ADDITION TO MY NOTE ON SEMI-SIMPLE RINGS

OSCAR GOLDMAN

In my Bulletin note on semi-simple rings[1] I made use of the following definition of the radical of a ring which I attributed to C. Chevalley: "The radical of a ring A is the intersection of the annihilators of all simple A-modules." Recently N. Jacobson has called my attention to the fact that the radical thus defined coincides with the one considered by him,[2] and that Chevalley's statement can easily be shown to be equivalent with the following characterization of the radical by Jacobson:[2,3] "If A is not a radical ring, then the radical of A is the intersection of all the primitive ideals contained in A."

To see the relation between the two statements, we need to make use of Jacobson's characterization of a primitive ideal as a proper ideal B such that the factor ring A/B is isomorphic with a simple ring of endomorphisms. From this it is clear that B is primitive if, and only if, B is proper and is the annihilator of a simple A-module. If the word "proper" is dropped from the definition of a primitive ideal, then the concept of primitive ideal is equivalent to that of annihilator of a simple A-module. Hence Chevalley's definition is essentially the same as Jacobson's characterization.[4]

PRINCETON UNIVERSITY

Received by the editors March 3, 1947.

[1] *A characterization of semi-simple rings*, Bull. Amer. Math. Soc. vol. 52 (1946) p. 1021.

[2] N. Jacobson, *The radical and semi-simplicity for arbitrary rings*, Amer. J. Math. vol. 67 (1945) p. 301.

[3] N. Jacobson, *A topology for the set of primitive ideals in an arbitrary ring*, Proc. Nat. Acad. Sci. U.S.A. vol. 31 (1945) p. 333.

[4] It is to be noted that, as a result of this equivalence, Theorems I and II of my note become superfluous. See Theorems V, IX, and XXV in footnote 2 above.