others, sought a new logic more adequate than the traditional one to modern science and practice. Prior to Frege or Russell, Hegel possessed logistic intent which was determining factor in development of his logic. He was familiar with historic roots of modern symbolic logic—with work of Lully, Bruno, Leibniz, Ploucquet, Euler, and Bardili—rejecting their primitive efforts as inadequate. He knew the Sancho Panza dilemma, recognized by Church as closely related to Russell's paradox, and even proposes solution resembling theory of types. His denial of so-called "laws of thought" antedates denial of law $A \cdot A = A^2$ in Boole's logical algebra, of parallel-axiom in non-Euclidean geometry, of commutative law in quaternion theory. When mathematicians and mathematical philosophers were seeking to avoid the infinite, Hegel restored it, with new interpretation, to a place of central importance in foundations of analysis. He speaks of infinite as involving equality of whole and part, distinguishes "bad" infinite (Cantor's variable finite) from "good" infinite (recognized as reflexive) and objects to the phrase "and so on to infinity," shown eliminable by Frege. Once influential, he was known to Boole, DeMorgan, Bolzano, Pierce, G. Cantor, and Russell, while Frege refers to Fischer's Hegelian logic. (Received March 23, 1946.)

STATISTICS AND PROBABILITY

195. Will Feller: *A limit theorem for random variables with infinite moments.*

Let $\{X_k\}$ be an arbitrary sequence of mutually independent random variables with the same distribution function $V(x)$. It is assumed that some moment of order less than two is infinite; the first moment may be infinite, but if it is finite it should be normed to zero. Let $S_n = X_1 + \cdots + X_n$ and let $\{a_n\}$ be a monotonic positive numerical sequence. It is shown that the probability that the inequality $|S_n| > a_n$ takes place for infinitely many n is the same as the probability that $|X_n| > a_n$ for infinitely many n; it is one or zero according as the series $\sum \{V(-a_n) + 1 - V(a_n)\}$ diverges or converges. (Received March 21, 1946.)

196. Will Feller: *The law of the iterated logarithm for identically distributed random variables.*

Let $\{X_n\}$ be a sequence of mutually independent random variables with the same distribution function $V(x)$ with vanishing first moment and unit variance. Suppose that (*) $\int_{|t|>x} t^2 dV(t) = O((\log \log x)^{-1})$, and let $\{\phi_n\}$ be an arbitrary monotonic sequence, $\phi_n > 0$. The probability that the inequality $X_1 + \cdots + X_n > n^{1/2}\phi_n$ will be satisfied for infinitely many n is shown to be zero or one according as the series $\sum \phi_n n^{-1} \exp(-\phi_n^2/2)$ converges or diverges. The condition (*) is in a certain sense the best possible. If it is not satisfied, the above exact analogue to the strict law of the iterated logarithm does not hold, but slightly more complicated necessary and sufficient conditions are given in the paper. (Received March 21, 1946.)

197. Casper Goffman: *Measures of fluctuation of a variable mean.* Preliminary report.

Suppose a finite order of random variables $x_1, \cdots, x_n$ is given, all normally distributed, with the same known standard deviation, but with unknown means $a_1, \cdots, a_n$, not necessarily alike. A measure of fluctuation of the means is defined as a function $f(a_1, \cdots, a_n)$ such that (1) for every real h, $f(a_1+h, \cdots, a_n+h) = f(a_1, \cdots, a_n)$, and (2) for every real c, $f(ca_1, \cdots, ca_n) = c^2 f(a_1, \cdots, a_n)$. It is

shown that, in an important sense, the only admissible measure of fluctuation, except for a constant factor, is the variance $f(a_1, \cdots, a_n) = \sum_{i=1}^{n}(a_i - n^{-1}\sum_{j=1}^{n}a_j)^2$. The method of maximum likelihood is applied to special functions to obtain tests for significant differences. These tests have applications to industrial problems. (Received March 22, 1946.)

198. Isaac Opatowski: *Average duration of transition in Markoff chains.*

Consider a chain of transitions $(i \to i+1)$, $(i+1 \to i)$, where $i = 0, 1, \cdots, n-1$. Let the usual conditional probabilities of these transitions within any time Δt be respectively $k_{i+1}\Delta t + o(\Delta t)$ and $g_i\Delta t + o(\Delta t)$, where the k_i's and g_i's are constant. Let the probability of any other transition during Δt be $o(\Delta t)$. The probability $P(t)$ of a transition $(0 \to n)$ within a time t is an increasing function of t and $P(\infty) = \prod_{i=1}^{i=n}k_i/K_i$, where $s = -K_i$ are the n roots of the secular equation $\|a_{r,c}\|/s = 0$ defined by $a_{r,r} = s + k_{r+1} + g_{r-1}, a_{r,r-1} = k_r, a_{r,r+1} = g_r, a_{r,c} = 0$ for $|r-c| > 1$; $r, c = 0, 1, \cdots, n$; $g_{-1} = 0$ (Bulletin of Mathematical Biophysics vol. 7 (1945) pp. 170–177). If $g_{n-1} = 0$ then $P(\infty) = 1$, if $g_{n-1} \neq 0$ then $P(\infty) < 1$. Consequently, the average duration of the transition $(0 \to n)$ is $E = \int_0^{P(\infty)} t \, dP/P(\infty)$. Its explicit expression is a simple symmetric function of the k_i's and K_i's. If all the g_i's are zero, $E = \sum_{i=1}^{i=n}k_i^{-1}$. By using the results of a previous paper (Proc. Nat. Acad. Sci. U.S.A. vol. 31 (1945) pp. 411–414) simple expressions of E are obtained also in the cases in which the k_i's and g_i's are independent of i. The paper is a part of an article to appear in the Bulletin of Mathematical Biophysics vol. 8 (1946). (Received March 19, 1946.)

199. Isaac Opatowski: *Markoff chains with variable intensities: average duration of transition.*

Consider a simple Markoff chain. Let $k_{i+1}\Delta t + o(\Delta t)$ be the conditional probability of a transition $(i \to i+1)$ within any time Δt, where $i = 0, 1, \cdots, n-1$ and $k_i = F(i)f(t)$. It is known that by changing t into a new time variable $T = \int_0^t f(t)dt$, the present chain may be treated as if its intensities k_i were constant and equal to $F(i)$. Let $t = \sum_m c_m T^m$ be a polynomial in T. Let $P(t)$ be the probability of a transition $(0 \to n)$ within t. It is shown that $\int_0^1 t \, dP$, the average duration of a transition $(0 \to n)$, equals $\sum_m m! c_m h_m$, where h_m is the complete homogeneous symmetric function of degree m of $1/F(1)$, $1/F(2), \cdots, 1/F(n)$. This formula is obtained by using a previous result on the moments of Markoff chains (Proc. Nat. Acad. Sci. U.S.A. vol. 28 (1942) pp. 83–88). The paper is a part of an article to appear in the Bulletin of Mathematical Biophysics vol. 8 (1946). (Received March 20, 1946.)

TOPOLOGY

200. E. E. Floyd: *On the extensions of homeomorphisms on the interior of a two cell.*

Let $f(I) = R$ be a homeomorphism of the interior I of a two cell with boundary C onto a bounded plane region R. It is shown that if f is extensible to $\bar{I}$, then the extension is non-alternating on the boundary C. A condition is also derived which is equivalent to the existence of an extension g of f, where $g(\bar{I}) = \bar{R}$, $g = f$ on I, and g is light and non-alternating on C. This is applied to conformal maps, yielding the following theorem: let $f(I) = R$ be a 1-1, conformal map of the interior I of the unit circle onto a