tion law. (2) For each $n \geqq 2$ there is an n-sided convex polygon of maximal length inscribed in such a curve. (Received August 9, 1945.)

237. M. M. Day: *Polygons circumscribed about closed convex curves.*

An earlier abstract (50-5-132) announced a proof without use of fixed point theorems of the following result. If C is a symmetric closed convex curve, there exists a parallelogram P circumscribed about C so that the midpoint of each side of C is on P. Using very elementary minimal area methods this result is extended to polygons of an arbitrary number of sides, and to polyhedra about convex bodies in Euclidean space of any finite dimension. In higher dimensions the phrase "midpoint of each side" is replaced by "centroid of each face." (Received August 9, 1945.)

238. H. P. Pettit: *The tangents at certain multiple points on a curve C_{2mn}.*

In a paper published in the Tôhoku Mathematical Journal in 1927, the author discussed the construction of a curve C_{2mn} of order $2mn$ by the use of two pencils of lines with centers A_1, A_3, related by means of two base curves C_n, C_m of orders n and m, respectively, and an auxiliary pencil with center A_2. It was shown that the line A_1A_2 meets C_m in m n-fold points of C_{2mn}. In the present paper, it is shown that certain projective relationships yield the following method of determining the tangents at such an n-fold point: Project the points in which A_1A_2 meets C_n from the point of intersection of A_1A_3 with the tangent to C_m at the point in question, thus determining n points on A_2A_3. These points are projected from the n-fold point in the desired tangents. (Received August 6, 1945.)

239. H. P. Pettit: *The tangents at certain ordinary points on a curve C_{2mn}.*

As shown by the author in a previous paper published in the Tôhoku Mathematical Journal in 1927, a curve of order $2mn$ is generated by means of two pencils of lines related by means of two base curves of orders m and n respectively and an auxiliary pencil. The generated curve was shown to pass through all common points of the base curves. This paper discusses a method of constructing the tangents to the generated curve at these common points. There is shown to be a projective relationship between the tangents to the base curves at the common point and the tangent to the generated curve, which results in the following. If A_1, A_2, A_3 are the vertices of the first, auxiliary, and second pencils, and P is the common point of the base curves C_1, C_3, determine K_1 in which the tangent to C_3 at P meets A_2A_3 and the point K_3 in which the tangent to C_1 at P meets A_1A_2. Draw the line K_1K_3 meeting A_1A_3 in T. The line PT is the required tangent to the generated curve. (Received May 26, 1945.)

Logic and Foundations

240. N. D. Nelson: *Recursive functions and intuitionistic number theory.* Preliminary report.

It is shown that the interpretation by the intuitionistic truth notion of realizability of Kleene (Bull. Amer. Math. Soc. abstract 48-1-85) satisfies certain formal systems of intuitionistic number theory. Further results are obtained which complete reasoning outlined by Kleene (loc. cit. and Trans. Amer. Math. Soc. vol. 53 (1943) pp. 41–73,

§16) to show that intuitionistic number theory admits, besides the extension which gives classical number theory, also an intuitionistic extension. Both extensions are simply consistent if the unextended system is, and the two extensions are contradictory to each other. The work involves formalization of the theory of certain primitive recursive predicates $R_k(e, x_1, \cdots, x_k, y)$ which in a formal intuitionistic system afford a representation of the theory of general and partial recursive functions and predicates. The present results combine with reasoning of Kleene (Abstract 48-1-85) to establish the independence of certain formulas of the intuitionistic predicate calculus, in particular of the formula $\neg\neg(x)(A(x) \vee \neg A(x))$. (Received August 8, 1945.)

Statistics and Probability

241. Reinhold Baer: *Sampling from a changing population.*

The stochastic limits of certain functions of random samples are determined where the samples are taken from different distributions belonging to a continuous family of distributions. (Received August 22, 1945.)

242. J. L. Doob: *Markoff chains—denumerable case.*

Let $p_{ij}(t)$, $i, j=1, 2, \cdots, 0 \leqq t < \infty$, be the transition probability functions of a Markoff process. Let $x(t)$ be the (integral) value assumed by the probability system at time t. Necessary and sufficient conditions are found that the $p_{ij}(t)$ satisfy the systems of first order differential equations (*) $p'_{ik}(t) = -q_i p_{ik}(t) + \sum_{j \neq i} q_{ij} p_{jk}(t)$, $p'_{ik}(t) = -p_{ik}(t) q_k + \sum_{j \neq i} p_{ij}(t) q_{jk}$, where $q_i = -p'_{ii}(0)$, $q_{ik} = p'_{ik}(0)$ $(i \neq k)$. A detailed analysis is made of the processes for which the discontinuities of $x(t)$ are well ordered. It is shown that if q_i, q_{ik} are specified arbitrarily except that $q_{ik} \geqq 0$, $q_i = \sum_j q_{ij}$, there is always a corresponding set of functions $\{p_{ij}(t)\}$ determining a Markoff process, but that in general there will be infinitely many such sets of functions, and even infinitely many satisfying (*), such that the discontinuities of $x(t)$ are well ordered. The initial conditions $p_{ij}(0) = \delta_{ij}$ are thus insufficient to determine uniquely the solutions to (*). (Received September 22, 1945.)

243. Mark Kac: *On the average of a certain Wiener functional.*

Let $x(t)$ be an element of the Wiener space. It is shown that the average of the functional $\exp\left(-z\int_0^1 |x(t)|\, dt\right)$ $(z>0)$ is given by the formula $\sum_1^\infty \kappa_j \exp(-(\delta_j/2)z^{2/3})$, where $\delta_1, \delta_2, \cdots$ are positive zeros of the derivative of $P(y) = (2y)^{1/2}\{J_{-1/3}((2^{3/2}/3)y^{3/2}) + J_{1/3}((2^{3/2}/3)y^{3/2})\}$ and $\kappa_j = (1 + \int_0^{\delta_j} P(y)dy)/\delta_j P(\delta_j)$. A related limit theorem in calculus of probability is discussed. In the course of the proof the following seemingly new result was also obtained: If r_j is the jth positive root of $J_\nu(x)$ $(\nu \geqq 0)$ then $r_j^2 > \nu^2 + (2\pi j)^{2/3}\nu^{4/3}$. For j fixed the estimate is weaker than a known asymptotic formula. The value of the estimate is due to the fact that j can depend on ν. (Received August 2, 1945.)

244. Isaac Opatowski: *Calculation of Markoff chains by incomplete gamma and beta functions and by Charlier polynomials.*

Several types of stochastic processes consisting of successive transitions between n states $\{i\}_1^n$ are considered (cf. Bull. Amer. Math. Soc. vol. 51 (1945) p. 665). Call $P_{r,s}(t)$ the probability of being a system at the time t in the state s if it is at $t=0$ in the state r. Let the only transitions possible during dt be $(i-1 \rightarrow i)(i+1 \rightarrow i)$ and the