^n into E^n. A point p is singular if the determinant of order n, $J=|\partial f_i/\partial x_j|$, vanishes at p. The rank of a singular point is that of the matrix defined by J. Let F_s denote the class of all mappings f for which $\partial J/\partial x_1, \cdots, \partial J/\partial x_n$ do not vanish simultaneously at any singular point of rank s. One shows that if f is a mapping of class C^3 of a region R of E^n into E^n, then there exists a function f' in F_{n-1} which approximates f arbitrarily closely through the second derivative. Let p be a singularity of rank $n-1$ of a mapping f in F_{n-1}. Then p is a fold, if, after a choice of a coordinate system making $\partial f/\partial x_1|_p=0$ (always possible), $\partial J/\partial x_1|_p\neq 0$. If $\partial J/\partial x_1|_p=0$, then p is a cusp. For $n=2$ any mapping may be approximated by one every singular point of which is either a fold or a cusp. If the origin is a fold of a mapping f in F_{n-1} of class C^4 then there exist curvilinear coordinate systems about 0 and $f(0)$ in terms of which f has the form $y_1=x_1^2$, $y_i=x_i$, $i=2, 3, \cdots, n$. The form for a cusp is less simple. For the image space E^m, $m\geqq 2n-1$, see H. Whitney (Duke Math. J. vol. 10 (1943) pp. 161–172; Ann. of Math. vol. 32 (1937) p. 818; ibid. vol. 37 (1936) p. 654). (Received March 12, 1948.)

305. L. H. Kanter: *On the zeros of the Jacobi polynomials.*

Applying the notation and method of Abstract 53-5-211 to the Jacobi polynomials, it is shown that for a weight function $(1-x/a)^\alpha(1+x/a)^\beta$, with $a\geqq 2$, $-1<\alpha(\tau)\leqq 0$, $-1<\beta(\tau)\leqq 0$, $\alpha'(\tau)\geqq 0$, $\beta'(\tau)\geqq 0$, $\alpha'(\tau)+\beta'(\tau)>0$, $\lambda_\nu(\tau)$ is a monotonically decreasing function of τ. Moreover, if the condition is changed to $a\geqq 2$, $\alpha(\tau)>\beta(\tau)>-1$, $\alpha'(\tau)\geqq 0$, $\beta'(\tau)\geqq 0$, $\alpha'(\tau)+\beta'(\tau)>0$, $x_\nu(\tau)>0$, $\lambda_\nu(\tau)$ will be a decreasing function of τ provided that $x_\nu(\tau)$ is an increasing function of τ. For the ultraspherical polynomials the conclusion is that $\lambda_\nu(\tau)$ is a monotonically decreasing function of τ if $-1<\tau\leqq 0$. By making use of equation (15.3.2), p. 343, Szegö, *Orthogonal polynomials*, Amer. Math. Soc. Colloquium Publications, vol. 23, it is shown that for the ultraspherical polynomials of degrees 2 and 3, respectively, $\lambda_\nu(\tau)$ is a decreasing function of τ for $-1<\tau$. (Received March 6, 1948.)

306. Morris Marden: *On the zeros of rational functions with prescribed poles.*

Biernacki and Dieudonné have proved that, if A is an arbitrary complex number and the m_j are positive integers and if all the points z_j lie in a circle C of radius r, then $F(z)=A+\sum_{j=1}^{n} m_j/(z-z_j)$ has at least $n-1$ zeros in the concentric circle of radius $2^{1/2}r$. This theorem was recently shown by Nagy to be valid also when the m_j are arbitrary positive numbers. In the present paper the theorem is generalized to functions $F(z)=\sum_{j=0}^{p} A_jz^j+\sum_{j=1}^{n} m_j/(z-z_j)$ in which the A_j are arbitrary complex numbers and the m_j are also complex. The main result is the following. Let $F(z)$ be a rational function which has at $z=\infty$ a pole of order p and has at the n finite points z_j $(j=1, 2, \cdots, n)$ simple poles with residues m_j such that $a\leqq\arg m_j\leqq a+b<a+\pi$. Let K be the smallest convex region containing all the z_j. Then $F(z)$ has at least $n-1$ zeros in the star-shaped region comprised of all points from which K subtends an angle of at least $(\pi-b)/(p+2)$. The proof is based upon the lemma (apparently new) that, if $Z_1, Z_2, \cdots, Z_{p+2}$ are any zeros of $F(z)$, then $\sum_{j=1}^{n}[m_j/(Z_1-z_j)(Z_2-z_j)\cdots(Z_{p+2}-z_j)]=0$. In the special case that the m_j are all positive integers, the theorem reduces to a result on the zeros of the function $P(z)f(z)+f'(z)$ where $f(z)$ is a given polynomial of degree n and $P(z)$ is an arbitrary polynomial of degree not exceeding p. (Received March 13, 1948.)

307*t*. H. L. Meyer: *Multiple integral problems in the calculus of variations. I.*

This paper considers the following problem $\mathcal{P}_{m,n}$ $(m>1, n>1)$ in the calculus of variations: to find conditions on a surface z_0 *sufficient* for z_0 to provide $I(z)=\int_G f(x, z, p)dx$ with relative minimum on a class of surfaces z having common boundary values with z_0. Here $x\in E_m$, $z\in E_n$ is a function of x, and $p=(\partial z^i/\partial x^j)$. General sufficiency theorems for strong and semi-strong minima are established for different classes of surfaces. These theorems use strengthened versions of the Weierstrass condition $E(p, p+\pi)>0$ and the Legendre condition $Q(\pi)>0$ which require these inequalities to hold for π's of arbitrary rank; the strengthened condition involving the second variation is taken in the form $I_2(\eta)>0$ or $I_2(\eta)\geqq h\int_G|\eta|^2dx$ for some $h>0$ and suitable variations η. The proofs are indirect, after a fashion introduced by McShane (Trans. Amer. Math. Soc. vol. 52 (1942) pp. 344–379) and developed by Hestenes (Trans. Amer. Math. Soc. vol. 60 (1946) pp. 93–118) for simple integral problems. Unpublished results of Hestenes and Karush are exploited. (Received March 11, 1948.)

308. E. J. Mickle and Tibor Rado: *On relationships between cyclic additivity theorems.*

Rado has given a generalization of a previous result of his on cyclic additivity (*A general cyclic additivity theorem*, abstract 54-7-313). Call the new result cyclic additivity in the strong sense and the previous result cyclic additivity in the weak sense. In this paper it is shown that in certain cases cyclic additivity in the weak sense implies cyclic additivity in the strong sense. In particular, this is so if the "initial space" is a 2-cell or a 2-sphere, or more generally any unicoherent Peano space. As a consequence, the result applies to the Lebesgue area of surfaces both in the 2-cell case and the 2-sphere case. (Received March 4, 1948.)

309. Josephine M. Mitchell: *Orthonormal systems defined on nonschlicht domains.*

The aim of this paper is to prove the existence of closed orthonormal systems on nonschlicht domains in the case of one and two complex variables. In the case of one complex variable the domain, which lies on a Riemann surface, may be locally uniformized at every point, and the closed orthonormal system constructed by a generalization of the method used by Bergman for schlicht domains (*Partial differential equations, Advanced Topics*, Brown University, Summer Session, 1941, Chapter VI). As an example, a closed orthonormal system is constructed for the doubly-covered domain defined by $w^2+z^2=a^2$, $|w|<1$, $0<a<1$. Similar results may be obtained for four-dimensional, nonschlicht domains embedded in the space of two complex variables, if it is assumed that the domains may be locally uniformized at every point (cf. Bergman, Mémorial des Science Mathématique, vol. 106). Closed orthonormal sets are constructed for several examples of such four-dimensional, nonschlicht domains. (Received March 10, 1948.)

310. V. C. Poor: *On residues of polygenic functions.*

This is a sequel to a paper published in vol. 32 of the Trans. Amer. Math. Soc., in which paper residues at a point of such functions were considered. Here our purpose is to extend these results to areas, generalizing the notion of classical residues of holomorphic functions. The general theory is developed and applied to several special cases. The Cauchy contour integral loses its meaning for polygenic functions. Two difficulties arise, the discontinuity and a continuous distribution over the area. In the "limit definition" used in the previous paper these difficulties were overcome by contracting to a point the contour integral considered as a circle. In the present paper these difficulties are obviated by subtracting an integral over the area, which integral contains the areal (or mean) derivative in its integrand. These new definitions conform better to the classical definition for residues. (Received January 14, 1948.)

311*t*. V. C. Poor: *On the two-dimensional derivative of a complex function.*

The two-dimensional derivative of a polygenic function treated by Cioranescu is monogenic, in that it is independent of direction. Two papers by Haskell and by Reade in recent numbers of the Bulletin consider this same subject. These papers are analyzed and synthesized in this paper using a new and different point of view. In extending these results, this class of monogenic functions is more explicitly characterized. Further, a second class of monogenic functions is defined and a dual relation between these two classes of functions is established. (Received January 14, 1948.)

312. G. B. Price: *Researches in the theory of functions of several real variables.* III. *Elements of a theory of functions.*

This paper contains applications of the theory of differentiation which was developed in Part II to the study of functions (mappings from an n-dimensional space onto an m-dimensional space). Among other topics it treats (a) extreme points and critical points, (b) the relation of the topological index to the derivative, (c) the law of finite increments, (d) a general theory for the evaluation of Riemann integrals, (e) the law of the mean and Rolle's theorem, and (f) a new Riemann-Stieltjes integral. The theory developed for the evaluation of multiple Riemann integrals includes, as special cases, Green's theorem and their evaluation by iterated integrals. Rolle's theorem, trivial in the one-dimensional case, has several forms in higher dimensions and is not simple. The treatment throughout the paper is based on topological methods and results. (Received March 11, 1948.)

313. Tibor Rado: *A general cyclic additivity theorem.*

The theorem proved in this paper is a generalization of a previous theorem of the writer (see Trans. Amer. Math. Soc. vol. 58, p. 452, §4.12). The type of generalization may be illustrated by stating an application. Notations: Q is the unit square $0 \leqq u \leqq 1$, $0 \leqq v \leqq 1$. M is an arbitrary Peano space. E_3 is Euclidean three-space. C is a generic notation for a proper cyclic element of M, and r_C is the monotone retraction from M onto C. f is a continuous mapping from Q into M, and g is a continuous mapping from M into E_3. S and S_C are the surfaces defined by the representations gf and gr_Cf respectively (products of mappings are read from right to left). A denotes Lebesgue area. Then we have the formula $A(S) = \sum A(S_C)$, $C \subset M$, if M is not a dendrite, and $A(S) = 0$ if M is a dendrite. The statement remains valid if Q is a 2-sphere. Previously known results correspond to the special case when g is light and f is monotone and onto. (Received February 16, 1948.)

314. E. H. Rothe: *Weak topology and nonlinear integral equations.*

Existence theorems for pairs of nonlinear integral equations of the form $y^*(s) + \int_D K(t, s) f(t, y)(t)) dt = 0$, $y(s) + \int_D K(s, t) f(t, y^*(t)) dt = 0$ and for systems of such pairs are derived by considering their left members as differentials of certain "scalars" in a suitable Hilbert space and applying to these scalars results about the existence of extrema which had been obtained in a previous paper (*Gradient mappings and extrema in Banach spaces*, Theorems 4.1 and 4.2, to appear soon in the Duke Math. J.) by the use of a "weak" topology. Specialized to the case of a symmetric and positive definite kernel $K(s, t)$ the results of the present paper contain as special cases theorems by Hammerstein (Acta Math. vol. 54 (1930), Theorems 1 and 2) and a theorem by M. Golomb (Publications Mathématiques de l'Université de Belgrade, vol. 5, 1936, Theorem 1). (Received February 16, 1948.)

315. H. M. Schaerf: *General problem of an invariant measure.* Preliminary report.

The problem is: Given an equivalence relation $\sim$ defined in a σ-field S of subsets of an arbitrary set, and given a family N of sets of S, to construct a nontrivial measure, defined on S, invariant under $\sim$ and vanishing exactly on N. A necessary and sufficient solubility condition is established if (i) $A \sim B$ implies that for any disjoint countable partition of A into sets $A_i \in S$, there is an analogous partition of B into sets $B_i \sim A_i$; (ii) for any sets A, B in $S - N$ ("positive" sets), there is a set $A' \sim A$ meeting B in a positive set. Then the solution is unique (to within a multiplicative constant) and any two sets of equal measure are almost congruent by countable partition. Any equivalence either under a group satisfying Weil's condition M or under σ-isomorphisms considered by D. Maharam fulfills (i) and (ii). These results are obtained by constructing an isomorphism of an algebraic system with countable summation onto a set of non-negative numbers. This partially solves a problem, the solubility of which had been doubted by Tarski. (Received March 31, 1948.)

316t. H. M. Schaerf: *On the functional equation* $f(\sum x_i) = \sum f(x_i)$. Preliminary report.

Assume that to every sequence $\{t_i\}$ of elements of an arbitrary set T there is ascribed an element $\sum t_i$ in T so that the following conditions are satisfied: (A) $\sum$ is associative and commutative; (B) T contains a zero-element θ; (C) $x \geqq y$ and $y \geqq x$ imply $x = y$ ($x \geqq y$ means $x = y + t$ for some t in T); (D) $\sum_1^n x_i \leqq y$ for $n = 1, 2, \cdots$ implies $\sum_1^\infty x_i \leqq y$; (E) for any two elements x, y different from θ, there exists an element $z \neq \theta$ such that $z \leqq x$ and $z \leqq y$. A necessary and sufficient condition for the existence of a nontrivial, non-negative solution of the functional equation $f(\sum t_i) = \sum f(t_i)$ is established. This solution is unique to within a multiplicative constant and its values are either multiples of a positive number or fill an interval. These results have applications in measure theory (see the preceding abstract). (Received March 31, 1948.)

317. W. R. Scott: *On the essential multiplicity function.*

Let T: $z=t(w)$ be a continuous plane transformation from the unit square in the w-plane into the z-plane. The essential multiplicity $\kappa(z, T)$ has been defined by Rado and Reichelderfer (Trans. Amer. Math. Soc. vol. 49 (1941) pp. 258–307). The corresponding definition of $\kappa(y, f)$ for continuous real-valued functions $y=f(x)$, $0 \leqq x \leqq 1$, is similar. The following inverse problem is considered. What are the necessary and sufficient conditions that a function $g(y)$ be equal to the essential multiplicity function $\kappa(y, f)$ for some continuous function $y=f(x)$, $0 \leqq x \leqq 1$? This question is answered. Next, a generalization of $\kappa(y, f)$ is given for general real-valued functions $y=f(x)$, $0 \leqq x \leqq 1$. The corresponding inverse problem is again solved. In the two-dimensional case, however, only a partial answer is obtained to the inverse problem. (Received February 26, 1948.)

Applied Mathematics

318. Nathaniel Coburn: *Two-dimensional non-steady irrotational flows of a compressible gas.*

In a previous paper (to be published in the Proceedings of the First Symposium of Applied Mathematics), Professor C. L. Dolph and the author developed some tensor methods for obtaining characteristic systems for the steady irrotational supersonic flow of a gas in three dimensions. The present paper indicates the manner in which the previous theory must be modified in order to treat the two-dimensional, non-steady, irrotational flow of a compressible gas. First, the potential equation of the flow is obtained. The characteristic surfaces associated with this potential equation lie in space-time. If one family of such surfaces consists of cylinders with generators parallel to the time axis, then the flow is called "proper supersonic." Such flows are characterized by the fact that the Mach number is independent of time. Further, it is shown that: (1) the flow is supersonic; (2) the characteristic system is an immediate generalization of that system for the steady case; (3) simple waves do not exist. Finally, by writing the potential equation in invariant form with respect to space-time coordinate transformations, one can obtain a characteristic system for general flows. (Received March 11, 1948.)

319. H. D. Huskey: *An example concerning rounding errors.*

The equation $y''=y$ was integrated on the ENIAC (Electronic Numerical Integrator and Computer, now at Aberdeen Proving Ground) over an interval of approximately one radian. Increments ranged from 0.001 to 0.000005 radians. An instance was found where the digit dropped in the process of rounding-off changed slowly from zero to four and back to zero again over about 250 steps of the integration. A second order formula was used in the integration process. (Received March 15, 1948.)

320. A. W. Jacobson: *A generalized convolution for the finite Fourier transformation.*

Consider a function $F(x, y)$ integrable over the square $0 \leqq x \leqq \pi$, $0 \leqq y \leqq \pi$. A generalized Fourier convolution $F^*(x)$ of $F(x, y)$ corresponding to the iterated finite sine transformation $S\{S\{F(x, y)\}\} = \int_0^\pi \int_0^\pi F(x, y) \sin nx \sin n'y \, dx \, dy = \bar{f}_{ss}(n, n')$ is defined to be $-\int_{-\pi}^{\pi} F_1(x-y, y)dy$, where F_1 is an odd periodic extension of F with respect to x and an odd extension with respect to y. When $n' = n$, it is shown that $S\{S\{F(x, y)\}\} = C\{F^*(x)/2\}$. In case $F(x, y) = F_1(x)G_2(y)$, the function $F^*(x)$ is the ordinary convolution of the two functions F_1 and G_2. Similar results are obtained for the iterated cosine, as well as for the sine-cosine, transformation. With the aid of this generalized convolution very general steady state boundary value problems are resolved into problems with simpler boundary conditions and source functions. Moreover, this method enables us to extend the Duhamel integral formula from time to space coordinates. A number of key functions are introduced and the solutions of some basic boundary value problems are expressed in terms of these key functions and in turn the solutions of more general problems in two and three dimensions are then expressed in terms of the solutions of the basic problems. (Received March 1, 1948.)

321. M. Z. Krzywoblocki: *On the so-called lost solutions in adiabatic potential flow equations.* Preliminary report.

The linearization of the quasi-linear equation of an adiabatic potential flow by means of the Molenbrock-Chaplygin transformation permitted to find the well known "boundary lines" of adiabatic flow beyond which the flow has no physical meaning. At the boundary line the adiabatic flow goes from one sheet of the flow plane to another sheet according to the way the Riemannian surface repeatedly overlaps. But there exists another way of the linearization of the mentioned quasi-linear equation, namely, by the application of a generalized Minkowski's function. In the present paper the author tries to find the lines analogous to boundary lines in Molenbrock-Chaplygin transformation. (Received March 4, 1948.)

Logic and Foundations

322. A. H. Copeland: *Implicative Boolean algebra.*

In conventional logic implication is a relation which holds between certain propositions and not between others, whereas in implicative Boolean algebra implication is an operation by which any pair of propositions can be combined to form a new proposition which may be regarded as either true or false. Implication is defined in terms of an associative but noncommutative operation called the cross product. This product admits left-hand cancellation and is distributive with respect to both conjunction and disjunction. It is interpreted in terms of certain transformations in the Boolean algebra and implication is interpreted in terms of remainder classes with respect to ideals. Not every Boolean algebra can be implicative but two examples are given, one in which the elements are increasing sequences of natural numbers and the other in which the elements are Lebesgue measurable sets in the unit interval. (Received March 8, 1948.)

Statistics and Probability

323. Benjamin Epstein: *A modified extreme value problem.*

There may arise situations where one is interested in knowing the distribution of smallest, largest, or more generally the nth smallest or nth largest value in samples taken from a population whose distribution is described by some probability law, and where the sample size is itself a random variable following some other specified probability law. In this paper the problem is solved for the case where the sample size is a random variable following the law of Poisson. The relationship between these results and the known results for the case of fixed sample size is considered and a practical example is given. (Received March 1, 1948.)

324t. M. S. Macphail and P. S. Dwyer: *Symbolic matrix derivatives.*

Let X be the matrix $[x_{mn}]$, t a scalar, and let $\partial X/\partial t$, $\partial t/\partial X$ denote the matrices $[\partial x_{mn}/\partial t]$, $[\partial t/\partial x_{mn}]$ respectively. Let $Y=[y_{pq}]$ be any matrix product involving X, X' and independent matrices, for example, $Y=AXBX'C$. Consider the matrix derivatives $\partial Y/\partial x_{mn}$, $\partial y_{pq}/\partial X$. Our purpose is to devise a systematic method for calculating these derivatives. Thus if $Y=AX$, we find that $\partial Y/\partial x_{mn}=AJ_{mn}$, $\partial y_{pq}/\partial X=A'K_{pq}$, where J_{mn} is a matrix of the same dimensions as X, with all elements zero except for a unit in the mth row and nth column, and K_{pq} is similarly defined, with the same dimensions as Y. We consider also the derivatives of sums, differences, powers, the inverse matrix and the function of a function, thus setting up a matrix analogue of elementary differential calculus. This is designed for application to statistics, and gives a concise and suggestive method for treating such topics as multiple regression and canonical correlation. (Received March 15, 1948.)

TOPOLOGY

325. Edwin Hewitt: *A class of topological spaces.*

A topological space X is said to be a K-space if it has a one-to-one continuous image which is a bicompact Hausdorff space. Let $\mathfrak{Z}(X)$ denote the family of all sets $Z(f)$ in X, where f is a continuous real-valued function on X and $Z(f)$ is the set of points in X where f vanishes. It is proved that under certain restrictions, a completely regular space X is a K-space if and only if there is a subfamily $\mathcal{A}$ of $\mathfrak{Z}(X)$ such that (1) $\prod_{F\in\mathcal{A}} F=0$, (2) any subfamily $\mathcal{B}$ of $\mathcal{A}$ with the finite intersection property has total intersection nonvoid. (Received March 8, 1948.)

326t. Edwin Hewitt: *Certain special function rings.*

Certain common rings of functions are examined from the point of view of their being isomorphic to rings of continuous real-valued functions on completely regular spaces. It is shown that the following rings cannot be rings of all continuous real-valued functions on any topological space: (1) the ring of all measurable real functions of a real variable; (2) the ring of all measurable real functions of a real variable modulo the ideal of all functions vanishing almost everywhere; (3) the rings generated by upper (lower) semicontinuous real functions of a real variable. It is shown that the following rings are isomorphic to rings of all continuous real-valued functions on appropriate spaces: (1) the ring of all real functions of a real variable, f, such that as $x\rightarrow x_0+$, $\lim f(x)=f(x_0)$ for all real numbers x_0; (2) the ring of all continuous real functions of a real variable which are bounded on the right (left) half-line. (Received March 2, 1948.)

327t. E. E. Moise: *A theorem on monotone interior transformations.*

B. Knaster has raised the question whether there is a compact metric continuum M, irreducible between two of its points, and a monotone interior transformation T throwing M into an arc, such that for each x of $T(M)$, $T^{-1}(M)$ is an arc. In the present note, this question is answered in the negative. (Received April 16, 1948.)

328*t*. H. M. Schaerf: *Regular measures.*

A measure defined on a σ-field of subsets of a topological space is called *regular* if its value on any measurable set E is the greatest lower bound of its values on open measurable sets containing E. A general *regularity criterion* is proved which implies that the following measures are regular: (a) every Radon measure in a locally compact topological space, (b) every measure defined on the σ-field of the Borel sets of a metric space which is the union of a sequence of open sets of finite measure. Theorem (a) generalizes Halmos' recent result that every Haar measure is regular (reported at the Conference on Topological Groups in Chicago, January 22–24, 1948). (Received March 5, 1948.)

329*t*. H. M. Schaerf: *Generalization of Lusin's theorem to regular measures.*

The following generalization of Lusin's theorem is proved: Let m be a measure defined and regular on a σ-field of subsets of a neighborhood space R which is the union of a sequence of measurable sets of finite measure. Let $f(x)$ be a mapping of a measurable set $E \subset R$ into a topological space T satisfying the second axiom of countability. If the image of any open set of T under f^{-1} is measurable, then for every positive number ϵ there is a measurable closed subset F_ϵ of E such that $m(E-F_\epsilon)<\epsilon$ and such that $f(x)$ is continuous on F. The scope of this generalization is apparent from the above abstract *Regular measures.* (Received March 5, 1948.)

330*t*. G. S. Young. *On continuous curves irreducible about compact sets.*

In partial answer to a question raised by Zippin (Fund. Math. vol. 20 (1933) p. 197), it is proved that in a connected and locally arcwise connected space every compact finite-dimensional set whose components are all locally connected, and whose nondegenerate components form a null sequence, lies in a compact continuous curve irreducible about it. (Received March 12, 1948.)

331. G. S. Young. *On product manifolds and fiberings.*

It is proved that if an n-dimensional topological manifold, not necessarily compact and possibly with boundary, is the product of two spaces A and B, and if dim $A=1$, 2, then A is itself a manifold. If a connected and locally connected space is a fiber bundle with fiber F, then all components of F are homeomorphic, and the fibering can be factored into a monotone fibering followed by a light fibering. Among other results, it is proved that there is no compact fibering of E^3 and E^4. Many of the methods used are from recent work of Borsuk. (Received March 12, 1948.)

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