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### TOPOLOGY

#### 185. R. H. Bing: *Collections filling up a simple plane web.*

It is shown that the compact continuum  $W$  is a simple plane web provided there exist an upper semicontinuous collection of mutually exclusive continua filling up  $W$  and another such collection  $H$  filling up  $W$  such that if  $g$  and  $h$  are elements of  $G$  and  $H$  respectively, then the common part of  $g$  and  $h$  exists and is totally disconnected. It is proved that if  $W$  has a bounded complementary domain and each of the collections  $G$  and  $H$  is a dendron with respect to its elements, then each is a non-equicontinuous collection. (Received June 25, 1945.)

#### 186. R. H. Bing: *Generalizations of two theorems of Janiszewski.*

Janiszewski proved that if  $H$  and  $K$  are continua neither of which cuts the plane, then the sum of  $H$  and  $K$  cuts the plane only if the common part of  $H$  and  $K$  is not connected. The present paper gives generalizations of this result by considering more general sets than continua. It is shown that if  $H$  and  $K$  are two sets of which neither cuts the point  $A$  from the point  $B$ ,  $H \cdot K$  is connected,  $H - H \cdot K$  is compact and  $H - H \cdot K$  and  $K - H \cdot K$  are mutually separated, then  $H + K$  does not cut  $A$  from  $B$ . Also, if  $H$  and  $K$  are connected sets one of which is compact,  $H$  is countinuumwise connected and  $\overline{H} \cdot K + H \cdot \overline{K}$  is the sum of two mutually separated sets each containing a point of  $H$ , then  $H + K$  cuts the plane. (Received July 23, 1945.)

#### 187. J. C. Oxtoby: *Invariant measures in groups which are not locally compact.*

In any complete separable metric group which is dense in itself it is possible to construct a left-invariant measure, defined for all Borel sets and zero for points; but such a measure cannot be locally finite, nor can every compact set have finite measure, unless the group is locally compact. This result is proved independently of Haar's theorem, and the measure constructed may have properties quite unlike Haar's measure. Other constructions, based on the idea of extending a Haar measure from a subgroup, or introducing a new topology, are considered and their limitations discussed. The results throw light on Weil's converse of Haar's theorem by giving examples of measures that fail to satisfy Weil's postulates, and by showing that there is a class of groups in which no Borel measure can satisfy them. (Received July 23, 1945.)

#### 188. Moses Richardson: *On weakly ordered systems.* Preliminary report.

A *weakly ordered system* is a system of elements  $a, b, \dots$  with a binary relation  $>$  such that (1)  $a > b$  implies  $a \neq b$ , and (2)  $a > b$  implies  $b$  not greater than  $a$ . Transitivity is not assumed. Such a system can be represented by an oriented 1-complex or linear graph in an obvious way. J. von Neumann and O. Morgenstern (*Theory of games and economic behavior*, Princeton, 1944) prove the existence of "solutions" for the case of a strictly acyclic system, and give a method of constructing such solutions. The main result of the present note is the existence and construction, by a

different method, of solutions in the non-acyclic case provided all unoriented cycles are even. (Received August 1, 1945.)

189. A. D. Wallace: *Another fixed point theorem.*

The author adheres in part to the notation of a recent paper (*A fixed point theorem*, Bull. Amer. Math. Soc. vol. 51 (1945) p. 414 et seq.). It will not be supposed that  $P$  is a topological space and (i) will be replaced by (i<sub>0</sub>): Each  $\Delta$ -simple set has a maximal element in  $P$ . Assume that (i<sub>0</sub>), (ii), (iii) and (iv) hold in  $P$ . Theorem: If  $T$  is a one-to-one transformation of  $P$  onto itself such that both  $T$  and  $T^{-1}$  preserve the relation  $\Delta$  then there is an  $f \neq e$  such that both  $f\Delta Tf$  and  $Tf\Delta f$  hold. (Received July 9, 1945.)

190. A. D. Wallace: *Dimensional types.*

Let  $H, S$  be topological spaces. The space  $H$  is said to be of dimensional type  $S$  if for each closed set  $X$  in  $H$  and mapping  $f: X \rightarrow S$  there is an extension  $\bar{f}: H \rightarrow S$ . When  $S$  is an  $n$ -sphere and  $H$  is separable metric then  $H$  is of dimensional type  $S$  if and only if it is of dimension at most  $n$ , a result of Hurewicz. Let  $H$  be normal and let  $S$  be an ANR. If  $H$  is the union of a countable family of closed sets of dimensional type  $S$  then  $H$  is of dimensional type  $S$ . The finite case of this theorem follows with no assumptions on  $H$  and  $S$  except that they are topological spaces. (Received July 9, 1945.)

191. G. T. Whyburn: *Boundary alternation of monotone mappings.*

Let  $R$  be a bounded plane region with connected boundary  $A$  and let  $f(\bar{R}) = B$  be any monotone mapping. In this paper it is shown that if (a) for each  $x \in A$ ,  $f^{-1}f(x)$  intersects every crosscut in  $R$  both of whose regions have  $x$  on their boundaries,  $f$  is non-alternating on  $A$ . Further, if  $B$  is a dendrite and  $R \cdot f^{-1}(y)$  is connected for each  $y \in B$ , then (a) is satisfied and hence the same conclusion holds. Also in case  $A$  is locally connected, it is noted that this same conclusion follows provided (b) the mapping  $fg(E) = B$  is monotone, where  $E$  is a 2-cell and  $g(E) = \bar{R}$  is the extended inverse of the relative distance mapping on  $R$ . In this latter case it is shown that conditions (a) and (b) are equivalent. (Received July 16, 1945.)

192. G. T. Whyburn: *On monotone retractability into simple arcs.*

In recent work on the area of surfaces Radó has had occasion to use the following properties as applied to locally connected continua  $A$ : ( $\pi$ ): Every simple arc in  $A$  is a monotone retract of  $A$ ; ( $\Pi$ ): Every monotone image of  $A$  has property ( $\pi$ ). Radó has noted that ( $\Pi$ ) implies ( $\pi$ ) and that the sphere and 2-cell each have ( $\Pi$ ). In this paper it is shown that (1) property ( $\Pi$ ) is equivalent to unicoherence for locally connected continua in general, (2) property ( $\pi$ ) is equivalent to unicoherence for plane locally connected continua, and (3) every closed 2-dimensional connected manifold has property ( $\pi$ ). (Received July 16, 1945.)

193. G. S. Young: *The introduction of local connectivity by change of topology.*

Let  $G$  be a collection of subsets of a space,  $S$ . A point  $P$  will be said to be a  $G$ -limit point of a subset  $M$  of  $S$  if every neighborhood of  $P$  contains an element of  $G$  which intersects  $P$  and  $M - P$ . Under certain conditions on  $G$  (for example, if  $G$  is the collection of all arcs of  $S$ ), in the topology for  $S$  thus defined,  $S$  is locally connected and still

has a topology much like before. Thus, if  $S$  were originally metric, it would remain metric. This concept permits use of known properties of non-compact continuous curves in certain problems concerning non-locally connected spaces. Several examples of this technique are given, one of which is the following: Let  $M$  be a set which contains no arc, let  $D$  be a dendrite, and let  $G$  be a minimal collection of arcs such that each point of  $M$  is joined to  $D$  by an arc of  $G$ . Then if  $M+D+G^*$  contains no simple closed curve, it has the fixed-point property. In order to obtain this last, a study is made of the properties of a type of generalized dendrite. (Received July 30, 1945.)

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### NEW PUBLICATIONS

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- CARTER, G. W. The simple calculation of electrical transients. New York, Macmillan; Cambridge University Press, 1945. 8+120 pp. \$1.75.
- DONNAY, J. D. H. Spherical trigonometry after the Cesàro method. New York, Interscience, 1945. 12+84 pp. \$1.75.
- KAGAN, V. F. Lobachevsky. Academy of Sciences of the U.S.S.R., 1944. 348 pp.
- SMITH, C. E. Applied mathematics for radio and communication engineers. New York and London, McGraw-Hill, 1945. 10+336 pp. \$3.50.
- TCHEBYCHEV, P. L. Collected works. Vol. 1. Academy of Sciences of the U.S.S.R., 1944. 342 pp.