condition $\sum_{k=1}^{n} a_k = 0$. But $x_2 - x_1$ shows that this condition is not sufficient. Pólya remarked that if (25) has infinitely many solutions we can not have $a_1 \geqq 0$, $a_1 + a_2 \geqq 0, \cdots, a_1 + a_2 + \cdots + a_n \geqq 0$. The characterization of the forms which satisfy (26) seems a difficult problem.

Finally we mention two more questions:

(1) Can the inequalities $p_{n+1} - p_n < p_{n+2} - p_{n+1} < \cdots < p_{n+k} - p_{n+k-1}$ have infinitely many solutions for every fixed k?

(2) Is it true that the number of solutions of $p_{k+1} - p_k > p_k - p_{k-1}$, $k \leqq n$ is $n/2 + o(n)$? As we already have stated we can show that the number of solutions in question is between $c_1 n$ and $(1 - c_1)n$.

SYRACUSE UNIVERSITY AND
UNIVERSITY OF BUDAPEST

ON MERSENNE'S NUMBER M_{227} AND COGNATE DATA

H. S. UHLER

When p equals one of the 55 primes 2, 3, 5, $\cdots$, 257 then, strictly speaking, $M_p = 2^p - 1$ is called a Mersenne number. To obtain a clear perspective of the history of this special subject the reader may consult the interesting accurate paper by R. C. Archibald.[1] Without any superior value of p, it has been shown by E. Lucas that the prime or composite character of a number of the form $2^p - 1$ (p prime) may be investigated by employing the sequence 3, 7, 47, 2207, $\cdots$ when p is of the form $4n - 1$, and the sequence 4, 14, 194, 37634, $\cdots$ when $p = 4n + 1$. In both cases the law of formation of the terms is $s_k = s_{k-1}^2 - 2$. However, it is no longer necessary to use the $4n - 1$ Lucasian series since D. H. Lehmer[2] stated and proved the following theorem: "*The number* $N = 2^n - 1$, *where n is an odd prime, is a prime if, and only if, N divides the* $(n-1)$*st term of the series*

$$S_1 = 4, \ S_2 = 14, \ S_3 = 194, \cdots, S_k, \cdots,$$

where $S_k = S_{k-1}^2 - 2$." This justifies the use by the present writer of the second progression although 227 falls in the $4n - 1$ class.

Received by the editors July 7, 1947.

[1] R. C. Archibald, *Mersenne's numbers*, Scripta Mathematica vol. 3 (1935) pp. 112–119.

[2] D. H. Lehmer, *On Lucas's test for the primality of Mersenne's numbers*, J. London Math. Soc. vol. 9–10 (1934–1935) pp. 162–165.

On June 4th, 1947 the writer finished calculating the 226th remainder of the Lucasian sequence 4, 14, 194, $\cdots$ as applied to the 69-digit number $2^{227}-1=2156$ 79573 33720 51183 57336 12069 61570 45389 09715 53803 24579 84882 88819 93727. The result was $r_{226}=1071$ 24133 67508 18344 33653 76892 98433 16050 93930 31886 49512 37078 23311 35950. Since this residual is not zero and since the calculations were performed with extreme care it follows that M_{227} is composite.

During the entire course of the work each arithmetical operation was checked with the auxiliary moduli 10^5+1 and 10^8+1. After the date given above all of the work-strips of the whole set were again examined and checked with the modulus 10^6+1, with a different computing machine, and in conformity with the formula of succession $r_{k-1}^2=M_{227}q_k+r_k+2$.

In addition to the theoretical probability of error indicated by the product of the three independent moduli 10^5+1, 10^6+1 and 10^8+1, the reliability of the writer's method and work has been subjected to an a posteriori acid test in each of two instances. In a letter dated September 12th, 1946, Professor Lehmer informed me that he had discovered[3] ("this July 4th weekend") the factor 2349023 for M_{167} and the factor 1504073 for M_{229}. He then wrote: "It might be interesting now to try to verify your . . . final residue in each of these tests by computing Lucas' series modulis 2349023 and 1504073 and then see if the results are the same as those obtained from casting these numbers out of your final remainders." This friendly suggestion was followed by me with the results that the two 166th residues were congruent to 2160517 (mod 2349023), and the two 228th remainders were identical with the value 465373. The heretofore intentionally unpublished value of r_{166} is 59077 89471 97183 05021 04043 18653 76339 69475 17591 49076.

As explained in an earlier paper[4] the essential figures of each of the terms above the 8th of the specified Lucasian sequence were multiplied in order by the reciprocal of the chief modulus, M_{227}, in preference to direct division by M_{227}. The approximation to this reciprocal was computed to be $(1/M_{227})_a=0.$(68 zeros) 46365 07688 35927 67321 64669 07693 45493 91709 45597 34472 38753 06629 88236 46998 17232 30809 13430 64583 44938 95187 64723 82742 71 $\cdots$. These figures were derived at once from the writer's earlier trustworthy

[3] D. H. Lehmer, *On the factors of* $2^n\pm 1$, Bull. Amer. Math. Soc. vol. 53 (1947) pp. 164–167.

[4] H. S. Uhler, *First proof that the Mersenne number* M_{157} *is composite*, Proc. Nat. Acad. Sci. U. S. A. vol. 30 (1944) pp. 314–316.

value of $(1/M_{229})_a$ and the obvious relation $M_{227}^{-1}=4M_{229}^{-1}+12M_{229}^{-2}$ $+\cdots$. Because $(1/M_{227})_a$ was to be used 218 times as multiplier, the value just given was verified twice by multiplication by the exact value of M_{227}, once using octad segregation of the digits and again with nonad grouping. It was proved that the last figure (1) recorded for $(1/M_{227})_a$ is too small by about 0.04 unit.

For future investigations it may be appropriate to record in this place the value of the tenth term of the sequence 4, 14, 194, $\cdots$ as carefully computed by the author. $s_{10}=s_9^2-2=$ 687 29682 40664 42772 38837 48623 17475 30924 24715 41086 46671 75219 26185 83088 48740 57909 57964 73288 30691 02561 04343 67796 63935 59517 20423 57306 59491 63446 06074 56471 28680 78287 60805 52030 24658 35943 90175 80883 91097 86661 85875 71741 55410 84494 92650 04751 67381 16850 59273 78181 89975 38392 60609 45226 53652 74850 90187 98812 03714. This term has 293 digits and it would be applicable to all odd primes less than and inclusive of $p=971$ in $2^p-1=M_p$. If the speed of present or future electronic computing machines should cause the operators to run out of problems it might be worth while to apply one of the machines to testing the character of M_{971} where $M_{971}+1$ $=2^{971}=$ 199 58403 09534 71981 16563 72713 03683 85660 67451 26043 54575 41502 54724 24372 11891 86896 40657 84957 96549 26357 01089 34244 68441 92495 24397 24379 88393 59366 07391 71798 28483 14203 20005 67295 10856 76517 53772 14443 62987 18265 33567 44543 92399 33308 10455 12087 03888 88855 26844 80441 57507 12090 68757 56041 64235 84952 30344 00992 78848. The accuracy of this power of 2 may be inferred from the following quotation of a sentence in a very recent letter from Dr. John W. Wrench, Jr. "Your value of 2^{971} has been collated with mine on several occasions in the past two days, and agreement is perfect."

The writer is now engaged in applying the sequence 4, 14, 194, $\cdots$ to the investigation of the sole remaining doubtful M_p within Mersenne's range of surmise ($p<263$), namely $2^{193}-1$.

YALE UNIVERSITY