# ON A NOTE OF GALBRAITH AND GREEN

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In a recent note [1][1] Galbraith and Green evaluate the integral

$$I_n = \frac{1}{\pi}\int_0^{\pi}\left[\frac{1+r^2}{1-r^2} - \frac{2r}{1-r^2}\cos\theta\right]^{-n-1} d\theta.$$

This may be identified with Laplace's second integral for the Legendre function. Thus if $\arg\,[(1+r^2)/(1-r^2)] < \pi/2$, by [2, §15.23],

$$I_n = P_n([1+r^2]/[1-r^2]). \tag{1}$$

By[2] [2, §§14.51, 15.22] this may be written

$$I_n = [\Gamma(-2n-1)/\Gamma^2(-n)](1/r^2-1)^{n+1}F(n+1,\ n+1;\ 2n+2;\ 1-1/r^2)$$
$$+ [\Gamma(2n+1)/\Gamma^2(n+1)](1/r^2-1)^{-n}F(-n,\ -n;\ -2n;\ 1-1/r^2).$$

From this it is readily seen that

$$\lim_{r\to 1^-} (1-r)^n I_n = 2^{-n}\Gamma(2n+1)/\Gamma^2(n+1) \quad \text{when } R(n) > -1/2, \tag{2}$$

$$\lim_{r\to 1^-} (1-r)^{-n-1} I_n = 2^{n+1}\Gamma(-2n-1)/\Gamma^2(-n) \quad \text{when } R(n) < -1/2. \tag{3}$$

The slight difference between (2) and Theorem 2 of Galbraith and Green's paper is due to an error in the latter. (3) does not appear in that paper.

## Bibliography

1. A. S. Galbraith and J. W. Green, *A note on the mean value of the Poisson kernel*, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 314–320.

2. Whittaker and Watson, *Modern analysis*, Cambridge University Press, 1935.

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[1] Numbers in brackets refer to the bibliography at the end of the paper.

[2] An error in §14.51 of the reference may be corrected by changing the signs of the arguments of the gamma functions of binomial argument in the formula at the top of p. 289.