

postulate systems for Boolean algebras and distributive lattices. (Received January 23, 1946.)

94. Archie Blake: *A Boolean derivation of the Moore-Osgood theorem.*

The process of proving a mathematical theorem is represented in symbolic logic by the transformation of logical expressions. This fact is illustrated in the case of the Moore-Osgood theorem, the central features of the derivation of which are shown to be representable in the *Prädikatenkalkül*. (Received January 12, 1946.)

95. Ira Rosenbaum: *Hegel's observations on the differential and integral calculus and its foundations.*

Attention is invited to an extended discussion of the differential and integral calculus and its foundations which appears in Hegel's *Science of logic*. The technical content, character, and interest of Hegel's discussion is indicated by citing authors, texts, methods, and problems with which Hegel dealt. The evidence relating to Hegel's knowledge of mathematics is presented and a picture of the development of Hegel's views is traced; relevant portions of the first and later editions of the *Logic* are compared. Hegel's relation to his contemporaries in mathematics is pointed out. The relevant literature is reviewed critically and after indicating the prevalent neglect of Hegel's discussion, it is concluded that such examination of Hegel's relation to the calculus as does exist is (1) dated, (2) incomplete and/or inadequate, (3) generally independent of earlier and contemporary work in the same field, and (4) limited in scope and point of view. An instance of the unsatisfactory state of the literature on this subject is considered. Hegel's observations on the calculus are examined, placed in their proper historical context, and his views compared with those of his predecessors, contemporaries, and successors. Evaluation from the standpoint of the modern logico-mathematical foundations of analysis is undertaken. (Received February 1, 1946.)

#### STATISTICS AND PROBABILITY

96. G. W. Brown and J. W. Tukey: *Some distributions of sample means.*

It is shown that certain monomials in normally distributed quantities have stable distributions with index  $2^{-k}$ . This provides, for  $k > 1$ , simple examples where the mean of a sample has a distribution equivalent to that of a fixed, arbitrarily large multiple of a single observation. These examples include distributions symmetrical about zero, and positive distributions. Using these examples, it is shown that any distribution with a very long tail (of average order greater than or equal to  $x^{-3/2}$ ) has the distributions of its sample means grow flatter and flatter as the sample size increases. Thus the sample mean provides *less* information than a single value. Stronger results are proved for still longer tails. (Received January 14, 1946.)

#### TOPOLOGY

97. R. H. Bing: *Generalization of a theorem of Janiszewski.*

Suppose that  $H$  and  $K$  are plane sets neither of which cuts the point  $A$  from the point  $B$ , that the boundary of  $H$  is compact, that the junction of  $H$  and  $K$  is equal to

$H \cdot K$  and that  $H$  is the sum of a collection of mutually exclusive sets no one of which contains either a limit point of the sum of the others or two components of  $H \cdot K$ . Then  $H+K$  does not cut  $A$  from  $B$ . (Received January 14, 1946.)

98. R. H. Bing: *Sets cutting the plane.*

Suppose that in the plane,  $K$  is a set with a connected complement and a bounded boundary, that  $W$  is either a countable set of points or a bounded closed set and that no component of  $W$  cuts the point  $A$  from the point  $B$  in  $K$ . Then  $W$  does not cut  $A$  from  $B$  in  $K$ . (Received January 19, 1946.)

99. C. H. Dowker: *Lebesgue dimension of a normal space.*

Let  $R$  be a normal space and let  $\dim R$  be the Lebesgue dimension of  $R$  defined as follows:  $\dim R \leq n$  means that every finite covering of  $R$  by open sets has a finite refinement of order not greater than  $n+1$ . It is shown that (1)  $\dim R \leq n$  if and only if every star-finite covering of  $R$  by open sets has a star-finite refinement of order not greater than  $n+1$ , (2)  $\dim R \leq n$  if and only if every locally finite covering of  $R$  by open sets has a locally finite refinement of order not greater than  $n+1$ , and (3)  $\dim R \leq n$  if and only if, for each closed set  $A$  of  $R$ , each continuous mapping  $f$  of  $A$  into the  $n$ -sphere  $S^n$  can be extended to a continuous mapping  $g$  of  $R$  into  $S^n$ . (Received January 19, 1946.)

100. Erik Hemmingsen: *Dimension theory for normal Hausdorff spaces.* Preliminary report.

In normal Hausdorff spaces it is shown that the two following definitions of dimension are equivalent: (I)  $\dim S \leq n$ , if to each finite covering of  $S$  by open sets there is an open refinement in which at most  $n+1$  sets intersect at a time; (II)  $\dim S \leq n$ , if to each mapping  $f$  of  $S$  into the  $n+1$  simplex,  $\sigma^{n+1}$ , there is a mapping  $g$  of  $S$  into the boundary of  $\sigma^{n+1}$ , written  $s^n$ , with  $f/f^{-1}(s^n) = g/f^{-1}(s^n)$ . The theorem  $\dim M \times N \leq \dim M + \dim N$  is proved for compact Hausdorff spaces. If  $A = \overline{AC}S$ ,  $S$  normal,  $\dim A \leq n$ ,  $f(S) \subset \sigma^{n+1}$ ,  $f$  continuous, and if  $\sigma_0^{n+1}$  is an  $n+1$  simplex contained in  $\sigma^{n+1}$  then it is shown that there is a mapping  $g$  of  $S$  in  $\sigma^{n+1}$  with  $g(A) \subset (\sigma^{n+1} - \text{Int } \sigma_0^{n+1})$  and  $g/B = f/B$  where  $B = f^{-1}(\sigma^{n+1} - \text{Int } \sigma_0^{n+1})$ ; the sum theorem for countably many closed sets is a corollary. (Received January 14, 1946.)

101. F. B. Jones: *A characterization of a plane semi-locally-connected continuum.*

Let space be a simple closed surface  $S$ . If  $G$  is a collection of mutually exclusive simple domains,  $G$  is said to contain a sequence of *folded* simple domains provided that there exist sets  $D_1, D_2, D_3, \dots, D_i, \dots$  ( $i=1, 2, 3, \dots$ ) such that (1) for each  $i$ ,  $D_i$  is an element of  $G$ , (2) for each  $i$ ,  $D_i$  contains a spanning arc-segment  $T_i$  separating  $D_i$  into two components  $C_i$  and  $E_i$  and (3) if  $\epsilon$  is a positive number and  $T$  is the sequential limiting set of  $T_1, T_2, T_3, \dots$ , there exist a point  $A$  not belonging to  $T$  and a number  $n$  such that if  $i > n$ , both  $C_i$  and  $E_i$  contain points at a distance from  $A$  less than  $\epsilon$ . The principle result of the paper is as follows: In order that a set  $M$  be a cyclic semi-locally-connected continuum it is necessary and sufficient that the components of  $S - M$  be a collection of mutually exclusive simple domains containing no sequence of folded simple domains. (Received January 29, 1946.)

102. G. W. Whitehead: *On families of continuous vector fields over spheres.*

Let  $f(n)$  be the maximum number of everywhere independent continuous fields of tangent vectors that can exist on the  $n$ -sphere  $S^n$ . It is well known that  $f(2n) = 0$ ,  $f(2n+1) \geq 1$ ,  $f(4n+3) \geq 3$ , and  $f(8n+7) \geq 7$ . It has been proved independently by B. Eckmann (Comment. Math. Helv. vol. 15 (1942) pp. 1-26) and the author (Ann. of Math. vol. 43 (1942) pp. 132-146) that  $f(4n+1) = 1$ . In this paper it is shown that  $f(8n+3) = 3$ . It follows from this and results of N. E. Steenrod (Ann. of Math. vol. 45 (1944) pp. 294-311) that if  $m > k$  and  $k = 2n$ ,  $4n+1$ , or  $8n+3$ , with  $n > 0$ , then  $S^m$  is not a  $k$ -sphere bundle over any complex  $B$ . (Received December 10, 1945.)

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### NEW PUBLICATIONS

A collection of papers in memory of Sir William Rowan Hamilton. (Scripta Mathematica Studies, no. 2.) New York, Scripta Mathematica, 1945. 82 pp. \$1.00.

Sequential analysis of statistical data: applications. Prepared by Statistical Research Group, Columbia University, for the Applied Mathematics Panel, National Defense Research Committee, Office of Scientific Research and Development. New York, Columbia University Press, 1945. 315 pp. \$6.25.