GEOMETRY

181. Reinhold Baer: *Projectivities with fixed points on every line of the plane.*

The class of projectivities under consideration comprises perspectivities and involutions. These two types exhaust our class in case the Theorem of Pappus holds. But already in the projective plane with coordinates from the field of real quaternions there may be found projectivities which are neither perspectivities nor involutions, and which still belong to our class. The properties of this class are found to be particularly striking, in case the projective plane under consideration is finite. (Received March 25, 1946.)

182. H. S. M. Coxeter: *The content of the general regular polytope.*

The regular solid $\{p, q\}$ has q p-gons at each vertex; that is, it has face $\{p\}$ and vertex figure $\{q\}$. Analogously, the four-dimensional polytope $\{p, q, r\}$ has cell $\{p, q\}$ and vertex figure $\{q, r\}$; and the n-dimensional polytope $\{p, q, \cdots, v, w\}$ has cell $\{p, q, \cdots, v\}$ and vertex figure $\{q, \cdots, v, w\}$. It is found that such a polytope of edge $2l$ has content $(g/n!)l^n b_1^{n/2}/b_1 b_2 \cdots b_{n-1} b_n^{1/2}$, wherein g is the order of the symmetry group and b_k the denominator of the kth convergent of the continued fraction $1/c_1 - 1/c_2 - 1/c_3 - \cdots$, $c_1c_2 = \sec^2 \pi/p$, $c_2c_3 = \sec^2 \pi/q, \cdots, c_{n-1}c_n = \sec^2 \pi/w$. In the case of the n-dimensional hypercube $\{4, 3, \cdots, 3, 3\}$, $g = 2^n n!$. Write $c_1 = 1$, $c_2 = c_3 = \cdots = 2$, whence every $b_k = 1$. (The kth convergent is simply $k/1$.) (Received March 6, 1946.)

183. John DeCicco: *Geodesic perspectivities upon a sphere.*

The following fundamental theorem is proved. If, under a perspective map of a surface Σ upon a sphere S, more than $3\infty^1$ geodesics of Σ correspond to the great circles of S, then every geodesic of Σ is mapped into a great circle of S, and Σ is a sphere or a plane. If Σ is a plane, the author obtains gnomic projection. Otherwise if Σ is a sphere, then both S and Σ are homothetic with respect to the point of perspectivity O and the corresponding points on S and Σ must be homothetic with respect to O. This may be contrasted with the corresponding result obtained by Kasner and the author, on conformal perspectivities of a sphere, where a point of S can correspond to either or both of the two perspective images on the sphere Σ. (Received March 7, 1946.)

184. Arnold Emch: *Dissection of two equivoluminal parallelotops into two finite series of equal numbers of congruent pieces in ordinary and higher Euclidean spaces.*

For the ordinary plane E_2 this problem has been known for a long time. Dehn has proved that in general it is not possible in E_3. By a new method based upon a particular affine relation between two equiareal rectangles it is shown how two equiareal polygons in E_2 can be dissected in the manner indicated above. In E_3 the designation "box" (Kiste by Schouten) is taken as a synonym for "rectangular parallelopipedon." The same term shall be used for rectangular parallelotop in $E_4, \cdots, E_n$, where the n edges on a vertex are mutually perpendicular. Based on an extension of the method for E_2 it is then successively shown how the problem can be solved for $E_3, E_4, \cdots, E_n$. The preliminary dissection of a parallelotop into an equivoluminal box in any E_n is not difficult. (Received February 19, 1946.)

185. Arnold Emch: *New properties of quadrics with umbilics based upon their stereographic projections.*

Nonsingular quadrics, with the exception of the hyperboloid of one sheet and the hyperbolic paraboloid, have real navel points or umbilics; that is, points such that the plane sections parallel to the tangent planes at these points are circles. Central quadrics have in general 12 such points of which four are real. An elliptical paraboloid has two real umbilics. In the case of quadrics of revolution they coincide with the vertices; that is, with the intersections of the axis of revolution. On a paraboloid of revolution, the infinite point of the axis must also be considered as an umbilic. The following theorem is proved: *Given a quadric Q with two or more umbilics (an infinite number in case of a sphere) $U, U', \cdots$, a real plane section S of Q and a plane of projection p, parallel to the tangent plane at one of the U's, intersecting Q in a circle E, the projection of S from U upon p is a circle C. If U' is a diametral umbilic of U, then from U', S is projected into a circle C', which is inverse to C with respect to E as a circle of inversion.* (Received February 20, 1946.)

186. G. B. Huff: *An arithmetic characterization of proper characteristics of linear systems.*

The characteristic $x=\{x_0; x_1, x_2, \cdots, x_\rho\}$ of a nondegenerate linear system of plane curves consists of the *order* x_0 and the *multiplicities* $x_1, x_2, \cdots, x_\rho$ at a set of ρ base points. If p, d are the *genus* and *dimension* of the system, x satisfies: (1) $x_0^2-x_1^2-x_2^2-\cdots-x_\rho^2=d+p-1$, and $3x_0-x_1-x_2-\cdots-x_\rho=d-p+1$; and a set of inequalities (2) $y_0x_0-y_1x_1-y_2x_2-\cdots-y_\rho x_\rho\geqq 0$, where y runs over the characteristics of all Bertini L-curves of order $y_0<x_0$ and defined at that base. A *proper* solution x of the Cremona equations (1) is one which is the characteristic of a nondegenerate linear system. It is shown for all ρ and certain values of p, d that a solution of (1) which also satisfies (2) is proper. (Received March 21, 1946.)

187. Edward Kasner and John DeCicco: *Comparison of union-preserving and contact transformations in space.*

In Bull. Amer. Math. Soc. vol. 50 (1944) pp. 98–107, the authors gave a general theory of union-preserving transformations in space from curve-elements of order n into lineal-elements. In the general case, these are defined by a directrix equation of the form: $\Omega(X, Y, Z, x, y, z, y', z', \cdots, y^{(n-2)}, z^{(n-2)})=0$, involving derivatives of order $(n-2)$. If $n=2$, the directrix equation $\Omega(X, Y, Z, x, y, z)=0$ involves no derivatives. Thus a single directrix equation in which a point in space corresponds to a surface gives rise to a union-preserving transformation, and also to a Lie contact transformation. In general, these transformations are distinct. A comparison of these two types of transformations is given. As an application the authors develop a new theory of parallel curves in space. In this theory, the parallel of a helix is a helix. Another example is that the parallel of a plane curve lies on the cylinder with elements orthogonal to the plane of the original curve and whose directorial curve is the evolute. The new curve is gauche in general. (Received March 7, 1946.)

188. Edward Kasner and John DeCicco: *Conformal perspectivities upon a sphere.*

Recently the authors proved that the only perspective conformalities upon a plane are Ptolemy's stereographic projection of a sphere and the obvious limiting case of a parallel plane. In the present work, all the prespective conformalities upon a sphere are obtained. If a surface Σ is conformally mapped upon a sphere S by a perspectivity with center at O, then Σ is a sphere inscribed in the cone (real or imaginary) whose vertex is at O and which is tangent to S. Thus a sphere S can be projected by a conformal perspectivity with center at O only into another sphere Σ; furthermore the centers of S and Σ are collinear with O and the distances of their centers from O are proportional to their radii. Two special cases are noteworthy. I. If a surface Σ admits a conformal perspectivity upon a sphere S from the center of S, then Σ is a sphere concentric with S. II. If a surface Σ admits a conformal perspectivity upon a sphere S from a point O on S, then Σ is a sphere tangent to S at O. Only in this last special case can Σ be a plane, in which event it is perpendicular to the diameter of S through O. (Received March 8, 1946.)

189. Edward Kasner and John DeCicco: *The distortion of angles in general cartography.*

Let a surface Σ be mapped in a point-to-point fashion (not conformal) upon a plane π. The azimuth ratio $\alpha = d\Theta/d\theta$ is defined as the rate of change of the inclination at a fixed point P on the surface Σ with respect to the inclination at the corresponding point p on the plane π. This is a function of (x, y, y'). An azimuth curve is the locus of a point along which the azimuth ratio does not vary. There are ∞^2 azimuth curves. These have many properties in common with the scale curves, which are defined as those along which the scale $\sigma = ds/dS$ is constant. Various geometrical properties of these azimuths are obtained. The case where the azimuths are all straight is studied in detail. (Received March 7, 1946.)

190. Harry Levy: *The projective geometry of Riemannian spaces of two dimensions.*

Cartan showed in 1937 that if K, the Gaussian curvature of a two-dimensional ds^2, is not constant, the orthogonal trajectories of the curves $K=$const. are invariant under geodesic preserving transformations. The author shows the existence of a second invariant family of curves, the integral curves of $dx^1/L^1 = dx^2/L^2$, where $L^i = K^i{}_{,j}K^j - (4/3)K^j{}_{,j}K^i$. The coincidence of these two families (or the vanishing of L^i) is itself an invariant property; the families, when distinct, determine a projective connection which is the basis for the development of an invariant projective theory. In terms of projective invariants of the K and L net of curves, necessary and sufficient conditions are established for the existence of transitive groups of geodesic transformations; such groups contain but two parameters; the equivalence problem is reduced to the problem of the existence of functional relations among a set of at most 12 invariants. Geodesics possess an "arc" determined to within an affine transformation; there exists an absolute "parallelism" of geodesics which is invariant, and on parallel geodesics the ratio of "arc" is preserved. (Received March 21, 1946.)

191. Abraham Schwartz: *Higher normal spaces for spaces of class greater than one.*

Consider a $V_m \subset R_n$ with a first normal space of m_1 dimensions at a particular point P. Call a direction at P "omniconjugate" if it is conjugate to all directions. By elementary algebraic methods one can show from the Gauss-Codazzi-Ricci equations that the second normal space at P vanishes unless (1) there are $m - m_1$ linearly independent omniconjugate directions at P, or (2) the V_m is contained within some V_{m+r},

$1 \leqq r < m_1$, in such a way that there are $m-r$ linearly independent omniconjugate directions at P with respect to the V_{m+r}. (The local R_{m+r} of the V_{m+r} at P is contained within the local R_{m+m_1}.) If there is a second normal space of m_2 dimensions, then $m_2 \leqq s(s+1)(s+2)/6 + t(t+1)/2$, where s and t are integers determined by $s(s+1)/2 \leqq m_1 \leqq (s+1)(s+2)/2$, $t = m_1 - s(s+1)/2$. Similar statements can be made about the vanishing of the third and other higher normal spaces. (Received March 19, 1946.)

192. Y. C. Wong: *Contributions to the theory of surfaces in a 4-space of constant curvature.*

A Riemannian 4-space of constant curvature and a surface in it are denoted by S_4 and V_2, respectively. The method of studying V_2 in S_4 in this paper is invariant and is similar to those of G. Ricci (*Lezioni sulla teoria della superficie*, Verona-Padova, 1898) and W. Graustein (Bull. Amer. Math. Soc. vol. 36 (1930)) for their studies of surfaces in a Euclidean 3-space. In essence, the method consists of setting up a suitable system of invariant fundamental equations for a V_2 in S_4, and expressing the required imbedding conditions of V_2 in S_4 in terms of the intrinsic properties of V_2. Curvature properties, especially those about the curvature conic, of a general V_2 in S_4 are first discussed. Then the V_2's whose curvature conic is of certain particular nature are studied. These include the minimal V_2, with the R-surface of K. Kommerell (Math. Ann. vol. 60 (1905)) as a special case, ruled V_2, and V_2 with an orthogonal net of Voss. The paper concludes with a complete determination of those V_2's in S_4 whose first fundamental form and one of whose second fundamental forms are respectively identical with the first and second fundamental forms of a surface in a 3-space of constant curvature. (Received March 11, 1946.)

193. Y. C. Wong: *Scale hypersurfaces for conformal-Euclidean space.*

This paper contains generalizations to n-space of some of the results obtained recently by E. Kasner and J. DeCicco (Amer. J. Math. vol. 67 (1945)) for the scale curves in conformal maps of a surface on a plane. The fundamental form $ds^2 = e^{2\sigma}(dx_1^2 + \cdots + dx_n^2)$, with $\sigma = \sigma(x_1, \cdots, x_n)$, represents a conformal-Euclidean n-space C_n, conformally mappable on the Euclidean n-space R_n with rectangular Cartesian coordinates $x_1, \cdots, x_n$. The hypersurfaces $\sigma = \text{constant}$ in R_n are the scale hypersurfaces in the mapping of C_n on R_n, and any simple family of hypersurfaces in R_n is called quasi-isothermal if it represents the scale hypersurfaces of a conformal mapping of some C_n on R_n such that the scalar curvature of C_n is constant over each of the scale hypersurfaces. A few theorems are proved concerning the cases when a family of quasi-isothermal hypersurfaces is a family of (a) ∞^1 hyperplanes, (b) ∞^1 generalized cylinders of rotation. This subject is closely connected with the subject of the isoparametric hypersurfaces of T. Levi-Civita and B. Segre (Rendiconti della Reale Accademia Nazionale dei Lincei (6) vol. 26 (1937), vol. 27 (1938)) and incidentally connected with that of the subprojective Riemannian space of B. Kagan and H. Schapiro (*Abhandlung des Seminars für Vektor- und Tensoranalysis*, vol. 1, 1933). (Received March 11, 1946.)

Logic and Foundations

194. Ira Rosenbaum: *Hegel, mathematical logic, and the foundations of mathematics.*

Hegel, like Boole, DeMorgan, Pierce, Frege, Peano, Russell, Whitehead and