229. C. A. Truesdell: *On the functional equation* $\partial F(z, \alpha)/\partial z = F(z, \alpha+1)$.

It is attempted to provide a theory which motivates and verifies many seemingly special relations among various familiar special functions. The recurrence relation $\partial F(z, \alpha)/\partial z = A(z, \alpha)F(z, \alpha)+B(z, \alpha)F(z, \alpha+1)$ satisfied by many familiar functions furnishes common ground for study. In all cases of interest this equation is reducible to the form $\partial F(z, \alpha)/\partial z = F(z, \alpha+1)$. If $\phi(\alpha)$ is bounded in a right half-plane a unique solution $F(z, \alpha)$, an integral function of z, exists such that $F(z_0, \alpha) = \phi(\alpha)$. Two solutions which agree in a right half-plane of α when $z = z_0$ agree for all values of z whether or not they are bounded functions of α. On the basis of these theorems it is possible to establish many relationships satisfied by certain classes of solutions of the F-equation: (1) power series solutions, (2) factorial and Newton series solutions, (3) contour integral solutions, (4) generating expansions, (5) definite integrals, (6) relations among various different solutions of the F-equation. Methods of discovery are stressed because the discovery of a relationship satisfied by some special function or functions is almost always more difficult and more interesting than the construction of an ad hoc rigorous proof. A number of these special relations are shown to be obtainable by substitution in general formulas. (Received August 7, 1945.)

Applied Mathematics

230. R. J. Duffin: *Nonlinear networks*. I.

A system of n nonlinear algebraic equations in n real variables is studied and shown to have a unique solution. A special case of this system is the equations which govern the distribution of current in a direct current electrical network when the conductors are *quasi-linear*. A quasi-linear conductor is one in which the potential drop across the conductor and the current through the conductor are nondecreasing functions of one another. It follows that the distribution of current among the conductors of a quasi-linear network is unique. (Received September 6, 1945.)

231. F. J. Murray: *Linear equation solvers*.

The theory and actual construction of certain devices for the solution of a system of linear equations $\sum_{j=1}^{n} a_{i,j}x_j = b_i$ is described. In these machines, the variables x_j are subject to the control of the operator and the machine indicates the value of $\mu = \sum_{i=1}^{n}(\sum_{j=1}^{n} a_{ij}x_j - b_i)^2$. The operator varies each x_j in turn to minimize this expression. This is equivalent to the Gauss-Seidel method applied to the symmetric system obtained by multiplying the given set of equations by the adjoint matrix. The expressions $\sum_{j=1}^{n} a_{i,j}x_j - b_i$ are realized (except possibly in sign) as the amplitudes of alternating current voltages by means of a combination of bell transformers and variable resistances. This can be done in a number of ways. These alternating currents are then rectified by means of diode vacuum tubes and the combined currents measured by a microammeter. The result is essentially μ. Emphasis is placed upon the possibility of amateur construction and standard radio parts are used. The cost of the part of the device associated with the coefficients is proportional to n^2. The sensitivity of modern vacuum tubes is utilized to minimize the constant of proportionality. Received August 3, 1945.)

232. H. E. Salzer: *Note on coefficients for numerical integration with differences*.

In a recent paper (A. N. Lowan and H. E. Salzer, *Table of coefficients for numerical integration without differences*, Journal of Mathematics and Physics vol. 24 (1945) pp. 1–21) quantities $B_i^{(n)}(p)$ are tabulated to 10 decimal places, for continuous numerical integration (that is, integration to various points within an interval of tabulation) by a Lagrangian formula which uses the tabular entries only. The present note indicates how those same quantities $B_i^{(n)}(p)$ can be employed as they stand, for continuous numerical integration using differences, in formulas obtained by integrating the interpolation formulas of Gregory-Newton, Newton-Gauss (two forms), Everett, and Steffensen. (Received August 6, 1945.)

Geometry

233. H. S. M. Coxeter: *Quaternions and reflections.*

Every quaternion $x=x_0+x_1i+x_2j+x_3k$ determines a point $P_x=(x_0, x_1, x_2, x_3)$ in Euclidean 4-space, and every quaternion a of unit norm determines a hyperplane $a_0x_0+a_1x_1+a_2x_2+a_3x_3=0$. The reflection in that hyperplane is found to be the transformation $x\rightarrow -a\bar{x}a$. This leads easily to the classical expression $x\rightarrow axb$ for the general displacement preserving the origin. If p and q are pure quaternions of unit norm, the transformation $x\rightarrow(\cos\alpha+p\sin\alpha)x(\cos\beta+q\sin\beta)$ represents the double rotation through angles $\alpha\pm\beta$ about the two completely orthogonal planes $P_0P_{p\mp q}P_{1\pm pq}$. (Received October 1, 1945.)

234. H. S. M. Coxeter: *The order of the symmetry group of the general regular hyper-solid.*

Schlafli defined $\{p, q, r\}$ as the regular four-dimensional polytope bounded by $\{p, q\}$'s, r at each edge; for example, $\{4, 3, 3\}$ is the hyper-cube. The order, g, of the symmetry group is found to be given by $16h/g=6/j_{p,q}+6/j_{q,r}+1/p+1/r-2$, where $\cos^2\pi/h$ is the greater root of the equation $x^2-(\cos^2\pi/p+\cos^2\pi/q+\cos^2\pi/r)x+\cos^2\pi/p\cos^2\pi/r=0$, and $j_{p,q}=[(2p+2q+7pq)/(2p+2q-pq)]^{1/2}+1$; for example, for the hyper-cube $\{4, 3, 3\}$, $128/g=6/8+6/6+1/4+1/3-2=1/3$. (Received October 1, 1945.)

235. H. S. M. Coxeter: *The Petrie polygon of a regular solid.*

Schlafli defined $\{p, q\}$ as the regular solid bounded by p-gons, q at each vertex; for example, $\{4, 3\}$ is the cube. The Petrie polygon of $\{p, q\}$ is a skew h-gon such that every two consecutive sides, but no three, belong to a face of the solid; for example, the Petrie polygon of the cube is a skew hexagon. It is found that $h=(g+1)^{1/2}-1$, where g is the order of the symmetry group (that is, four times the number of edges). Since $4/g=1/p+1/q-1/2$, we deduce an expression for h in terms of p and q. Moreover, the solid has $3h/2$ planes of symmetry. (Received October 1, 1945.)

236. M. M. Day: *Note on the billiard ball problem.*

In his book *Dynamical systems*, G. D. Birkhoff proves by the rather deep Poincaré ring theorem the fact that for each convex cornerless billiard table and each integer n there is a closed path around the table of precisely n sides such that a billiard ball will follow this path around and around if it satisfies the usual reflection law that the angle of incidence equals the angle of reflection whenever the ball hits a side. An elementary proof of this result is given by the following two statements: (1) Any n-sided convex polygon of stationary length inscribed in a convex cornerless curve satisfies the reflec-