fraction $K_1^\infty(1/b_p)$ is that at least one of the following three statements holds: (a) $\sum|b_{2p+1}|$ diverges, (b) $\sum|b_{2p+1}s_p^2|$ diverges, where $s_p=b_2+b_4+\cdots+b_{2p}$, (c) $\lim_{p\to\infty}s_p=\infty$. This condition, which first appeared in a theorem of Hamburger, is called condition (H). In particular, they show that the continued fraction diverges if $\sum b_{2p}$, $\sum b_{2p+1}$ converge, at least one absolutely, thus extending a result of Stern and von Koch. The condition (H) is sufficient for convergence in the case where $b_{2p-1}=k_{2p-1}z_p$, $b_{2p}=k_{2p}$, $k_1>0$, $k_{2p+1}\geqq 0$, $R(k_{2p})\geqq 0$, $R(z_p)\geqq\delta$, $|z_p|<M$ ($\delta>0$, $M>0$, $p=1, 2, \cdots$). This result includes theorems of Stieltjes, Van Vleck, Hamburger and Mall. (Received August 19, 1946.)

375. I. E. Segal: *The group algebra of a locally compact group.*

Earlier results of the author (see Bull. Amer. Math. Soc. Abstract 46-7-366 and Proc. Nat. Acad. Sci. U.S.A. vol. 27 (1940) pp. 348–352) are extended and refined. (Received August 10, 1946.)

376. J. E. Wilkins: *The converse of a theorem of Tchaplygin on differential inequalities.*

If $y(x)$ is a solution of the equation $L[y]\equiv y''-p_1y'-p_2y-q=0$, such that $y(x_0)=y_0$, $y'(x_0)=y_0'$, and if $v(x)$ is such that $L[v]>0$, $v(x_0)=y_0$, $v'(x_0)=y_0'$, then $v(x)>y(x)$ when $x_0<x\leqq x_1$ provided that x_1 is the first zero to the right of x_0 of the solution $u(x)$ of the equations $u''-p_1u'-p_2u=0$, $u(x_0)=0$, $u'(x_0)=1$. This is a best possible result in the sense that either x_1 does not exist or there exists a function $v(x)$ satisfying the above requirements for which $v(x)-y(x)$ vanishes at a point arbitrarily close to x_1. (Received September 27, 1946.)

Applied Mathematics

377. H. W. Becker: *Circuit algebra.*

By means of the symbols $+$, $\|$, and $\perp$, any passive electrical network is representable on the linotype. They denote series, parallel, and bridge connections respectively, and have inverses $-$, $\_$, and $\top$. The definition $R=a\|b=ab/(a+b)$ generates a system parallel to ordinary arithmetic, except that infinity and zero exchange roles, and so on, hence called paraarithmetic. The number of integer solutions of this equation depends only on the prime factor structure, not magnitude, of R. If $R=p_1^{n_1}\cdots p_m^{n_m}$, this number is $\Psi_m(C+1)$, where $C_0=1$, $C_v=2^{v-1}$, and Ψ is an expansion with generalized binomial coefficients (m, v), the sum of the products of the n's v at a time. Where all the n's equal 1, this reduces to $(3^m+1)/2$. Considered as a static structure, a SP network is collapsible. Rigidity is imparted by bridge connections, or trusses. The elementary model is the Wheatstone bridge $(a_1+a_2)\|(b_1+b_2)\perp\beta=[(a_1+a_2)\cdot(b_1+b_2)\beta+a_1a_2(b_1+b_2)+(a_1+a_2)b_1b_2]/[(a_1+a_2+b_1+b_2)\beta+(a_1+b_1)(a_2+b_2)]$. In this algebra, shorting or opening an operand effects remarkable transformations amongst the operators; as, $(a_1+a_2)\|(b_1+b_2)\perp 0=a_1\|b_1+a_2\|b_2$. In general, $\perp$ connections are specified by subscripts at the pluses involved. Thus the network whose components are the edges of a cubic lattice energized at two opposite vertices is $[(a_1+_1a_2)\|(b_1+_2b_2)+a_3]\|[c_1+(c_2+_1c_3)\|(d_1+_2d_2)]\perp\beta_1\perp\beta_2$. By a method of combinatory synthesis, the total and transfer conductances are then formulated, alternatively to the Kirchhoff method. (Received September 9, 1946.)

378. Herman Chernoff: *A note on the inversion of power series.*

The author presents a method of calculating the coefficients of the inverse of a given power series. This method has the advantage of being compact, requiring only one page for calculations, of being systematic, and of not requiring the substitution in complicated formulae. In fact the only operations are cumulative multiplications and only occasional divisions. The method can be extended to many problems of calculations in power series. (Received August 10, 1946.)

379. R. E. Gaskell: *An extension of the finite Fourier transformation.*

Applications of the finite Fourier transformation (see R. V. Churchill, *Modern operational mathematics in engineering*, p. 267) are limited to problems involving special linear differential operators, $\sum a_i \partial^{2i}/\partial x^{2i}$, where the coefficients, a_i, do not involve the independent variable, x. Further, the boundary conditions used must be of a special form. A more general transformation which can be used in problems involving any linear differential operator, L, of order r, is introduced. The transformation is defined by $T\{F\} \equiv \int_a^b \psi_n F dt$, where ψ_n is a solution of the equation $\overline{L}(\psi) = \lambda_n^2 \psi$, involving the adjoint operator. The transformation can be inverted with the help of ϕ_n, a solution of $L(\phi) = \lambda_n^2 \phi$. The homogeneous end conditions $\Phi_i = 0$ for ϕ_n are taken from the boundary conditions of the problem, those for ψ_n are determined so that the bilinear concomitant has the special form $\sum \Phi_i \Psi_{2r-i+1}$. An elementary example requiring the steady state temperature in a thick-walled cylindrical tube of finite length, with radiation at one end, serves to illustrate the more general transformation. (Received September 3, 1946.)

380. H. E. Goheen: *A bound for the error in computing the Bessel functions of the first kind by recurrence.*

A method is determined for obtaining a close upper bound to the absolute value of the error in the computation of $J_n(x)$, the Bessel function of the first kind, the method of computation assumed being the obvious one of using the recurrence relation on approximate values of $J_0(x)$ and $J_1(x)$. It is shown that while in general the absolute value of the error increases without limit, it is less than an easily determined function of n, x and an upper bound to the absolute values of the errors in $J_0(x)$ and $J_1(x)$. (Received September 16, 1946.)

381. Harry Polachek and R. J. Seeger: *On the existence of solutions for three-shock Prandtl-Meyer configurations in the case of weak reflected shocks.*

Simple analytical expressions are obtained for three-shock Prandtl-Meyer configurations for the case where the pressure ratio across the reflected shock ξ' approaches unity. For the presence of Prandtl-Meyer waves in the regions between (a) the reflected shock and the density discontinuity and (b) the incident and reflected shocks, these expressions are identical. By means of this expression, a theorem concerning the existence of Prandtl-Meyer waves in the above regions is derived. The ω-ξ plane (defined as the angle between the direction of material flow and the incident shock, and the pressure ratio across the incident shock, respectively) is divided into four regions by the curves $f=0$ and $g-f=0$, where g and f are known functions of ω and ξ. In one of these regions no solutions are possible, while two of these regions contain solutions only for case (a) above, and one region contains solutions only for case (b). (Received September 16, 1946.)

382. R. C. Roberts: *On the lift of a triangular wing at supersonic speeds.*

In a recent paper (Quarterly of Applied Mathematics vol. 4 (1946)) H. J. Stewart has found the lift on a flat plate wing, where the plan form of the wing is an isosceles triangle whose base is perpendicular to the free stream flow, and the wing lies entirely within the Mach cone through the vertex opposite to the base. In the present paper, the author has extended the method to the case where the plan form of the wing is any triangle under similar conditions. The result obtained shows that the slope $dC_L/d\alpha$ of the lift coefficient curve is $\{2\pi k/[E(k')(M^2-1)^{1/2}\}\{(k_1+k_2)/2k\}^{1/2}$. In this formula, M is the Mach number of the free stream, $E(k')$ is the complete elliptic integral of the second kind with modulus $k'=(1-k^2)^{1/2}$, k_1 and k_2 are related to the angles ω_1, ω_2 between the perpendicular to the base and the sides of the triangle by the formulae $k_1=(M^2-1)^{1/2}\tan\omega_1$, $k_2=(M^2-1)^{1/2}\tan\omega_2$, and $k=[1+k_1k_2-((1-k_1^2)(1-k_2^2))^{1/2}]/(k_1+k_2)$. When $k_1=k_2$ the result reduces to the formula $dC_L/d\alpha=2\pi\tan\omega_1/E(k')$, obtained by Stewart. (Received September 18, 1946.)

383. H. E. Salzer: *An alternative definition of reciprocal differences.*

In place of Thiele's reciprocal difference $\rho_n(x_1\cdots x_nx_{n+1})$ which has the advantage of symmetry in the x_i's, a new definition for the nth reciprocal difference is employed, namely $P_n(x_{n+1}x_n\cdots x_1)\equiv\rho_n(x_1\cdots x_nx_{n+1})-\rho_{n-2}(x_1\cdots x_{n-2}x_{n-1})$. Although nonsymmetric in the x_i's, $P_n(x_{n+1}x_n\cdots x_1)$ is generated by a simpler recursion formula than the $\rho_n(x_1\cdots x_nx_{n+1})$ and enables Thiele's continued fraction interpolation formula to be expressed in a more concise and natural form. Both these properties lead to a saving of numerical work. The proof that the reciprocal difference (new definition) of a certain order of a rational function is constant is shown to be more direct than the proof of the same statement for Thiele's reciprocal differences. This new definition is also applied to the completely confluent case, where all the x_i's approach x_1. (Received September 7, 1946.)

384. H. E. Salzer: *Tables for facilitating the use of Chebyshev's quadrature formula.*

Chebyshev's n-point quadrature formula $\int_{-1}^{+1}f(z)dz=(2/n)\sum_{i=1}^{n}f(z_i)+R_n$, where z_i are the zeros of $T_n(z)\equiv$polynomial part of $z^n\exp(-n/2\cdot 3z^2-n/4\cdot 5z^4-\cdots)$, has the advantage of being equally weighted, but the disadvantage that the z_i's are irregularly spaced. For a partial check on all the $f(z_i)$ taken together, coefficients $D_i^{(n)}$ are given to obtain the $(n-1)$th divided difference as $\sum_{i=1}^{n}D_i^{(n)}f(z_i)$, for $n=3(1)7$, 9, $i=1, 2, \cdots, n$, mostly to $9S$. The calculations on $D_i^{(n)}$ were checked by the relations $D_i^{(n)}=D_{n-i+1}^{(n)}$, n odd, and $D_i^{(n)}=-D_{n-i+1}^{(n)}$, n even, and by use in examples. Use of the checking coefficients $D_i^{(n)}$, especially for the larger n's, is more likely to detect an error nearer to the central $f(z_i)$. Also for the same range of n and i, the zeros of the polynomials $T_n(z)$ are given to $10D$, extending all previous calculations. Since Chebyshev's formula is of practical use only when the zeros of $T_n(z)$ are all real, an investigation showed that $T_n(z)$ does not have all real roots for $n=11$ through 15 ($n=8$ and 10 already noted in the literature) and raises the interesting question whether $T_n(z)$ has all real roots for any $n>15$. The exact expressions for $T_n(z)$ are given for $n=1(1)12$. (Received September 12, 1946.)

385. S. A. Schaaf: *A cylinder cooling problem.*

The temperature distribution is obtained for a heat conducting region consisting of an infinitely long cylinder $0 \leqq r < a$ initially at temperature T_0, immersed in an infinite medium $r > a$ composed of a different substance initially at zero temperature, with a contact resistance condition at the interface $r = a$. The Laplace transform is used and, in inverting, it is necessary to show that $D(z) = I_0'(\alpha z)K_0(\beta z) - \lambda I_0(\alpha z)K_0'(\beta z) - \mu z I_0'(\alpha z)K_0'(\beta z)$, where $I_0(z)$ and $K_0(z)$ are Bessel functions and α, β, λ and μ are positive real numbers, does not vanish for $|\arg z| \leqq \pi/2$. This is done by considering the integral of $D'(z)/D(z)$ around a contour consisting of two semicircular arcs $|z| = R_1$, R_2 and the segments of the imaginary axis joining them. (Received September 26, 1946.)

386. Fred Supnick: *Cooperative phenomena.* I. *Structure of the linear Ising model.*

The partition function $f(T)$ (the physical term) plays an important part in the theory of crystal statistics (cf. C. H. Wannier, Review of Modern Physics (1945) pp. 50–60). Let the set of spins $u_1, \cdots, u_n$ each capable of two orientations be characterized by $u_i = +1$ or $u_i = -1$, and arranged in cyclic order. It is assumed that only adjacent elements interact. To evaluate $f(T)$ the interaction energy E must be found. E involves the calculation of $\Sigma \equiv \sum_{i=1}^{n} u_i u_{i+1}$ where $u_{n+1} \equiv u_1$. All spin distributions are considered in evaluating $f(T)$. The author calls Σ the interaction constant of the spin distribution. Now, let Σ_i be any integer with $|\Sigma_i| \leqq n$. In this paper the set of all possible spin distributions with interaction constants equal to Σ_i is determined. A method is given for constructing each spin distribution with $\Sigma = \Sigma_i$. Results involving the number of spin distributions with the same interaction constant are obtained. Both cyclic and non-cyclic cases are considered. (Received September 28, 1946.)

GEOMETRY

387. Germán Ancochea: *Zariski's proof of the theorem of Bertini-Enriques in the case of an arbitrary ground field.*

Zariski (Trans. Amer. Math. Soc. vol. 50 (1941)) gave a new proof of the theorem of Bertini-Enriques on reducible linear systems of V_{r-1}'s on an algebraic V_r, by considering this theorem in a larger sense than the customary since irrational pencils are also included. The proof, given for the case of ground fields of characteristic zero, is based on several lemmas concerning the behavior, with respect to irreducibility, of an algebraic variety under ground field extensions. Most of these lemmas have been extended by Chevalley to the case of arbitrary ground fields (Trans. Amer. Math. Soc. vol. 55 (1944)). In the present paper the theorem of Bertini-Enriques, in the sense of Zariski, is extended to ground fields of characteristic $p \neq 0$. The auxiliary lemmas are reconsidered from a different standpoint than that of Chevalley, by using the Chow-van der Waerden concept of an associated form of an algebraic variety. It also has been found necessary to incorporate in Zariski's definition of an irreducible pencil on V_r the extra requirement that the field of functions on V be separably generated over the ground field. With these changes the theorem of Bertini-Enriques is proved essentially as in Zariski's paper, provided that the ground field be an infinite field for the case of linear systems. (Received August 2, 1946.)