336. H. E. Salzer: *Further remarks on the approximation of numbers as sums of reciprocals.*

The present work consists of three main parts. (1) Comparison of R-expansions with simple continued fractions for rational numbers a/b leads to the analogue of the Euclidean algorithm, with a multiple of the g.c.d. of a and b in place of the g.c.d. One practical result is that for a/b, as a rule, fewer partial fractions are required in the R-expansion than in the s.c.f. (2) Proof that if p/q is an approximation to x obtained by the R-expansion, then the remainder $(x-p/q)<1/q$. (3) Proof that if p/q is an approximation to x obtained by the $\overline{R}$-expansion, then $|x-p/q|<1/2q$ except when $x=3/4$, when the $\leqq$ relation may hold. Both theorems are best possible ones. It is shown that when in an R- or $\overline{R}$-expansion the denominator of the $(n+1)$th partial fraction is about k times the minimum value that could arise in an R- or $\overline{R}$-expansion, then not only the closeness of the nth, but of all ensuing approximations p/q will be about $1/kq$ or $1/2kq$ respectively. A critical bibliography is provided, which reviews all work by other authors on R- or $\overline{R}$-expansions. (Received June 18, 1947.)

337. H. E. Salzer: *Polynomials of best approximation in an infinite interval.* Preliminary report.

Chebyshev polynomials $C_n(x)$ are useful for approximating polynomials of high degree in a finite interval $[a, b]$, by polynomials of much lower degree, because of the property that of all polynomials with leading coefficient 1, $C_n(x)$ has the least value of the greatest deviation from 0 in the interval $[-1, 1]$. To approximate functions of the form $e^{-x}p(x)$ and $e^{-x^2}q(x)$, where $p(x)$, $q(x)$ are polynomials, over $[0, \infty]$ and $[-\infty, \infty]$ respectively, it is useful to know: (I) polynomials $P_n(x)$, degree n, leading coefficient 1, such that the greatest absolute value of $e^{-x}P_n(x)$ differs least from 0 in $[0, \infty]$; (II) polynomials $Q_n(x)$, degree n, leading coefficient 1, such that the greatest absolute value of $e^{-x^2}Q_n(x)$ differs least from 0 in $[-\infty, \infty]$. $P_n(x)\equiv x^n+a_{n-1}x^{n-1}+\cdots+a_0$ satisfies $2n$ transcendental equations $P_n'(x_i)=P_n(x_i)$, $e^{-x_i}P_n(x_i)=(-1)^i a_0$, $i=1, \cdots, n$, where x_i are the abscissae of the extrema of $e^{-x}P_n(x)$. $Q_n(x)\equiv x^n+b_{n-1}x^{n-1}+\cdots+b_0$ satisfies $2n+1$ equations $Q_n'(x_i)=2x_iQ_n(x_i)$, $e^{-x_i^2}Q_n(x_i)=(-1)^{i-1}e^{-x_1^2}Q_n(x_1)$, $i=1, \cdots, n, n+1$. When suitably normalized, $P_n(x)$ and $Q_n(x)$ are characterized by being tangent alternately to $\pm e^x$ and $\pm e^{x^2}$ respectively. $Q_n(x)\equiv P_{n/2}(x^2)$ for n even, and is an odd function for n odd. (Received July 11, 1947.)

Geometry

338. W. R. Utz: *The properties of geodesics on certain n-dimensional manifolds.* Preliminary report.

Let S denote the interior of an $(n-1)$-dimensional unit sphere in Euclidean n-space and let G denote a properly discontinuous group of homeomorphisms of S onto itself that preserve the hyperbolic metric $\int(dx_idx_i)^{1/2}/(1-x_ix_i)$. The action of this group is discussed in a manner similar to that of Poincaré (*Théorie des groupes Fuchsiens*, Acta Math. vol. 1 (1882) pp. 1–62) for the case $n=2$. By the identification of congruent points of S under G an n-dimensional manifold, Σ, is secured. An investigation of the geodesics on Σ leads to results concerning geodesics and hyperbolic lines of the same type, asymptotic geodesics and limit geodesics analogous to certain results of Morse (*A fundamental class of geodesics on any closed surface of genus greater than one*, Trans. Amer. Math. Soc. vol. 26 (1924) pp. 25–60)) in the case of certain two-dimensional manifolds. (Received May 24, 1947.)