shown that, in an important sense, the only admissible measure of fluctuation, except for a constant factor, is the variance $f(a_1, \cdots, a_n) = \sum_{i=1}^{n}(a_i - n^{-1}\sum_{j=1}^{n} a_j)^2$. The method of maximum likelihood is applied to special functions to obtain tests for significant differences. These tests have applications to industrial problems. (Received March 22, 1946.)

198. Isaac Opatowski: *Average duration of transition in Markoff chains.*

Consider a chain of transitions $(i \to i+1)$, $(i+1 \to i)$, where $i = 0, 1, \cdots, n-1$. Let the usual conditional probabilities of these transitions within any time Δt be respectively $k_{i+1}\Delta t + o(\Delta t)$ and $g_i \Delta t + o(\Delta t)$, where the k_i's and g_i's are constant. Let the probability of any other transition during Δt be $o(\Delta t)$. The probability $P(t)$ of a transition $(0 \to n)$ within a time t is an increasing function of t and $P(\infty) = \prod_{i=1}^{i=n} k_i/K_i$, where $s = -K_i$ are the n roots of the secular equation $\|a_{r,c}\|/s = 0$ defined by $a_{r,r} = s + k_{r+1} + g_{r-1}$, $a_{r,r-1} = k_r$, $a_{r,r+1} = g_r$, $a_{r,c} = 0$ for $|r-c| > 1$; $r, c = 0, 1, \cdots, n$; $g_{-1} = 0$ (Bulletin of Mathematical Biophysics vol. 7 (1945) pp. 170–177). If $g_{n-1} = 0$ then $P(\infty) = 1$, if $g_{n-1} \neq 0$ then $P(\infty) < 1$. Consequently, the average duration of the transition $(0 \to n)$ is $E = \int_0^{P(\infty)} t dP/P(\infty)$. Its explicit expression is a simple symmetric function of the k_i's and K_i's. If all the g_i's are zero, $E = \sum_{i=1}^{i=n} k_i^{-1}$. By using the results of a previous paper (Proc. Nat. Acad. Sci. U.S.A. vol. 31 (1945) pp. 411–414) simple expressions of E are obtained also in the cases in which the k_i's and g_i's are independent of i. The paper is a part of an article to appear in the Bulletin of Mathematical Biophysics vol. 8 (1946). (Received March 19, 1946.)

199. Isaac Opatowski: *Markoff chains with variable intensities: average duration of transition.*

Consider a simple Markoff chain. Let $k_{i+1}\Delta t + o(\Delta t)$ be the conditional probability of a transition $(i \to i+1)$ within any time Δt, where $i = 0, 1, \cdots, n-1$ and $k_i = F(i)f(t)$. It is known that by changing t into a new time variable $T = \int_0^t f(t)dt$, the present chain may be treated as if its intensities k_i were constant and equal to $F(i)$. Let $t = \sum_m c_m T^m$ be a polynomial in T. Let $P(t)$ be the probability of a transition $(0 \to n)$ within t. It is shown that $\int_0^1 t dP$, the average duration of a transition $(0 \to n)$, equals $\sum_m m! c_m h_m$, where h_m is the complete homogeneous symmetric function of degree m of $1/F(1)$, $1/F(2), \cdots, 1/F(n)$. This formula is obtained by using a previous result on the moments of Markoff chains (Proc. Nat. Acad. Sci. U.S.A. vol. 28 (1942) pp. 83–88). The paper is a part of an article to appear in the Bulletin of Mathematical Biophysics vol. 8 (1946). (Received March 20, 1946.)

Topology

200. E. E. Floyd: *On the extensions of homeomorphisms on the interior of a two cell.*

Let $f(I) = R$ be a homeomorphism of the interior I of a two cell with boundary C onto a bounded plane region R. It is shown that if f is extensible to $\bar{I}$, then the extension is non-alternating on the boundary C. A condition is also derived which is equivalent to the existence of an extension g of f, where $g(\bar{I}) = \bar{R}$, $g = f$ on I, and g is light and non-alternating on C. This is applied to conformal maps, yielding the following theorem: let $f(I) = R$ be a 1-1, conformal map of the interior I of the unit circle onto a plane bounded region R. A necessary and sufficient condition that f be extensible to $\bar{I}$ is that the boundary of R be locally connected. If the extension exists, it is light and non-alternating on the unit circle. (Received March 5, 1946.)

201. W. H. Gottschalk: *Almost periodicity, equi-continuity and total boundedness.*

Let f: $X\times T\rightarrow X$ be a transformation group, where X is a uniform space and T a topological group. A set E in T is called ***relatively dense*** if there is a compact set A in T such that each left translate of A meets E. A point x of X is called ***almost periodic*** if to each neighborhood U of x corresponds a relatively dense set E in T such that $f(x, E)\subset U$. The transformation group f is called ***almost periodic*** if to each index α corresponds a relatively dense set E in T such that $x\in X$ implies $f(x, E)\subset U_\alpha(x)$. Let G denote $[f^t|t\in T]$, provided with the strong uniformity. The following theorems are proved: (1) If the family G is equi-continuous at x, if f_x is continuous on T, and if x is almost periodic, then the orbit of x is totally bounded. Conversely, if the family G is equi-uniformly continuous and if the orbit of x is totally bounded, then x is almost periodic. (2) If X is compact and if f is continuous on $X\times T$, then the following statements are pairwise equivalent: (i) the transformation group G is almost periodic; (ii) the family G is equi-uniformly continuous; (iii) the space G is totally bounded. Bochner's characterization of almost periodic functions and a sharpened form of the Dirichlet-Kronecker theorem are corollaries of (1) and (2), respectively. (Received March 11, 1946.)

202. D. W. Hall and J. W. T. Youngs: *Comments on the cores of certain classes of spaces.*

This paper considers a class of spaces $\mathfrak{X}$, a class of spaces $\mathfrak{Y}$ containing $\mathfrak{X}$, and a class of transformations τ each of which maps certain elements of $\mathfrak{X}$ onto certain elements of $\mathfrak{Y}$. For any subclass $\mathfrak{A}$ of $\mathfrak{X}$ the closure $\overline{\mathfrak{A}}$ of $\mathfrak{A}$ (with respect to τ) is defined as the subclass of $\mathfrak{Y}$ consisting of all possible images of spaces taken from $\mathfrak{A}$ under transformations taken from τ. Under a mild restriction on τ this closure satisfies the Kuratowski axioms for a topological space. Indeed, the stronger result holds that the closure of the union of *any* collection of subclasses of $\mathfrak{X}$ is the union of their closures. For a given subclass $\mathfrak{A}$ of $\mathfrak{X}$ the set $C(\mathfrak{A})$, called the core of $\mathfrak{A}$, denotes the largest closed subclass of $\mathfrak{A}$ with respect to τ. This paper studies the problem of characterizing the cores of certain classes of spaces under particular families of mappings. Examples are given showing that this approach provides a new setting for certain results of T. Radó, G. T. Whyburn, and R. H. Bing. (Received March 22, 1946.)

203. R. G. Helsel: *A theorem on surface area.*

J. W. T. Youngs (Ann. of Math. vol. 45 (1944) pp. 753–785) has proved that the Lebesgue area is Kerékjártó invariant in the 2-sphere case, that is, two Kerékjártó equivalent continuous mappings from a 2-sphere into 3-space determine Fréchet surfaces with equal Lebesgue area. The present paper proves the Kerékjártó invariance of both the Lebesgue area and the lower area in the 2-cell case (see Radó, Proc. Nat. Acad. Sci. U. S. A. vol. 31 (1945) pp. 102–106, for the definition of the lower area). The argument is based on recent topological results of Radó (Trans. Amer. Math. Soc. vol. 58 (1945) pp. 420–454). (Received March 13, 1946.)

204. Edwin Hewitt: *A characterization of rings of continuous functions.*

It is well known that the ring of all continuous real-valued and bounded functions on an arbitrary topological space X can be considered as a normed ring with real scalars, if $\|f\| = \sup |f(p)|$, where p runs through all points of X. The following algebraic characterization of rings of continuous functions, with this norm and the usual definitions of addition and multiplication, is established. A commutative normed ring $\mathfrak{R}$ with unit and with real scalars can be mapped onto the ring of all continuous real-valued functions defined on an appropriate bicompact Hausdorff space by a mapping which is both an isometry and an algebraic isomorphism if and only if $(x^2+e)^{-1}$ exists for all $x \in \mathfrak{R}$ and $(\|x\|e - X)^{-1}$ exists for no $x \in \mathfrak{R}$. The proof is carried out by topologizing the set of all maximal ideals in the ring R and is similar to corresponding proofs for rings with complex scalars given by Gelfand (Rec. Math. (Mat. Sbornik) N.S. vol. 9 (1941) pp. 3–23). (Received February 19, 1946.)

205. Edwin Hewitt: *A generalization of the concept of complete regularity.* Preliminary report.

A generalization of complete regularity for topological spaces is introduced by the following definition. A topological space X is said to be completely regular with respect to the topological space T if: (1) for every pair of distinct points p and q in X, there is a continuous mapping f of X into T such that $f(p) \neq f(q)$; (2) for every $p \in X$ and every neighborhood U_p, there exist a point $s \in T$, a neighborhood V_t, and a continuous mapping g of X into T such that $g(p) = s$, and $g(q) \in V_t'$ for all $q \in U_p'$. If T is the closed interval $[0, 1]$, then complete regularity with respect to T is ordinary complete regularity. It is proved that X is homeomorphic to a subset of a certain Cartesian product of spaces T if X is completely regular with respect to T. It has been proved by M. H. Stone that every T_0-space is completely regular with respect to the T_0-space containing two points and having exactly one nonvoid proper subset which is open. It is shown that this result cannot be extended to T_1-spaces: there is no T_1-space such that all T_1-spaces are completely regular with respect to it. (Received February 19, 1946.)

206. Edwin Hewitt: *An effective construction of the Stone-Čech beta for a countable discrete space.*

Stone (Trans. Amer. Math. Soc. vol. 41 (1937) pp. 461–465) and Čech (Ann. of Math. vol. 38 (1937) pp. 831–837) have proved that for every completely regular space X there exists a unique bicompact Hausdorff space βX such that X is homeomorphic to a dense subset X^* of βX and such that every bounded continuous real-valued function defined on X^* can be extended over βX so as to remain continuous. Both proofs are nonconstructive, and very few spaces βX, for non-bicompact X, have been exhibited. By standard topological methods, an effective construction is given which exhibits β for a countable discrete space as a subset of the space of all characteristic functions on the closed unit interval. (Received February 19, 1946.)

207. Edwin Hewitt: *On rings of continuous functions.* Preliminary report.

It has been proved by Stone (Trans. Amer. Math. Soc. vol. 42 (1937) p. 475) and Gelfand and Kolmogoroff (C.R. (Doklady) Acad. Sci. URSS. vol. 22 (1939) pp. 11–15) that two bicompact Hausdorff spaces are homeomorphic if and only if the rings of real-valued continuous functions defined on the two spaces are algebraically isomorphic. Also, two completely regular spaces satisfying the first axiom of countability are homeomorphic if and only if the rings of bounded real-valued and continuous functions on the two spaces are algebraically isomorphic. It is to be expected that many topological properties of a space can be described in terms of the algebraic structure of the function ring. Algebraic characterizations are given of the dimension of an arbitrary normal space and of the Čech homology groups for an arbitrary completely regular space. The former depends upon an algebraic characterization of the ring of real continuous functions on the n-sphere, while the latter has a simple characterization in terms of elements in the function ring which can be represented as squares. (Received March 19, 1946.)

208. G. B. Huff: *Rational-distance sets in the plane.*

Erdös and Anning recently exhibited (Bull. Amer. Math. Soc. vol. 51 (1945) p. 598) examples of planar configurations with the property that the distance between any two was rational. In this note such a configuration is called a rational-distance set. It is shown that if S is a rational-distance set, and I_c is an inversion in a circle such that the square of the radius is rational, with center c at a point of S, then the transform of S by I_c is a rational-distance set. Examples of rational-distance sets are given, including one consisting of an infinite number of points on a line and two points off the line. (Received March 21, 1946.)

209. J. L. Kelley and Everett Pitcher: *Applications of natural homomorphism sequences.* II.

This is a continuation of the work reported in I of the same title (Bull. Amer. Math. Soc. Abstract 52-1-49). A principal theorem relates the homology groups of a complex to the homology groups of a finite set of subcomplexes which form a covering, the homology groups of intersections, and the homology groups of the nerve. This extends the Mayer-Vietoris formulas on coverings by two subcomplexes and the theorem of Helly on coverings by acyclic subcomplexes. (Received February 21, 1946.)

210. J. H. Roberts: *Open transformations and dimension.*

Suppose A and B are separable metric spaces and $f(A)=B$, where f is open; that is, if U is open in A then $f(U)$ is open in B. It is not assumed that f is continuous. If for every $y\in B$ the set $f^{-1}(y)$ is countable, then $\dim B \leqq \dim A$, a result obtained by P. Alexandroff in case f is continuous. For any A, if $\dim A=n$ and $-1<m<n$ then there exists a B and an open f such that $f(A)=B$, $\dim B=m$, and for every $y\in B$, $f^{-1}(y)$ is a single point. Thus open transformations cannot raise dimension if $f^{-1}(y)$ is countable, but open transformations exist which lower dimension at will, even with $f^{-1}(y)$ a single point. (Received March 30, 1946.)

211. G. E. Schweigert and G. S. Young: *Remarks concerning invariants for certain finite transformations.*

The transformations considered are either exactly k to 1, $T_1(A)=B$, or finite to one and interior, denoted by $T_2(A)=B$. The space A is compact and metric. It is assumed that A contains sets $M(i)$ converging to a set M and that each of these contains more than one point and is perfect. It is also assumed that these sets are pairwise disjoint. One theorem states that in the image of $M+\sum M(i)$ a similar situation of convergence of perfect sets prevails, the images being taken under T_1 or T_2. In the case in which the sets M and $M(i)$ are connected, the above shows that continua which are not hereditarily locally connected cannot be mapped onto continua which are; in other words, hereditarily locally connected continua are invariant under the inverses of T_1 and T_2. Other instances of invariance are also considered. (Received March 21, 1946.)

212. N. E. Steenrod: *Extensions of maps and products of cocycles.*

The Pontrjagin algebraic enumeration of the homotopy classes of maps of a 3-complex K^3 in a 2-sphere S^2 is generalized to a classification of maps of K^{n+1} in S^n. This is achieved using new products of cocycles. A product of order $i \geqq 0$ of a p-cochain and a q-cochain is defined which has dimension $p+q-i$ and is the ordinary product if i is zero. The square of order i of a p-cocycle is a cocycle (cocycle mod 2) if $p-i$ is odd (even). This leads to invariant squaring operations on cohomology classes. A square is always of order 2. Let f map the n-section K^n of a complex K in S^n and let z be the image in K of the basic n-cocycle of S_n under the cochain mapping induced by f. Then f can be extended to K^{n+2} if and only if z is a cocycle in K and its square of order $n-2$ is zero. The homotopy classification is obtained from this extension theorem. (Received March 21, 1946.)

213. Fred Supnick: *Rectilinear deformation.*

A linear graph in which each edge is a line segment is called a rectilinear graph. A finite planar nonsingular rectilinear graph G is called a simple graph. Two simple graphs G_1 and G_2 in the same plane, equivalent as linear graphs, are said to be strongly equivalent if the cyclic order of the edges in the stars of any two corresponding vertices, and of any two corresponding cycles of G_1 and G_2, is the same, and if corresponding vertices are inside corresponding cycles. A rectilinear deformation of a simple graph G is a "continuous deformation of G such that at each instant (during which it is being deformed) the graph is strongly equivalent to G." The theorem is proved: If G_1 is strongly equivalent to G_2, then G_1 can be deformed rectilinearly into G_2. This is done by giving a method for carrying out the deformation. The above theorem is based on the theorem: Let $[v_i, v_j]$ be an edge of a simple G which has no chain of less than three edges joining v_i to v_j besides $[v_i, v_j]$, then G can be deformed rectilinearly so that the vertices v_i and v_j coincide. Thus, suspended chains can be "straightened." Constructive methods are given throughout. (Received April 1, 1946.)

214. Fred Supnick: *Equivalent rectilinear graphs.*

Let G_1 be a rectilinear graph which is not disconnected by the removal of any one vertex. That simple closed cycle such that each vertex of the graph is either inside or on it is called the maximal cycle. The following theorem is established: There exists a graph G_2 strongly equivalent to G_1 with its maximal cycle convex. It is shown how to construct G_2. As a consequence of the deformation theorem G_1 can be rectilinearly deformed into G_2. Also it is shown that if two vertices u and v of a simple graph G_1 can be joined by a Jordan arc which does not otherwise intersect G_1, then a graph G_2 strongly equivalent to G_1 can be constructed such that the correspondents of u and v in G_2 can be joined by a line segment which does not otherwise intersect G_2. Again applying the deformation theorem G_1 can be rectilinearly deformed into G_2. Thus, the corollary at the end of the previous abstract has been established by a different method. Also, an example is given to show that the latter reasoning can not be directly extended to infinite graphs. (Received March 16, 1946.)

215. G. W. Whitehead: *A generalization of the Hopf invariant.*

A theorem of J. H. C. Whitehead (Ann. of Math. vol. 42 (1941) pp. 409–428) is generalized as follows: if S^p and S^q are spheres with one point in common, then $\pi_n(S^p \cup S^q) \approx \pi_n(S^p) + \pi_n(S^q) + \pi_n(S^{p+q-1})$ for $n < p+q+\min(p, q) - 3$. This theorem is used to construct a homomorphism H of $\pi_n(S^r)$ into $\pi_n(S^{2r-1})$ for $n < 3r-3$. If $\alpha \in \pi_{2r-1}(S^r)$, then $H(\alpha)$ has degree equal to the invariant of α defined by H. Hopf (Math. Ann. vol. 104 (1931) pp. 637–665). If $\alpha \in \pi_n(S^r)$ and $\beta \in \pi_r(X)$, an element $\beta \cdot \alpha \in \pi_n(X)$ is determined by superposition of representatives of α and β; it is shown that $(\beta_1 + \beta_2) \cdot \alpha = \beta_1 \cdot \alpha + \beta_2 \cdot \alpha + [\beta_1, \beta_2] \cdot H(\alpha)$, where $[\beta_1, \beta_2]$ is the product defined by J. H. C. Whitehead (ibid.). It is further shown that the homotopy groups $\pi_{8n}(S^{4n})$ and $\pi_{16n+2}(S^{8n})$ are different from zero; in fact, they contain elements α with $H(\alpha) \neq 0$. (Received March 8, 1946.)

216. R. L. Wilder: *A generalization of local co-connectedness and its applications.*

Let U be an open subset of a space S. Let r, k be non-negative integers such that if $\mathcal{E}$ is a fcos of S, then there exists a refinement $\mathcal{D}$ of $\mathcal{E}$ such that if an open set D meets U and lies in an element of $\mathcal{D}$, then there exists $E \in \mathcal{E}$ containing D such that exactly k r-cocycles in $U \cap D$ are linearly independent rel. cohomology in $U \cap E$. One then says that $p_r(U, x)$ is *uniformly* equal to k over U. In order that U be r-coulc, it is nas that $p_r(U, x)$ be uniformly equal to 0 over U. For open subsets of an n-gcm, the properties of being 0-ulc and of $p_n(U, x)$ being uniformly equal to 1 over U are dual; for $0 < r < n$, the r-ulc and $(n-r)$-coulc properties are dual. This renders much simpler the treatment of ulc properties in an orientable n-gcm. For example, a fundamental technique for dealing with an open ulc^k U is that of displacing a cycle Z^k on $\bar{U}$ into a "nearby" homologous cycle in U. This is easily accomplished by a co-realization Z_{n-k} in U, of a coordinate of Z^k based on the "co-properties" of U, the intersection of Z_{n-k} with the fundamental n-cycle of S being the desired cycle. Details are given in chapter 10 of the author's forthcoming book. (Received March 21, 1946.)

217. R. L. Wilder: *Certain topological properties in the large and their applications.*

A subset M of a space S is said to have Property $(P, Q)_r[(P, Q)^r]$ if for every pair of open sets P, Q such that $\bar{Q}$ is compact and $P \supset \bar{Q}$, at most a finite number of r-cocycles [compact r-cycles] of M in Q are linearly independent rel. cohomologies on M in P. The corresponding property when only cocycles that cobound in S are considered is denoted by $(P, Q, \sim)_r$. For subsets of compact spaces, Property $(P, Q)_r$ = Property S_r (Amer. J. Math. vol. 61 (1939) pp. 823–832). If a locally compact S has Property $(P, Q)_{r+1}$ and $p_r(x) \leqq \omega$ for all $x \in S$, then S has Property $(P, Q)_r$. An application of this shows that if S is n-dimensional, then a nasc that S be lc^n is that $p_r(x) \leqq \omega$ for all $x \in S$ and $r \leqq n$. (This shows, incidentally, condition (b) in the n-gm definition of Begle, Amer. J. Math. vol. 67 (1945) p. 63, to be unnecessary.) Properties $(P, Q, \sim)^{r-1}$ and $(P, Q, \sim)_r$ are dual; and if S has Properties $(P, Q, \sim)_r$ and $(P, Q, \sim)_{r+1}$ and M is a closed subset of S, then Property $(P, Q, \sim)_r$ of M is dual to Property $(P, Q, \sim)_{r+1}$ of $S-M$. When S is a spherelike n-gcm, Properties $(P, Q, \sim)_r$ and $(P, Q, \sim)^{n-r}$, $r \leqq n-2$, are dual for open sets, and if M is a closed subset of S, Property $(P, Q)_r$ of M is dual to Property $(P, Q)^{n-r-1}$ of $S-M$, $1 \leqq r \leqq n-1$. These properties and their applications are given in detail in chapter 11 of the author's forthcoming book *Topology of manifolds.* (Received March 21, 1946.)

218. R. L. Wilder: *Concerning generalized n-cells.*

Basic among the open n-gms and n-gms with boundary are the generalized cells: A *generalized n-cell* is a non-compact n-gm whose compact homology groups of dimension less than n reduce to the identity. A *generalized closed n-cell* is a compact space consisting of a spherelike $(n-1)$-gcm K and a generalized n-cell A such that (1) $K\cap A=0$, (2) if Z^{n-1} is the fundamental cycle of K, then $Z^{n-1}\sim 0$ on $K\cup A$, and $K\cup A$ is an irreducible membrane rel. to this homology; and (3) $P_r(K\cup A, x)=0$ for $r\leqq n$ and all $x\in K$. (In the separable metric case, the generalized 2-cell is an ordinary 2-cell and the generalized closed 2-cell is an ordinary closed 2-cell.) For a spherelike n-gcm S, the Jordan-Brouwer separation theorem may be stated thus: If M is a spherelike $(n-1)$-gcm in S, then $S-M$ is the union of two disjoint generalized n-cells A_i, $i=1, 2$; moreover, $\bar{A}_i=A_i\cup M$ is a generalized closed n-cell. However, of greater interest is the fact that if two generalized closed cells are joined along their boundaries, the result is a manifold; specifically, if a space S is the union of two generalized closed n-cells $C_i=K\cup A_i$, $i=1$, 2, where K and A_i satisfy the conditions relative to K and A of the above definition, and $A_1\cap A_2=0$, then S is a spherelike n-gcm. It follows that if $K\cup A$ is a generalized closed n-cell as in the definition above, then the generalized n-cell A is a ulc^{n-1} subset of $K\cup A$. Details will be given in chapters 9 and 10 of the author's forthcoming book. (Received March 21, 1946.)

219. R. L. Wilder: *Homology groups of perfectly normal spaces.*

Let $\mathfrak{J}$ be an arbitrary algebraic field. If S is a locally compact, lc^r space and M a compact G_δ subset of S, then the group $H^r(M, \mathfrak{J})$ is a complete metric space with countable base $\{Z_i^r\}$ such that lim $Z_i^r=0$ and every coset is uniquely expressible in the form $\sum_{i=1}^{\infty}a^iZ_i^r$, $a^i\in\mathfrak{J}$; the vector subspace generated by the elements Z_i^r is dense in $H^r(M, \mathfrak{J})$ and uniquely determines the latter. (Compare L. Vietoris, Math. Ann. vol. 97 (1927) pp. 454–472, where an analogous result is given for the case where M is compact metric and hence may be considered a subset of the Hilbert fundamental parallelopiped.) When M has infinite Betti number $p^r(M, \mathfrak{J})$ and is a subset of an orientable n-gcm S, the Alexander type of duality may be obtained via collections $\{Z_i^r\}$, $\{Z_r^j\}$, where the Z_r^j are cocycles mod $S-M$ such that $Z_i^r\cdot Z_r^j=\delta_i^j$, the $(n-r-1)$ cycles obtained from the intersections of δZ_r^i with the fundamental n-cycle of S forming a system whose elements are geometrically linked with the corresponding Z_i^r (cf. E. Begle, Amer. J. Math. vol. 67 (1945) pp. 59–70). For an open orientable n-gm which is the union of a countable collection of compact sets, a similar countable base of infinite cycles is obtainable and the Poincaré duality may be expressed as a homeomorphism between the Betti groups of infinite r-cycles and compact $(n-r)$-cycles. (Received March 21, 1946.)

220. G. S. Young: *Interior and border transformations on surfaces.*

Let M be a compact 2-manifold, with or without boundary. Let $f(M)=N$ be either an interior or border (G. E. Schweigert, Bull. Amer. Math. Soc. Abstract 47-1-104) transformation. Then either N is a 2-manifold and f is light, or N is a set obtained from a dendrite by a finite number of finite identifications, and f is not light. If for each point x in N, each component of the inverse of x is locally connected, and f is not light, then N is either an arc or a simple closed curve. This last result generalizes results of Whyburn and of Puckett. Extensions of these theorems are made to noncompact manifolds and to pseudo-manifolds. It is shown that each of these possible images of M can be realized by an interior monotone transformation. (Received March 21, 1946.)

221. J. W. T. Youngs: *The topological theory of Fréchet surfaces* (*2-cell case*).

The principal result of this paper is a solution of the representation problem: given one representation of a surface, obtain the totality of its representations. The solution is a consequence of a characterization of positive Fréchet equivalence in terms of a suitably modified Kerékjártó equivalence. If, for $i=1, 2$, X_i is an oriented 2-sphere containing a 2-cell H_i, then two mappings $f_1(H_1)=Y=f_2(H_2)$ are positively Kerékjártó equivalent if there exist: (1) a cactoid $\mathfrak{X}$ containing a hemicactoid $\mathfrak{H}$; (2) a pair of monotone mappings $m_1(X_1)=\mathfrak{X}=m_2(X_2)$ such that $m_1(H_1)=\mathfrak{H}=m_2(H_2)$, m_i is one-to-one on X_i-H_i, and each true cyclic element $\mathfrak{S}$ of $\mathfrak{X}$ can be oriented so that, simultaneously, Dgr $m_i(\mathfrak{S}; X_i)=1$, for $i=1, 2$ (the mapping m_i $(\mathfrak{S}; X_i)$ is m_i followed by the retraction of $\mathfrak{X}$ onto $\mathfrak{S}$); and (3) a light map $l(\mathfrak{H})=Y$, such that $f_i(x_i)=lm_i(x_i)$, $x_i \in H_i$, for $i=1, 2$. The mappings are positively Fréchet equivalent if for every $\epsilon>0$ there is a homeomorphism $h_\epsilon(H_1)=H_2$ such that $\rho\{f_1(x_1), f_2h_\epsilon(x_1)\}<\epsilon$. It is shown that positive Fréchet equivalence is the same as positive Kerékjártó equivalence. (Received March 21, 1946.)